Examination

TA 4780, ‘Flow and Transport in Fractured Rock Masses’

Monday 22 January 2001, room 234

Remark: The questions may be answered either in English or in Dutch, or in a mixture, EngDutch.

Question series 1

The most basic principle describing fluid flow through fractured porous media is the mass balance equation.

1a. Explain the mass balance equation using the words ‘inflow,’ ‘outflow’ and ‘storage.’

In flow and transport problems there are two important mass balances:
(i) the mass balance of the groundwater (e.g. saline groundwater), and
(ii) the balance of dissolved mass (e.g. salt).

1b. Give the equation for the mass balance of the groundwater (the continuity equation including the storage term). Which parameter(s) occur in this equation? Assuming that this parameter is known, how many unknowns occur in this equation?

1c. The continuity equation does not suffice to determine the groundwater flow. Give the additional equations that are required to solve the flow. How many equations are there for how many unknowns? Explain the meaning of these unknowns and give a unit in which they are measured. What is the relevant parameter and in which unit is it expressed?

1d. What is the difference between heterogeneity and homogeneity? What is the difference between anisotropy and isotropy? Explain in words and/or with a picture why a block of fractured porous rock that is isotropic and heterogeneous on the fine scale, is anisotropic and homogeneous on the coarse scale.

1e. Explain in words at least four processes that play a role in the mass balance of the dissolved mass.

1f. What is the physical basis of (i) molecular diffusion and (ii) of mechanical dispersion?

Question series 2

The equations governing groundwater flow and transport are *partial differential equations* in *continuous* space and time.

2a. Why do we need numerical approximation methods?

2b. Into what type of equations do numerical methods transform the partial differential equations?

2c. What are direct methods, what are iterative methods, and why do we need them?

2d. What is the basic idea behind Finite Difference Methods?

2e. What is the basic idea behind Finite Element Methods?

2f. For a ‘mathematician’ a numerical method is an *approximation* method, since such a method converges to the exact solution only in the limit of vanishingly small discretization intervals. This is the case even for partial differential equations with constant coefficients (for instance, the equation $\nabla^2 h = 0$).
However, for an engineering geologist or a geohydrologist there is another reason why numerical methods are considered as *approximation* methods. This reason has to do with the parameters that occur in the equations. Explain the kind of engineering geological approximations that have to be introduced. Use the *scale* concept in your explanation.

Question series 3

Suppose we want to calculate (i) the groundwater pressure, (ii) the groundwater flux density, and (iii) the salt concentration in a porous and fractured part of the subsurface.

Let us consider an isotropic aquifer. The four equations for the four unknowns q_x, q_y, q_z, h are given by

$$q_x = -k\frac{\partial h}{\partial x}$$
$$q_y = -k\frac{\partial h}{\partial y}$$
$$q_z = -k\frac{\partial h}{\partial z}$$
$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0$$

In aquifers the above 3D flow equations are often simplified to a 2D approximation. This approximation is sometimes stated as: “In aquifers the flow is two-dimensional, with zero vertical flux component $q_z = 0$.”

3a. Show that the above stated simplification ($q_z = 0$) results in three 2D equations for three unknowns. Is this a useful approximation when the aquifer is recharged by precipitation?

3b. Consider again the 3D equations and show that replacement of the third component of Darcy’s law, $q_z = -k\partial h/\partial z$, by $\partial h/\partial z = 0$ (Dupuit’s approximation) results in four equations for four unknowns. Is this a useful approximation when the aquifer is recharged by precipitation? What is the advantage of using these approximate equations above the original 3D equations?

3c. Derive the 2D equation for the hydraulic head and introduce the transmissivity. Show that in this equation the vertical flux component $q_z \neq 0$. How can we introduce recharge in this equation?

3d. In the continuity equation we have neglected the storage term $s\partial h / \partial t$. This term takes the compressibility of the water and the changes in porosity caused by changes in water pressure into account. However, even when neglecting these effects, there is still storage in phreatic aquifers. What kind of storage is meant, and explain how it can be included in the 2D equation for groundwater flow in phreatic aquifers.

Question series 4

The ‘permeability’ / ‘conductivity’ / ‘mobility’ $\underline{\underline{k}}$ is a tensor in Darcy’s law relating the flux density vector $\underline{q}$ to the head gradient vector $\mathrm{grad}\ h$ in the following way

$$\underline{q} = -\underline{\underline{k}} \bullet \mathrm{grad}\ h$$

Here Darcy’s law is written in vector-tensor notation, which is a notation that is *independent* of the choice of a coordinate system. Let us now limit the discussion to two-dimensional flow. In an arbitrary Cartesian *xy* coordinate system this vector-tensor equation is then given by

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = -\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} \bullet \begin{pmatrix} \dfrac{\partial h}{\partial x} \\ \dfrac{\partial h}{\partial y} \end{pmatrix}$$

(In a coordinate system, a vector is represented by a column of components, and a tensor is represented by a matrix of components.) Under the assumption that the permeability is symmetric, i.e., $k_{xy} = k_{yx}$, there exists a *uv* coordinate system in which the matrix of permeability components has a diagonal form

$$\begin{pmatrix} q_u \\ q_v \end{pmatrix} = -\begin{pmatrix} k_u & 0 \\ 0 & k_v \end{pmatrix} \bullet \begin{pmatrix} \dfrac{\partial h}{\partial u} \\ \dfrac{\partial h}{\partial v} \end{pmatrix}$$

This coordinate system is called the *principal coordinate* system. The following expression holds

$$\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} = \begin{pmatrix} k_1 c^2 + k_2 s^2 & (k_1 - k_2)cs \\ (k_1 - k_2)cs & k_1 s^2 + k_2 c^2 \end{pmatrix}$$

where $c = \cos \omega$ and $s = \sin \omega$, in which ω is the angle over which the principal *uv* coordinate system has been rotated to obtain the general *xy* coordinate system.

4a. Consider a fractured block of rock with plane parallel fractures that make an angle of −45° with the horizontal *x* direction. The composite (coarse-scale) conductivity of the fractured block is 1 m/day in the direction parallel with the fractures and is 0 (negligibly small) in the direction normal to the fractures. Write Darcy’s law in the ‘horizontal-vertical’ *xy* coordinate system.

4b. Many commercially available groundwater flow models codes cannot handle off-diagonal terms in the conductivity tensor (especially codes based on the finite difference method, like Modflow). In order to be able to use such codes, the off-diagonal terms in the conductivity matrix are neglected (Parsons’s approximation). Write Darcy’s law in the ‘horizontal-vertical’ *xy* coordinate system using Parsons’s approximation.

4c. Suppose there is a potential gradient over the block with components $\left(\dfrac{\partial h}{\partial x}, \quad \dfrac{\partial h}{\partial y} \right) = -(1, \quad 1)$.

Calculate the flux components in *x* and *y* direction, first using the exact equations and then using

Parsons's approximation. What is your opinion about the applicability of Parsons's approximation when dealing with flow through fractured rocks?

Question 5

Give in a text of approximately ½ to 1 page your opinion about the usefulness of a course on flow and transport in fractured porous media for engineering geologists. Please be honest, but at the same time, constructive.