Fundamentals of Borehole Logging (AES1500), April 13, 2012 at 14:00-17:00h.

Responsible lecturer: B. Vogelaar (bvogelaar@gmail.com)

The maximum number of credit points per question is indicated in brackets.

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1) For linear elastic isotropic media, part of Hooke's law can be written as follows:

$$\sigma_{ii} = \lambda\varepsilon_{kk} + 2\mu\varepsilon_{ii} \qquad (i = x, y, z). \tag{1}$$

a) (2p) Explain to which stress state equation (1) is applicable. What are ε_{kk} and ε_{ii}?

b) (2p) Write down the <u>remaining</u> part of Hooke's law for $i \neq j$.

c) (2p) Use $\sigma_{xx} \neq 0$ and $\sigma_{yy} = \sigma_{zz} = 0$ in equation (1) to show that

$$\varepsilon_{yy} = -\frac{\lambda}{2(\lambda+\mu)}\varepsilon_{xx}. \tag{2}$$

What is the name of the proportionality factor?

d) (2p) Use equation (2) to derive the Young's modulus:

$$E = \frac{\mu(3\lambda+2\mu)}{\lambda+\mu}. \tag{3}$$

e) (2p) In another situation, pressure p is equivalent to stresses $\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = -p$. Use these relationships into equation (1) to derive for the bulk modulus:

$$K_b = \lambda + \frac{2}{3}\mu.$$

2) Stoneley waves may propagate in a fluid-filled borehole. Boundary conditions require that normal stress and displacement are continuous and that the tangential stress in the solid vanishes at the solid-fluid interface.

a) (2p) Justify these boundary conditions on physical grounds.

Consider a homogeneous fluid with density ρ_f in a cylindrical borehole (radius r) penetrating a porous isotropic medium having shear modulus μ (Figure 1). We assume an axially P-wave propagating tube wave mode. Using p for pressure and u_z for vertical displacement, Newton's second law applied to a volume element of fluid, $V = \pi r^2 \Delta z$, is

$$-\frac{\partial p}{\partial z}\Delta z \pi r^2 = \rho_f \pi r^2 \Delta z \frac{\partial^2 u_z}{\partial t^2}. \tag{4}$$

The change in fluid volume ΔV due to expansion along the axis and radially, is

$$\Delta V = \pi r^2 \frac{\partial u_z}{\partial z}\Delta z + 2\pi r u_r \Delta z, \tag{5}$$

where u_r is the change in the radius of the borehole.

b) (2p) Show that equation (5) is valid only for $u_r << r$.

c) (2p) Use the definition of the bulk modulus of the fluid K_f to show that we get

$$p = -K_f\left(\frac{\partial u_z}{\partial z} + \frac{2u_r}{r}\right). \qquad (6)$$

The relation between u_r and p for an annulus of innner and outer radii r and R is

$$\frac{u_r}{r} = \frac{p}{E}\frac{(1+\nu)(R^2+r^2)-2\nu\, r^2}{R^2-r^2}, \qquad (7)$$

where $E = 2\mu(1+\nu)$ is the Young's modulus and ν is Poisson's ratio.

e) (1p) How can we obtain u_r/r for our borehole configuration from equation (7)? Give the expression in μ.

f) (2p) Use equations (4) and (6) and derive the expression of the squared tube wave velocity in terms of K_f and μ.

g) (1p) Explain which contribution to the volume change of the fluid is not taken into account in equation (5). Give a practical application of the implied technique.

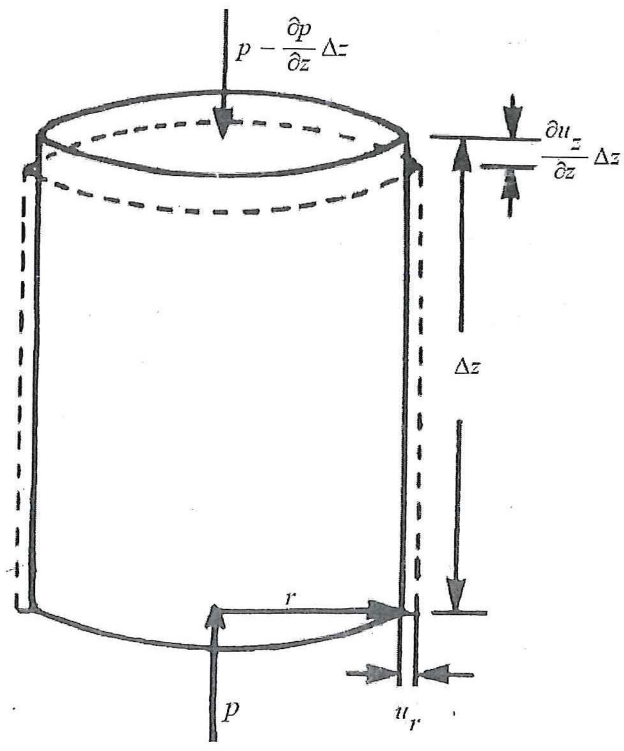

Figure 1

3) Consider a fully saturated rock sample immersed in a pressure tank containing the saturating liquid. A piston causes an external pressure change dp_e. The so-called Gedanken experiments involve a jacketed and unjacketed test.

The change in bulk volume as a result of the change in pore pressure and normal intergranular stress is the sum of the change in bulk volume of the jacketed and unjacketed test:

$$-\frac{dV_b}{V_b} = \frac{dp}{K_s} + \frac{d\sigma_0}{K_b}, \tag{8}$$

in which σ_0 is the isotropic component of the intergranular stress. Another equation of the relative change in bulk volume is obtained from the Gedanken experiment and the definition of the bulk modulus of the fluid:

$$-\frac{dV_b}{V_b} = \left[\frac{\phi}{K_f} + \frac{1-\phi}{K_s}\right] dp + \frac{1}{K_s} d\sigma_0. \tag{9}$$

a) (2p) Use equations (8) and (9) and the definition of the Gassmann modulus

$$K_G = -\frac{V_b}{dV_b}\bigg|_{\text{sat}} dp_e \quad \text{and} \quad dp_e = d\sigma_0 + dp, \tag{10}$$

to show that for incompressible grains

$$K_G = K_b + \frac{K_f}{\phi}. \tag{11}$$

b) (2p) Explain in detail whether the rock sample will be easier or more difficult to compress according to equation (11) if we take a percolation porosity into account. Is this expected?

For poroelastic rocks the Gassmann velocity is

$$v_P = \sqrt{\frac{K_G + \frac{4}{3}\mu}{\rho}}, \tag{12}$$

where ρ is the total weighted density and μ is the shear modulus. Fluid substitution is an important part of seismic attribute work. The most commonly used technique for doing this involves the application of Gassmann's equations.

c) (2p) Explain in detail what will happen with the Gassmann velocity if we replace the water in the pore space by air.

The applicability of Gassmann's equation is for frequencies far below the so-called critical Biot frequency ω_B:

$$\omega << \frac{\phi\eta}{k_0 \alpha_\infty \rho_f} = \omega_B, \tag{13}$$

with porosity ϕ, fluid density ρ_f, permeability k_0, viscosity η, and tortuosity α_∞.

d) (2p) Give the units of these five variables and demonstrate as a result that the unit of frequency is Hertz.

e) (2p) Use the reasoning from a Carman-Kozeny relationship to compare the permeabilities of two different rocks having the same porosity and microstructure, but different average grain size and show that the critical Biot frequency of the rock with the larger average grain size is lower (or higher) than the one with the smaller average grain size.

4) The Biot momentum equation for the fluid in an elastic porous material is

$$\phi\rho_f \frac{\partial w}{\partial t} = -\phi\frac{\partial p}{\partial x} + \phi\rho_f(\alpha_\infty - 1)\frac{\partial}{\partial t}(v-w) + \frac{\eta\phi^2}{k_0}(v-w), \tag{14}$$

with porosity ϕ, fluid density ρ_f, permeability k_0, viscosity η, and tortuosity α_∞. We only consider the 1D compressional case, so the wave propagates in the x-direction and the average solid and fluid particle velocities in the x-direction are v and w.

a) (1p) Show that we may find Darcy's law from equation (14).

We can rewrite expression (14) in the frequency domain using harmonic wave propagation in the x-direction for the relevant variables, e.g. $v = \hat{v}\exp i\,(\omega\, t - k\, x)$. The full Biot equations can then be cast in the following form:

$$\begin{aligned} \hat{v}\left[\hat{\rho}_{11}c^2 - P\right] &= \hat{w}\left[Q - \hat{\rho}_{12}c^2\right] \\ \hat{v}\left[\hat{\rho}_{12}c^2 - Q\right] &= \hat{w}\left[R - \hat{\rho}_{22}c^2\right] \end{aligned} \tag{15}$$

where P, Q, and R are elastic moduli and $\hat{\rho}_{11}, \hat{\rho}_{12}$, and $\hat{\rho}_{22}$ are complex densities.

b) (2p) Show that set (15) gives the dispersion relation

$$d_2c^4 + d_1c^2 + d_0 = 0, \tag{16}$$

and give the expressions for d_2, d_1, and d_0.

c) (2p) Solve (16) and explain why may we observe both waves in the laboratory but not in the field?

An ultrasonic bench-top experiment is performed on a dry rock sample to investigate the predictive power of the Biot theory. The Spectral Ratio method is used to calculate the frequency-dependent properties of the porous rock sample.

d) (2p) Make a schematic drawing of the acquisition set-up with the needed equipment.

e) (1p) Explain why tortuosity is dominant (or not) in this experiment.

f) (2p) Explain why the output will worsen in the frequency domain if we do not adjust the window length in the time domain?