Examination

ta4780

'Flow and Transport in

Fractured Rock Masses'

9 April 2001, 9:00-12:00 a.m., room 227

Remark 1: You may answer the questions in English, in Dutch, or in a mixture 'Dutchlish'

Remark 2: Each well-answered question will gives you 10/20 point. Since there are 20 questions to be answered, you can have a maximum of 10 points.

Question series 1 (60/20 point)

The most basic equation describing fluid flow through fractured porous media is the mass balance equation.

1a. Explain the mass balance equation using the words 'inflow,' 'outflow' and 'storage.'

In flow and transport problems there are two important mass balances:
(i) the mass balance of the solute-water mixture (e.g. saline groundwater), and
(ii) the balance of dissolved mass (e.g. salt).

1b. In the momentum balance (Darcy's law), nine parameters (coefficients) occur. These parameters are generally not considered as unknowns in the mathematical sense. Nevertheless, these parameters are quite often unknown. Describe these nine coefficients and give at least one unit in which they are measured.

1c. It is the mathematician's task to solve the continuity equation and Darcy's law under the condition that the conductivity distribution is known. What is in this context the main problem of the engineering geologist or geohydrologist?

1d. What is the difference between heterogeneity and homogeneity? What is the difference between anisotropy and isotropy? Explain in words and/or with a picture why a block of fractured porous rock that is isotropic and heterogeneous on the fine scale, is anisotropic and homogeneous on the coarse scale.

1e. Explain in words at least four processes that play a role in the mass balance of the dissolved mass.

1f. What is the physical basis of (i) molecular diffusion and (ii) of mechanical dispersion?

Question series 2 (60/20 point)

The equations governing groundwater flow and transport are *partial differential equations* in *continuous* space and time. In such equations four partial differential operators play a role: (1) the time derivative, $\partial_t = \partial / \partial t$; (2) the gradient. $\text{grad} = \nabla$; (3) the curl, $\text{rot} = \text{curl} = \nabla \times$; and (4) the divergence, $\text{div} = \nabla \bullet$.

2a. Give an expression for the gradient and for the divergence, explain their physical meaning, and give the equations relevant to groundwater flow in which these operators occur.

2b. Give the essential difference between exact analytical solution methods and numerical solution methods.

2c. Into what type of equations do numerical methods transform the partial differential equations?

2d. What are direct methods, what are iterative methods, and why do we need them?

2e. What is the basic idea behind the Finite Difference Method?

2f. What is the basic idea behind the Finite Element Method?

Question series 3 (40/20 point)

Suppose we want to calculate (i) the groundwater head, (ii) the groundwater flow velocity, and (iii) the salt concentration in a porous and fractured part of the subsurface. Let us, therefore, consider an aquifer for which the groundwater flow is described by the equations $\text{div}\,\underline{q} = 0$, $\underline{q} = -k\,\text{grad}\,\varphi$. In Cartesian coordinates:

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0$$

$$q_x = -k \frac{\partial \varphi}{\partial x}$$

$$q_y = -k \frac{\partial \varphi}{\partial y}$$

$$q_z = -k \frac{\partial \varphi}{\partial z}$$

3a. Is this aquifer isotropic or anisotropic? Why? Is this aquifer homogeneous or heterogeneous? Why?

In aquifers the above 3D flow equations are generally simplified to a 2D approximation. This approximation is sometimes stated as: “In aquifers the flow is two-dimensional, with zero vertical flux component $q_z = 0$.”

3b. Introduce the above-stated simplification ($q_z = 0$). How many equations result for how many unknowns? Is this a useful approximation when the aquifer is recharged by precipitation?

3c. Consider again the 3D equations and replace the third component of Darcy’s law, $q_z = -k\partial\varphi/\partial z$, by the Dupuit approximation $\partial\varphi/\partial z = 0$. How many equations result for how many unknowns? Is this a useful approximation when the aquifer is recharged by precipitation?

3d. Derive the 2D equation for the potential and introduce the transmissivity. Show that in this equation the flux component $q_z \neq 0$. How do we introduce recharge in this equation?

Question series 4 (30/20 point)

The 'permeability' / 'conductivity' / 'mobility' $\underline{\underline{k}}$ is a tensor in Darcy's law relating the flux vector $\underline{q}$ to the potential gradient vector $\text{grad } \varphi$ in the following way

$$\underline{q} = -\underline{\underline{k}} \bullet \text{grad } \varphi$$

Here Darcy's law is written in vector-tensor notation, which is a notation that is *independent* of the choice of a coordinate system. Let us now limit the discussion to two-dimensional flow. In an arbitrary Cartesian *xy* coordinate system this vector-tensor equation is then given by

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = -\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} \bullet \begin{pmatrix} \partial\varphi / \partial x \\ \partial\varphi / \partial x \end{pmatrix}$$

(In a coordinate system, a column of components represents a vector, and a matrix of components represents a tensor.) Under the assumption that the permeability is symmetric, i.e., $k_{xy} = k_{yx}$, there exists a *uv* coordinate system in which the matrix of permeability components has a diagonal form

$$\begin{pmatrix} q_u \\ q_v \end{pmatrix} = -\begin{pmatrix} k_u & 0 \\ 0 & k_v \end{pmatrix} \bullet \begin{pmatrix} \partial\varphi / \partial u \\ \partial\varphi / \partial v \end{pmatrix}$$

This coordinate system is called the *principal coordinate* system. The following expression holds

$$\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} = \begin{pmatrix} k_1 c^2 + k_2 s^2 & (k_1 - k_2)cs \\ (k_1 - k_2)cs & k_1 s^2 + k_2 c^2 \end{pmatrix}$$

where $c = \cos \omega$ and $s = \sin \omega$, in which ω is the angle over which the principal *uv* coordinate system has been rotated to obtain the general *xy* coordinate system.

4a. Consider a fractured block of rock with plane parallel fractures that make an angle of 30° with the horizontal *x* direction. The composite (coarse-scale) conductivity of the fractured block is 0.1 m/day in the direction parallel with the fractures and is 0.0001 m/day in the direction normal to the fractures. Write Darcy's law in the 'horizontal-vertical' *xy* coordinate system.

4b. Many commercially available groundwater flow models codes cannot handle off-diagonal terms in the conductivity tensor (especially codes based on the finite difference method cannot). In order to be able to use these codes, the off-diagonal terms in the conductivity matrix are neglected (Parsons's approximation). Write Darcy's law in the 'horizontal-vertical' *xy* coordinate system using Parsons's approximation.

4c. Suppose there is a potential gradient over the block with components $(\partial\varphi / \partial x, \partial\varphi / \partial y) = (\frac{1}{2}, \frac{1}{2}\sqrt{3})$, calculate the flux components in *x* and *y* direction. Calculate the flux (using the exact equations). Make a picture of the block of rock with its fractures and the flow and its direction. Now use Parsons's approximation and make a picture of the rock and the Parsons's flow direction. What is your opinion about the applicability of Parsons's approximation when dealing with flow through fractured rocks?

Question 5 (10/20 point)

Give in a short text your opinion about the usefulness of a course on flow and transport in fractured porous media for engineering geologists. Please be honest, but at the same time constructive. Your answer will help in making improvements for new students.