1) A sponge and an oil reservoir are two examples of a poroelastic material, comprising solid material and fluid-filled pores. Mass conservation for the solid and the fluid gives:

$$\frac{\partial}{\partial t}(1-\phi)\rho_s + \frac{\partial}{\partial x}(1-\phi)\rho_s v = 0, \tag{1}$$

$$\frac{\partial}{\partial t}\phi\rho_f + \frac{\partial}{\partial x}\phi\rho_f w = 0 \tag{2}$$

where ϕ, v, and w are the porosity, the velocity of the solid, and the velocity of the fluid. The solid and the fluid densities are denoted ρ_s and ρ_f, respectively.

a) Are (1) and (2) one-dimensional equations and why (not) ?

b) Linearize (1) and (2) by substitution of $\rho_s = \rho_{s0} + \rho'_s$, $\rho_f = \rho_{f0} + \rho'_f$, $v = v'$, $w = w'$, $\phi = \phi_0 + \phi'$.

d) Under what condition can (1) can be rewritten as

$$\frac{\partial\phi'}{\partial t} = (1-\phi_0)\frac{\partial v'}{\partial x}? \tag{3}$$

e) Write the consolidation equation, which is obtained from substitution of (3) into the linearized version of (2).

f) Write the constitutive relation for the fluid, which relates the change in fluid pressure to the change in fluid density.

2) Momentum conservation for an elastic medium:

$$\rho \frac{\partial^2 \underline{u}}{\partial t^2} = \nabla \cdot \underline{\underline{\sigma}} \tag{4}$$

a) What is the meaning of $\underline{u}$, ∇ and $\underline{\underline{\sigma}}$ and what units are they expressed in ?

b) Is equation (4) linearized or not ? Why ?

c) Elaborate (4) for the z-component of $\underline{u}$.

d) Hooke's law:

$$\sigma_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} \tag{5}$$

$$e_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) \tag{6}$$

Use (5) and (6) to elaborate (4) for the z-component of $\underline{u}$.

e) For plane wave propagation in the z-direction, we know that $\frac{\partial}{\partial x} = \frac{\partial}{\partial y} = 0$. How can we finally write the equation for the z-component of $\underline{u}$?

f) Are we now dealing with a compressional wave (P) or a shear wave (S) ? Why ?

g) Now derive the expression for the wave speed.