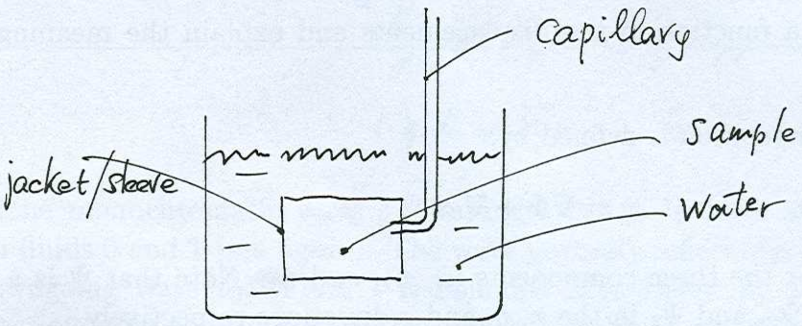
Fundamentals of Borehole Logging TA3460 24/3/04 900-1200



1) We consider a jacketed test on a water-saturated porous sample with a volume of 1 dm^3 and a porosity of 30 %. The bulk modulus K_b=20 GPa, the grain bulk modulus K_s=40 GPa, and the water bulk modulus K_f=2.2 GPa. The diameter of the capillary connnecting the sample to the outside world is 1 mm. The water in the capillary has height h. We exert a pressure change dp_e of 100 bars on the sample.

a) Give the relation between the change in the sample volume and the pressure change dp_e.

b) Give the relation between the porosity change and dp_e.

c) Compute the height change dh in the capillary.

d) Now the pressure change dp_e is also applied to the capillary. What is the new water level height ?

e) A new experiment on the same sample is performed. This time there is no capillary at all. What is the change in porosity this time ?

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2) Hooke's law gives the relation between stresses and displacements:

$$\sigma_{ij} = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} \tag{1}$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{2}$$

a) Write σ_{31} en σ_{33} as a function of the displacements and explain the meaning of the subscripts $_{31}$ and $_{33}$.

b) The Helmholtz decomposition is defined by:

$$\underline{u} = \nabla\Phi + \nabla \times \underline{\Psi} \tag{3}$$

Write the expressions for the three components u_1, u_2, and u_3. Note that $\underline{\Psi}$ is a vector, having components Ψ_1, Ψ_2, and Ψ_3 in the x, y, and z directions respectively.

c) For 2D wave propagation in the XOZ plane, we know that $u_2 = 0$, and that $\partial/\partial y = 0$. So we now know that

$$\underline{u} = \begin{pmatrix} \dfrac{\partial\Phi}{\partial x} - \dfrac{\partial\Psi}{\partial z} \\ 0 \\ \dfrac{\partial\Phi}{\partial z} + \dfrac{\partial\Psi}{\partial x} \end{pmatrix}, \tag{4}$$

where Ψ is a shorthand notation for Ψ_2. Write down σ_{31} and σ_{33} as a function of the potentials Φ en Ψ.

d) For a plane monochromatic wave, we may write

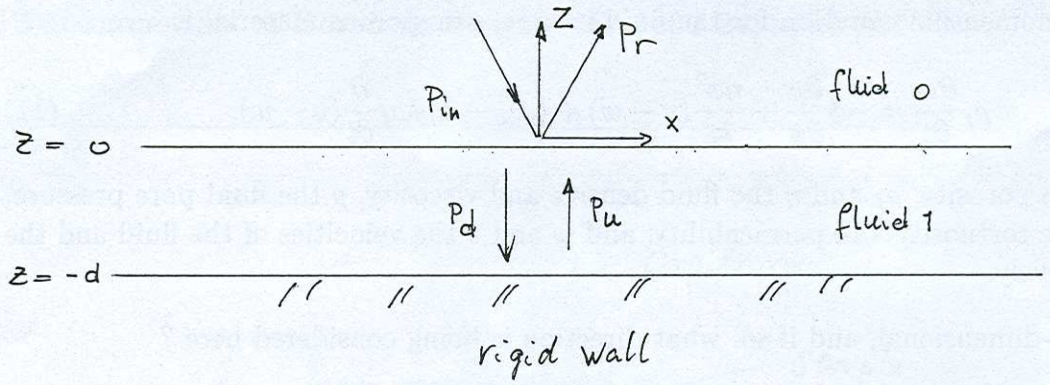
$$\Phi = \hat{\Phi}\exp i(\omega t - k_x x - k_{1z}z) \tag{5}$$

$$\Psi = \hat{\Psi}\exp i(\omega t - k_x x - k_{3z}z) \tag{6}$$

What is the meaning of k_{1z} en k_{3z} ?

e) Now rewrite σ_{31} and σ_{33}

f) On the interface $z = 0$ between an elastic medium and a vacuum, it holds that $\sigma_{31} = \sigma_{33} = 0$. Both boundary conditions are met by a Rayleigh wave. Derive the dispersion relation for this wave (Hint: write both boundary conditions in terms of the matrix relation $\underline{\underline{A}}\,\underline{x} = 0$, and calculate the determinant of matrix $\underline{\underline{A}}$).



3) A plane monochromatic wave p_{in} impinges obliquely from above upon the interface between fluids 0 and 1 (see figure). The wave partially reflects (p_r) and partially transmits as a downgoing wave p_d. Fluid 1 is bounded from below by a rigid wall. Because of the reflection from the rigid wall, there must also be an upgoing wave p_u. Omitting the common factor $\exp i(\omega t - k_x x)$ in all terms, the pressure in fluid 0 is written as:

$$p_0 = p_{in} \exp(ik_{0z}z) + p_r \exp(-ik_{0z}z), \tag{7}$$

and in fluid 1 as

$$p_1 = p_d \exp(ik_{1z}z) + p_u \exp(-ik_{1z}z). \tag{8}$$

a) What is the difference between wave velocity and fluid velocity ?

b) Momentum conservation yields for the z-components of the fluid velocities in layers 0 and 1:

$$i\omega\rho_0 w_{0z} = -\frac{\partial p_0}{\partial z}, \qquad i\omega\rho_1 w_{1z} = -\frac{\partial p_1}{\partial z}. \tag{9}$$

Give the expressions for w_{0z} en w_{1z}, using the impedance definitions $Z_0 = \omega\rho_0/k_{0z} = \omega\rho_0/(k_0 \cos\theta_0)$ and $Z_1 = \omega\rho_1/k_{1z} = \omega\rho_1/(k_1 \cos\theta_1)$

c) At the interface $z = -d$, the boundary condition is that $w_{1z} = 0$. Give the expression for p_d/p_u.

d) Give the boundary conditions for pressure and velocity at the interface between the two fluids (z=0), and show that the expression for the reflection coefficient

$$R = p_r/p_{in} = \frac{Z_1 - iZ_0 \tan(k_{1z}d)}{Z_1 + iZ_0 \tan(k_{1z}d)}. \tag{10}$$

Hint: $\tan(x) = -i[\exp(ix) - \exp(-ix)]/[\exp(ix) + \exp(-ix)]$.

e) Snell's law tells us that $\sin(\theta_1)/c_1 = \sin(\theta_0)/c_0$, where c_0 and c_1 are the wave velocities in the fluids 0 en 1. For $c_0/c_1 = 1/\sqrt{2}$ and $\theta_0 = 30°$, compute θ_1.

f) Prove that $|R|$ is always 1.

4) The Biot momentum equation for the fluid in an elastic porous material is

$$\phi\rho_f \frac{\partial w}{\partial t} = -\phi\frac{\partial p}{\partial x} + \frac{\eta\phi^2}{k_0}(v-w) + (\alpha_\infty - 1)\phi\rho_f\frac{\partial}{\partial t}(v-w), \quad (11)$$

where ϕ is the porosity, ρ_f and η the fluid density and viscosity, p the fluid pore pressure, α_∞ and k_0 the tortuosity and permeability, and w and v the velocities of the fluid and the solid, respectively.

a) Is (11) one-dimensional, and if so, what direction is being considered here ?

b) Give the SI units for all variables in the equation (11)

c) What is the physical meaning of the tortuosity ?

d) For a rigid porous material, $v = 0$. Now rewrite (11) until only three terms are left.

e) Under what condition can we find back Darcy's law ? Write Darcy's law.

f) For a typical sandstone ($k_0 = 200$ mD, $\phi = 0.2$) filled with water ($\eta = 0.001$ Pa.s), compute the necessary pressure gradient for a water velocity of 1 mm/s.
