Fundamentals of Borehole Logging (AES1500), July 4, 2013 at 14:00-17:00h.

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The maximum number of credit points per question is indicated in brackets.

1) Hooke's law for a linear elastic isotropic solid is

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu\varepsilon_{ij} \quad \text{with} \quad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{1}$$

a) (2p) Explain the meaning of <u>all</u> variables and give their unit.

b) (2p) Give the expressions for σ_{xx}, σ_{yx}, and σ_{zx} in terms of u.

The linearized momentum equation for an isotropic homogeneous solid is

$$\rho\frac{\partial^2 u}{\partial t^2} = \nabla \cdot \sigma \tag{2}$$

c) (1p) What does "isotropic" mean? And what does "homogeneous" mean?

d) (2p) Write the linearized momentum equation in the x-direction only.

e) (2p) Insert the expressions for σ_{xx}, σ_{yx}, and σ_{zx} in the linearized momentum equation in the x-direction to show that

$$\rho\frac{\partial^2 u_x}{\partial t^2} = (\lambda + \mu)\frac{\partial}{\partial x}(\nabla \cdot u) + \mu\nabla^2 u_x \tag{3}$$

Now, assume wave propagation in the z-direction with particles moving in the x-direction.

f) (1p) Derive the corresponding wave equation and give the expression for the wave velocity. What is the name of this wave?

2) Stoneley waves may propagate in a fluid-filled borehole. Consider a homogeneous fluid with density ρ_f in a cylindrical borehole (radius r) penetrating a homogeneous isotropic medium having shear modulus μ (Figure 1).

a) (2p) Discuss the effect on the Stoneley wave if the fluid in the borehole is lowered.

We assume an axially P-wave propagating tube wave mode. Using p for pressure and u_z for vertical displacement, Newton's second law applied to a volume element of fluid, $V = \pi r^2 \Delta z$, is

$$-\frac{\partial p}{\partial z}\Delta z\pi\, r^2 = \rho_f\pi\, r^2\Delta z\frac{\partial^2 u_z}{\partial t^2}. \tag{4}$$

b) (1p) Identify Newton's second law in equation (4).

We develop equation (4) by introducing the bulk modulus of the fluid K_f.

c) (1p) Give the expression of K_f in terms of p and give its unit.

The change in fluid volume ΔV is due to expansion along the axis and radially

$$\Delta V = \pi \, r^2 \, \frac{\partial u_z}{\partial z} \Delta z + 2\pi \, r \, u_r \Delta z, \tag{5}$$

where u_r is the change in the radius of the borehole.

d) (2p) Explain why equation (5) is only valid for $u_r \ll r$.

e) (1p) Show that we get

$$p = -K_f \left(\frac{\partial u_z}{\partial z} + \frac{2u_r}{r} \right). \tag{6}$$

The relation between u_r and p for our borehole configuration is

$$\frac{u_r}{r} = \frac{p}{2\mu}. \tag{7}$$

f) (1p) Use equations (4), (6), and (7) to derive the expression of the squared tube wave velocity in terms of K_f and μ.

g) (2p) Explain how dispersive tube waves can be used to detect fractures in a cemented well casing.
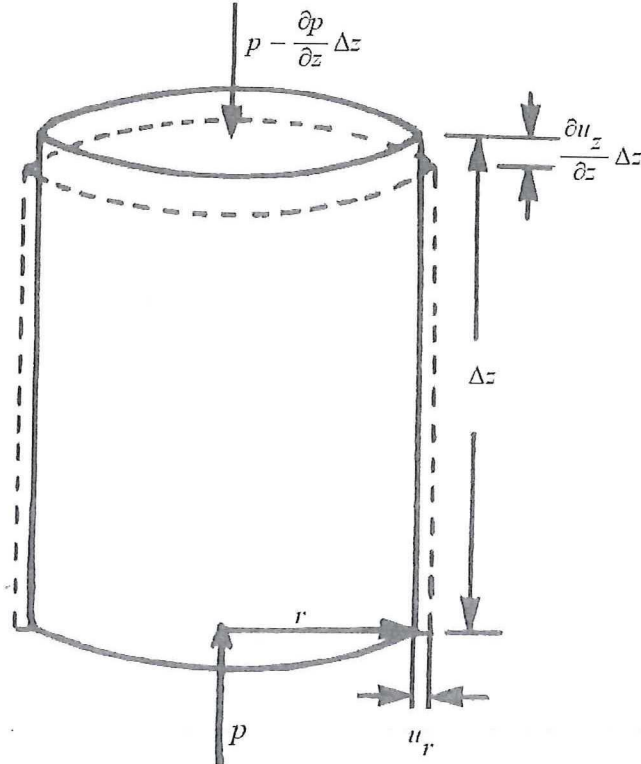

Figure 1

3) Consider a fully saturated rock sample immersed in a pressure tank containing the saturating liquid. A piston causes an external pressure change dp_e. The so-called Gedanken experiments involve a jacketed and unjacketed test.

The change in bulk volume as a result of the change in pore pressure and normal intergranular stress is the sum of the change in bulk volume of the jacketed and unjacketed test:

$$-\frac{dV_b}{V_b} = \frac{dp}{K_s} + \frac{d\sigma_0}{K_b}, \tag{8}$$

in which σ_0 is the isotropic component of the intergranular stress. Another equation of the relative change in bulk volume is obtained from the Gedanken experiment and the definition of the bulk modulus of the fluid:

$$-\frac{dV_b}{V_b} = \left[\frac{\phi}{K_f} + \frac{1-\phi}{K_s}\right]dp + \frac{1}{K_s}d\sigma_0. \tag{9}$$

a) (2p) Use equations (8) and (9) and the definition of the Gassmann modulus

$$K_G = -\frac{V_b}{dV_b}\bigg|_{sat} dp_e \quad \text{and} \quad dp_e = d\sigma_0 + dp, \tag{10}$$

to show that for incompressible grains

$$K_G = K_b + \frac{K_f}{\phi}. \tag{11}$$

b) (2p) Write the bulk moduli $K_{s,b,f,G}$ in ascending (or descending) order (under normal borehole conditions) and motivate your choice briefly.

For poroelastic rocks the Gassmann velocity is

$$v_P = \sqrt{\frac{K_G + \frac{4}{3}\mu}{\rho}}, \tag{12}$$

where ρ is the total weighted density and μ is the shear modulus. Fluid substitution is an important part of seismic attribute work. The most commonly used technique for doing this involves the application of Gassmann's equations.

c) (2p) If we replace the water in the pore space by air the Gassmann modulus and weighted density obviously change. Justify on physical grounds why we may assume that the shear modulus remains constant.

The applicability of Gassmann's equation is for frequencies far below the so-called critical Biot frequency ω_B:

$$\omega << \frac{\phi\eta}{k_0\alpha_\infty\rho_f} = \omega_B, \tag{13}$$

with porosity ϕ, fluid density ρ_f, permeability k_0, viscosity η, and tortuosity α_∞.

d) (2p) Give the units of these five variables and demonstrate as a result that the unit of frequency is Hertz.

e) (2p) Insert ballpark values for these variables and show that the critical Biot frequency is in the ultrasonic range.

4) The Biot momentum equation for the fluid in an elastic porous material is

$$\phi \rho_f \frac{\partial w}{\partial t} = -\phi \frac{\partial p}{\partial x} + \phi \rho_f (\alpha_\infty - 1) \frac{\partial}{\partial t}(v - w) + \frac{\eta \phi^2}{k_0}(v - w), \tag{14}$$

with porosity ϕ, fluid density ρ_f, permeability k_0, viscosity η, and tortuosity α_∞. We only consider the 1D compressional case, so the wave propagates in the x-direction and the average solid and fluid particle velocities in the x-direction are v and w.

a) (1p) Show that we may find Darcy's law from equation (14).

We can rewrite expression (14) in the frequency domain using harmonic wave propagation in the x-direction for the relevant variables, e.g. $v = \hat{v} \exp i (\omega t - k x)$. The full Biot equations can then be cast in the following form:

$$\hat{v}\left[\hat{\rho}_{11} c^2 - P\right] = \hat{w}\left[Q - \hat{\rho}_{12} c^2\right]$$
$$\hat{v}\left[\hat{\rho}_{12} c^2 - Q\right] = \hat{w}\left[R - \hat{\rho}_{22} c^2\right], \tag{15}$$

where P, Q, and R are elastic moduli and $\hat{\rho}_{11}, \hat{\rho}_{12}$, and $\hat{\rho}_{22}$ are complex densities.

b) (2p) Show that set (15) gives two solutions for c.

c) (1p) Explain the difference between the four velocities (two c's and v and w).

d) (2p) Although a slow <u>shear</u> wave has yet never been observed, its existence in porous media is predicted by some researchers. Explain how the Biot momentum equation for the <u>solid</u> should be modified accordingly.

An ultrasonic bench-top experiment is performed on a dry rock sample to investigate the predictive power of the Biot theory. The Spectral Ratio method is used to calculate the frequency-dependent properties of the porous rock sample.

e) (2p) Make a schematic drawing of the acquisition set-up with the needed instruments and cables.

f) (2p) Write the following processing steps in proper order for the Spectral Ratio method: load data; select window in time domain, zero-padding, apply Fourier Transform, unwrap phase, calculate wave number, select window in frequency domain.