Examination

ta4780

'Flow and Transport in

Fractured Rock Masses'

20 January 2003, 2:00-5:00 a.m., room 227

Remark 1: **You may answer the questions in English or Dutch**

Remark 2: **For the examination in 'new style' all answers have to be answered, but no report has to be delivered.**

For the examination in 'old style' the questions marked with asterisks (**) need not be answered. Instead, a report has to be delivered.

Question series 1

The most basic equation describing fluid flow through fractured porous media is the mass balance equation.

1a. Explain the principle of mass conservation using the words inflow, outflow and storage (or accumulation).

1b. Explain (make a picture of) a block centered finite difference mesh and indicate the volumes over which the mass balance is satisfied.

1c. In Darcy's law, nine parameters (coefficients) occur. These parameters are generally obtained form measurements and are, therefore, specified before performing a calculation. Describe the nine coefficients and give at least one unit in which they are measured.

1d. Assume that the hydraulic conductivity is isotropic. Explain how in a block centered finite difference method the flux through the faces is computed from the heads in the grid block centers. Explain why not the arithmetic averages, but the harmonic averages of the hydraulic conductivities play a role.

1e. The block centered finite difference method represents a discrete analog of the continuum equations. The mesh centered finite difference method is another discrete analog. Make a picture of the mesh and explain why in this method the arithmetic average of the hydraulic conductivities occurs and not the harmonic average.

1f. At present, discrete analogs are the most popular methods to perform groundwater flow and transport calculations. However, in the past, before the advent of computers, continuum methods were used. Explain the meaning of continuum equations and present the mass conservation equation and Darcy's law in continuum form.

1g. The block centered finite difference method is a lower bound method since it underestimates the averaged hydraulic conductivity. The mesh centered finite difference method is an upper bound method since it overestimates the averaged hydraulic conductivity. Give an argument why this is the case.

1h. There exist two types of geohydrological problems: (i) groundwater *quantity* problems and (ii) groundwater *quality* problems. Give an example of each problem.

1i. What is the difference between heterogeneity and homogeneity?
What is the difference between anisotropy and isotropy?
Explain in words and/or with a picture why a block of fractured porous rock that is isotropic and heterogeneous on the fine scale, is anisotropic and homogeneous on the coarse scale.

Question series 2

Suppose we want to calculate both the groundwater head and the groundwater flow velocity in a porous and fractured rock. Let us, therefore, consider an aquifer for which the groundwater flow is described by the following equations

$$\operatorname{div} \vec{q} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0,$$
$$q_x = -k_{//} \frac{\partial h}{\partial x},$$
$$q_y = -k_{//} \frac{\partial h}{\partial y},$$
$$q_z = -k_{\perp} \frac{\partial h}{\partial z},$$

where $k_{//} >> k_{\perp}$; *x* and *y* are horizontal coordinates, while *z* is the vertical coordinate.

2a. Is this aquifer isotropic or anisotropic? Why?

2b. Is this aquifer homogeneous or heterogeneous? Why? Make a picture of the most probable fracture pattern.

In aquifers the above 3D flow equations are generally simplified to a 2D approximation. This approximation is sometimes stated as: "In aquifers the flow is two-dimensional, with zero vertical flux component $q_z = 0$."

2c. Introduce $q_z = 0$ in the above equations. Is this a useful approximation when the aquifer is recharged by precipitation?

2d. Consider again the above equations and replace the third component of Darcy's law, $q_z = -k_{\perp} \partial h / \partial z$ by the Dupuit approximation (or vertical equilibrium approximation) $\partial h / \partial z = 0$. How many equations result for how many unknowns? Is this a useful approximation when the aquifer is recharged by precipitation? Are the resulting 2D equations isotropic or anisotropic?

For circular symmetry along a vertical well the equations are:

$$\operatorname{div} \vec{q} = \frac{1}{r} \frac{\partial (r q_r)}{\partial r} + \frac{\partial q_z}{\partial z} = 0,$$
$$q_r = -k_{//} \frac{\partial h}{\partial r},$$
$$q_z = -k_{\perp} \frac{\partial h}{\partial z},$$
$$r = \sqrt{x^2 + y^2}.$$

2e. Groundwater flow in horizontal direction is caused by a well with discharge rate Q per unit depth. There is, at the same time, groundwater flow in vertical direction with specific discharge $q_z = P_e$ (= effective precipitation).

What is the solution for the specific discharge in horizontal direction, q_r, as a function of *x* and *y*? Give the expression for the head h as a function of *x*, *y* and *z*.

Question series 3

3a. Explain in words at least four processes that play a role in the transport dissolved mass.

3b. What is the physical basis of (i) molecular diffusion and (ii) of mechanical dispersion? Explain why mechanical dispersion does not exist on the scale of the pore scale, and why it does exist on the scale of a 'representative elementary volume' (i.e., a volume containing many solid grains).

3c. Explain the difference between micro dispersion and macro dispersion.

3d. What is the difference in description between dispersion in porous media and dispersion in fractured media?

3e. Give a practical example in which transport problems play a role.

3f. Present the advection-dispersion equation and explain the different terms

3f. Transport of saline groundwater, for instance in the dune area, can also be described by the advection-dispersion equation. Are the flow equations as given in question series 2 sufficient to describe flow of saline water? If not, what is lacking in the flow equations of question series 2?

**** Question series 4 (not to be answered for examination old style)**

The equations governing groundwater flow and transport are *continuum equations* in *continuous* space and time. In such equations four partial differential operators play a role: (i) the time derivative, $\partial_t = \partial / \partial t$; (ii) the divergence, $\mathrm{div} = \nabla \bullet$; (iii) the curl, $\mathrm{rot} = \mathrm{curl} = \nabla \times$; and (iv) the gradient. $\mathbf{grad} = \boldsymbol{\nabla}$.

**** 4a.** Explain the physical meaning of these four operators, and give continuum equations relevant to groundwater flow in which at least two of these operators occur.

**** 4b.** What is the difference between a system of continuum equations and a discrete analog? Make a picture. Give an example of the discretized operators div and grad (for instance in a block centered finite difference mesh).

**** 4c.** Make a picture of the mesh of a conventional finite element method. What are volumes, faces, edges and nodes? Suppose that the volumes are tetrahedrons. Are there more faces than thetrahedrons? Are there more nodes than tetrahedrons? Are there more edges than tetrahedrons?

**** 4d.** Does a conventional finite element methods conserve mass at the faces? Why? For what type of problems do conventional finite element methods yield good solutions?

**** 4e.** The conventional finite element method transforms the continuum equations into a system of algebraic equations $\mathbf{M}\,\mathbf{x} = \mathbf{r}$, where $\mathbf{M}$ is a $N \times N$ matrix, (N is number of nodes), $\mathbf{x}$ is the column of heads at the nodes (the unknowns) and $\mathbf{r}$ is the known right-hand side containing boundary conditions, information about wells etc. Matrix $\mathbf{M}$ is sparse; explain what 'sparse' means and explain why the matrix is sparse.

**** 4f.** What are direct solvers, what are iterative solvers?