Fundamentals of Borehole Logging (AES1500), April 12, 2013 at 14:00-17:00h.

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The maximum number of credit points per question is indicated between brackets.

1) Rock properties might be obtained by laboratory measurements on recovered cores. The rock strength is measured in a tensile test, where Young's modulus E is given by the ratio of axial stress to axial strain in an uniaxial stress state.

a) (1p) Give the unit of E, σ_{xx}, and ε_{xx}. Why does E not have a subscript?

Hooke's law for a linear elastic isotropic solid is

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij}. \qquad (1)$$

b) (1p) Write down the full expressions for σ_{xx} and σ_{yy}.

c) (2p) Show that this stress state dictates that $\varepsilon_{yy} = \varepsilon_{zz}$.

It can be shown that $$\varepsilon_{yy} = -\frac{\lambda}{2(\lambda+\mu)}\varepsilon_{xx}. \qquad (2)$$

d) (2p) Explain why ε_{xx} and ε_{yy} have opposite sign. What is the name of their ratio?

e) (2p) Derive the Young's modulus:

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}.$$

In seismic and borehole acoustics we have body and surface waves, where the velocity is given by the square root of some elastic modulus over density. One might argue a similar relationship using Young's modulus:

$$v_E = \sqrt{\frac{E}{\rho}}, \qquad (3)$$

where ρ is the density.

f) (2p) Discuss the significance of equation (3).

2) Stoneley waves may propagate in a fluid-filled borehole. Consider a homogeneous fluid with density ρ_f in a cylindrical borehole (radius r) penetrating a homogeneous isotropic medium having shear modulus μ (Figure 1).

a) (2p) Discuss the effect of lowering the borehole fluid.

We assume an axially P-wave propagating tube wave mode. Using p for pressure and u_z for vertical displacement, Newton's second law applied to a volume element of fluid, $V = \pi r^2 \Delta z$, is

$$-\frac{\partial p}{\partial z}\Delta z \pi\, r^2 = \rho_f \pi\, r^2 \Delta z \frac{\partial^2 u_z}{\partial t^2}. \qquad (4)$$

b) (1p) Identify the appropriate terms of Newton's second law from equation (4).

We develop equation (4) by introducing the bulk modulus of the fluid K_f.

c) (1p) Give the expression of K_f in terms of p and give its unit.

The change in fluid volume ΔV is due to expansion along the axis and radially

$$\Delta V = \pi\, r^2 \frac{\partial u_z}{\partial z} \Delta z + 2\pi\, r\, u_r \Delta z, \tag{5}$$

where u_r is the change in the radius of the borehole.

d) (1p) Explain why equation (5) is only valid for $u_r << r$.

e) (1p) Show that we get

$$p = -K_f \left(\frac{\partial u_z}{\partial z} + \frac{2u_r}{r} \right). \tag{6}$$

The relation between u_r and p for our borehole configuration is

$$\frac{u_r}{r} = \frac{p}{2\mu}. \tag{7}$$

f) (2p) Use above equations to derive the expression of the squared tube wave velocity in terms of K_f and μ.

g) (2p) Why is the tube wave velocity in practice close to the velocity in the borehole fluid?

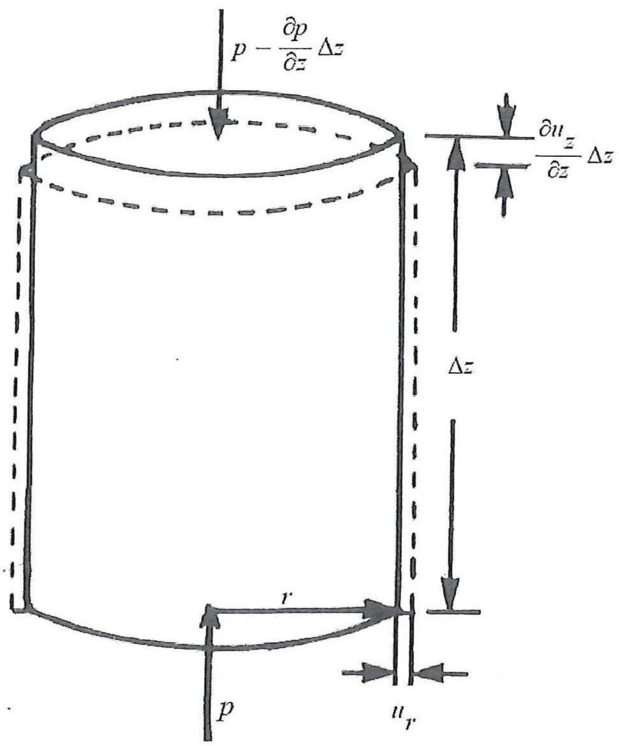

Figure 1

For poroelastic rocks the Gassmann velocity is

$$v_P = \sqrt{\frac{K_G + \frac{4}{3}\mu}{\rho}}, \qquad (13)$$

where ρ is the total weighted density and μ is the shear modulus. Fluid substitution is an important part of seismic attribute work. The most commonly used technique for doing this involves the application of Gassmann's equations.

e) (2p) Discuss what will happen with the Gassmann velocity if we replace the water in the pore space by light oil (having a bulk modulus of half that of water).

f) (2p) Write all given bulk moduli in ascending order (under normal borehole conditions) and justify your choice on physical grounds.

4) The Biot momentum equation for the fluid in an elastic porous material is

$$\phi\rho_f \frac{\partial w}{\partial t} = -\phi \frac{\partial p}{\partial x} + \phi\rho_f(\alpha_\infty - 1)\frac{\partial}{\partial t}(v-w) + \frac{\eta\phi^2}{k_0}(v-w), \qquad (14)$$

with porosity ϕ, fluid density ρ_f, permeability k_0, viscosity η, and tortuosity α_∞. We only consider the 1D compressional case, so the wave propagates in the x-direction and the average solid and fluid particle velocities in the x-direction are v and w.

a) (2p) Give the units of all variables of equation (14).

Karman-Cozeny relates permeability and porosity to characteristic grain size.

b) (2p) Explain whether grain size then plays a dominant role at high or low-frequencies in equation (14).

c) (2p) Show that we may find Darcy's law for the case of a rigid matrix.

Henceforth, we assume harmonic wave propagation in the x-direction for the relevant variables, e.g. $v = \hat{v} \exp i\,(\omega\, t - k\, x)$.

The full Biot equations can then be written in the frequency domain:

$$\begin{aligned} \hat{v}\left[\hat{\rho}_{11}c^2 - P\right] &= \hat{w}\left[Q - \hat{\rho}_{12}c^2\right] \\ \hat{v}\left[\hat{\rho}_{12}c^2 - Q\right] &= \hat{w}\left[R - \hat{\rho}_{22}c^2\right], \end{aligned} \qquad (15)$$

where P, Q, and R are elastic moduli and $\hat{\rho}_{11}$, $\hat{\rho}_{12}$, and $\hat{\rho}_{22}$ are complex densities.

d) (2p) Explain the different meaning of velocity v and velocity c.

Set (15) gives the dispersion relation $d_2c^4 + d_1c^2 + d_0 = 0$, with solutions.

e) (2p) What is the physical implication of the fact that the term 'complex densities' refers to their frequency-dependent nature?

3) The so-called Gedanken experiments can be used to derive an expression for the Gassmann modulus of fully saturated rocks. Consider a fully saturated rock sample immersed in a pressure tank containing the saturating liquid. A piston causes an external pressure change dp_e. We perform two tests.

Upon changing the hydrostatic pressure, the volume of the solid grains of an open sample changes, so that we can measure the bulk modulus of the grains (unjacketed test).

Assume now that we maintain a constant pore pressure (e.g. atmospheric pressure) by means of a capillary tube in a closed sample (jacketed test). Then, the amount of fluid that is squeezed out of the sample is measured by the fluid rise in the capillary tube.

	dp	$d\sigma_0$	dV_s	dV_b	$d\phi$
Unjacketed	dp_e	0	$-\frac{V_s}{K_s}dp_e$	$-\frac{V_b}{K_s}dp_e$	0
Jacketed	0	dp_e	$-\frac{V_s}{(1-\phi)K_s}dp_e$	$-\frac{V_b}{K_b}dp_e$	$-\left(\frac{1-\phi}{K_b}-\frac{1}{K_s}\right)dp_e$

The change in bulk volume as a result of the change in pore pressure and normal intergranular stress is the sum of the change in bulk volume of the jacketed and unjacketed test.

$$-\frac{dV_b}{V_b}=\frac{dp}{K_s}+\frac{d\sigma_0}{K_b}, \tag{8}$$

in which σ_0 is the isotropic component of the intergranular stress.

a) (2p) Explain how σ_0 is related to σ_{ij} in equation (1).

b) (1p) Show that also

$$\frac{dV_b}{V_b}=(1-\phi)\frac{dV_s}{V_s}+\phi\frac{dV_f}{V_f}. \tag{9}$$

c) (1p) Use the table and the definition of the bulk modulus of the fluid to show that expression (9) gives

$$-\frac{dV_b}{V_b}=\left[\frac{\phi}{K_f}+\frac{1-\phi}{K_s}\right]dp+\frac{1}{K_s}d\sigma_0. \tag{10}$$

Henceforward, we assume incompressible grains.

d) (2p) Use equations (8) and (10) and the definition of the Gassmann modulus

$$K_G=-\frac{V_b}{dV_b}\bigg|_{\text{sat}} dp_e \quad \text{and} \quad dp_e=d\sigma_0+dp, \tag{11}$$

to show that for incompressible grains

$$K_G=K_b+\frac{K_f}{\phi}. \tag{12}$$