Fundamentals of Borehole Logging (AES1500), April 7, 2011 at 14:00-17:00h.

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The maximum number of credit points per question is indicated between brackets.

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1) Hooke's law for a linear elastic isotropic solid is

$$\sigma_{ij} = \lambda \varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \qquad \text{with} \qquad \varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{1}$$

a) (2p) Explain the meaning of <u>all</u> variables and give their unit.

b) (2p) Give the expressions for σ_{xx}, σ_{yx}, and σ_{zx} in terms of u.

The momentum equation for isotropic homogeneous solids is

$$\rho\left[\frac{\partial^2 u}{\partial t^2} + \left(\frac{\partial u}{\partial t}\cdot\nabla\right)\frac{\partial u}{\partial t}\right] = \nabla\cdot\sigma \tag{2}$$

c) (1p) What does "isotropic" mean? And what does "homogeneous" mean?

d) (2p) Write the <u>linearized</u> momentum equation in the x-direction only.

e) (2p) Insert the expressions for σ_{xx}, σ_{yx}, and σ_{zx} in the linearized momentum equation in the x-direction to show that

$$\rho\frac{\partial^2 u_x}{\partial t^2} = (\lambda + \mu)\frac{\partial}{\partial x}(\nabla\cdot u) + \mu\nabla^2 u_x \tag{3}$$

Now, assume wave propagation in the z-direction with particles moving in the x-direction.

f) (1p) Derive the corresponding wave equation and give the expression for the wave velocity. What is the name of this wave?

2) Consider an effective medium composed of two horizontal isotropic layers, say dolomite and shale, with the following layer properties (ρ is density, d is layer thickness, v_P and v_S are P- and S-wave velocity):

$\rho_1 = 2450$ kg/m^3; $d_1 = 7.5$ m; $v_{P1} = 5200$ m/s; $v_{S1} = 2700$ m/s.

$\rho_2 = 2440$ kg/m^3; $d_2 = 5.0$ m; $v_{P2} = 2900$ m/s; $v_{S2} = 1400$ m/s.

The <u>effective</u> elastic shear moduli are given by

$$\bar{c}_{66} = \left\langle \rho v_S^2 \right\rangle \qquad \text{and} \qquad \bar{c}_{44} = \frac{1}{\left\langle \dfrac{1}{\rho v_S^2} \right\rangle} \tag{4}$$

The brackets indicate the averages of the enclosed properties weighted by their volumetric proportions.

a) (2p) Explain why we have two effective shear moduli. How many independent elastic moduli do we need to describe full wave propagation in this medium?

b) (2p) Apply the weighted averaging operator in equations (4) and insert the values of the layers to show that S-wave anisotropy exists. (There is no need to use a calculator.)

Now, we increase both layer thicknesses until we have two semi-infinite solid media.

c) (2p) What is the name of the wave that may propagated along the interface? Which boundary conditions (in text or equations) have to be fulfilled to obtain a solution for wave propagation along the interface?

d) (2p) Use figure 1 and the given material properties to explain whether or not this wave type exists under current conditions.

Finally, we replace the semi-infinite top layer with vacuum to obtain yet another wave propagating along the interface and obeying the following wave equations

$$\frac{\partial^2 \phi}{\partial t^2} = v_P^2 \nabla^2 \phi \qquad \text{and} \qquad \frac{\partial^2 \chi_V}{\partial t^2} = v_S^2 \nabla^2 \chi_V \tag{5}$$

Appropriate potentials for this wave are

$$\phi = A e^{-mkz} e^{ik(x-v\,t)} \qquad \text{and} \qquad \chi_V = B e^{-nkz} e^{ik(x-v\,t)} \tag{6}$$

where m and n are real, positive, constants.

e) (2p) Show that the velocity of this wave is always less than the S-wave velocity of the semi-infinite bottom layer.

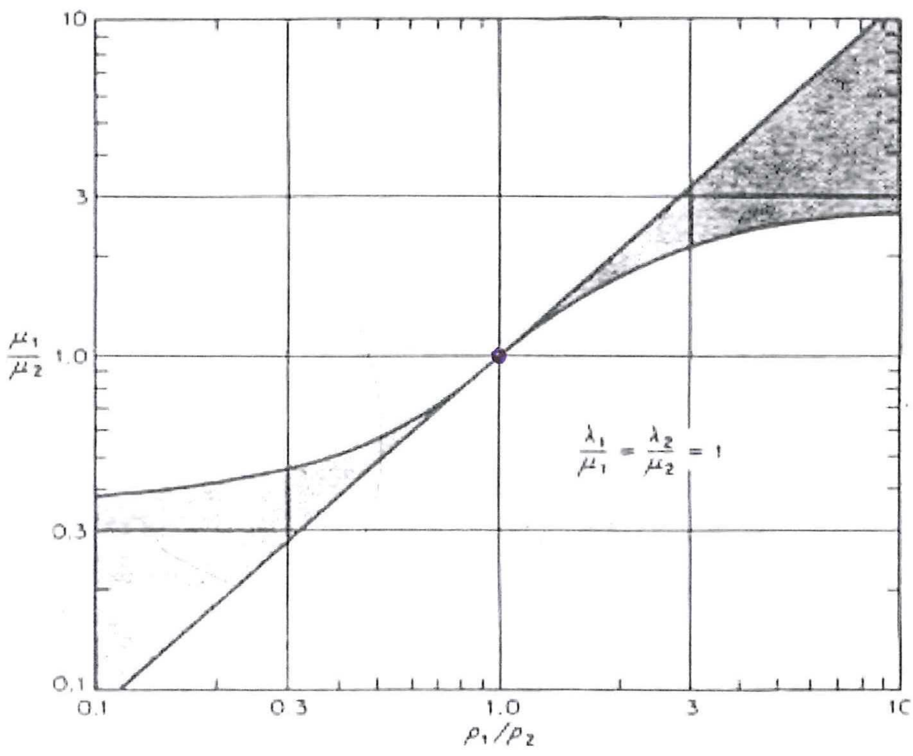

Figure 1

f) (1p) Use the table and the definition of the bulk modulus of the fluid to show that expression (10) gives

$$-\frac{dV_b}{V_b} = \left[\frac{\phi}{K_f} + \frac{1-\phi}{K_s}\right] dp + \frac{1}{K_s} d\sigma_0 \tag{11}$$

g) (2p) Use equations (7) and (11) and the definition of the Gassmann modulus

$$K_G = -\left.\frac{V_b}{dV_b}\right|_{\mathrm{sat}} dp_e \quad \text{and} \quad dp_e = d\sigma_0 + dp \tag{12}$$

to show that for incompressible grains

$$K_G = K_b + \frac{K_f}{\phi}. \tag{13}$$

4) The Biot momentum equation for the fluid in an elastic porous material is

$$\phi\rho_f \frac{\partial w}{\partial t} = -\phi\frac{\partial p}{\partial x} + \phi\rho_f(\alpha_\infty - 1)\frac{\partial}{\partial t}(v-w) + \frac{\eta\phi^2}{k_0}(v-w) \tag{14}$$

with porosity ϕ, fluid density ρ_f, permeability k_0, viscosity η, and tortuosity α_∞. We only consider the 1D compressional case, so the wave propagates in the x-direction and the average solid and fluid particle velocities in the x-direction are v and w.

a) (1p) Explain whether tortuosity is dominant at high or low-frequencies.

b) (2p) Show that we may find Darcy's law for the case of a rigid matrix.

c) (2p) Write expression (14) in the frequency domain using harmonic wave propagation in the x-direction for the relevant variables, e.g. $v = \hat{v}\exp i(\omega t - kx)$.

The full Biot equations can then be cast in the following form:

$$\begin{aligned} \hat{v}\left[\hat{\rho}_{11}c^2 - P\right] &= \hat{w}\left[Q - \hat{\rho}_{12}c^2\right] \\ \hat{v}\left[\hat{\rho}_{12}c^2 - Q\right] &= \hat{w}\left[R - \hat{\rho}_{22}c^2\right] \end{aligned} \tag{15}$$

where P, Q, and R are elastic moduli and $\hat{\rho}_{11}, \hat{\rho}_{12}$, and $\hat{\rho}_{22}$ are complex densities.

d) (2p) Show that set (15) gives the dispersion relation

$$d_2c^4 + d_1c^2 + d_0 = 0 \tag{16}$$

and give the expressions for d_2, d_1, and d_0.

e) (2p) Solve (16) and indicate which wave corresponds to which solution.

f) (1p) Why may we observe both waves in the laboratory but not in the field?

3) Consider a fully saturated rock sample immersed in a pressure tank containing the saturating liquid. A piston causes an external pressure change dp_e. We perform two tests.

Upon changing the hydrostatic pressure, the volume of the solid grains of an open sample changes, so that we can measure the bulk modulus of the grains (unjacketed test).

Assume now that we maintain a constant pore pressure (e.g. atmospheric pressure) by means of a capillary tube in a closed sample (jacketed test). Then, the amount of fluid that is squeezed out of the sample is measured by the fluid rise in the capillary tube.

The jacketed and unjacketed tests can be used to find Hooke's law for poroelastic materials.

	dp	$d\sigma_0$	dV_s	dV_b	$d\phi$
Unjacketed	dp_e	0	$-\frac{V_s}{K_s}dp_e$	$-\frac{V_b}{K_s}dp_e$	0
Jacketed	0	dp_e	$-\frac{V_s}{(1-\phi)K_s}dp_e$	$-\frac{V_b}{K_b}dp_e$	$-\left(\frac{1-\phi}{K_b}-\frac{1}{K_s}\right)dp_e$

a) (2p) Make a schematic drawing of each test.

The change in bulk volume as a result of the change in pore pressure and normal intergranular stress is the sum of the change in bulk volume of the jacketed and unjacketed test.

$$dV_b = -\frac{V_b}{K_s}dp - \frac{V_b}{K_b}d\sigma_0 \tag{7}$$

in which σ_0 is the isotropic component of the intergranular stress.

b) (1p) Show that equation (7) can be rewritten as

$$-\sigma_0 = K_b\varepsilon_{kk} + \frac{K_b}{K_s}p\delta_{ij} \tag{8}$$

Hooke's law for a nonporous elastic solid is here written as (minus sign for compression)

$$-\sigma_{ij} = \lambda\varepsilon_{kk}\delta_{ij} + 2\mu\varepsilon_{ij} \tag{9}$$

c) (2p) Use equation (9) to write the sum of the normal stress components. For which extreme rock property is this result equal to expression (8)?

d) (1p) How many independent components has σ_{ij} for a general nonporous homogeneous isotropic solid?

e) (1p) Show that

$$\frac{dV_b}{V_b} = (1-\phi)\frac{dV_s}{V_s} + \phi\frac{dV_f}{V_f} \tag{10}$$