Partial examination

October 30, 1998

4:00 - 7:00 p.m.

room 227 Building ‘Mijnbouw’

mp4780

‘Flow and Transport in

Fractured Rock Masses’

The questions may be answered in any language

(dus ook in de Nederlandse taal)

Question 1

The most basic equation describing fluid flow through fractured porous media is the equation expressing conservation mass (balance equation).

1a. Explain the mass conservation equation using the words ‘inflow,’ ‘outflow’ and ‘storage.’

In flow and transport problems there are two important mass balances:
(i) the mass balance of the solute-water mixture (*e.g.* saline groundwater), and
(ii) the mass balance of dissolved mass (*e.g.* the salt).

1b. Give the partial differential equation for the mass balance of the solute-water mixture (the continuity equation) including the storage term. How many unknowns occur in this equation? What is the name of the parameter that occurs in this equation? Give a unit in which this parameter is measured.

1c. The continuity equation does not suffice to uniquely determine the flow of the solute-water mixture (the fluid, the groundwater). Give the additional equations that are required to solve the flow uniquely. Provided that the parameters in this equation are known, what kind of additional data has to be specified to solve these equations uniquely?

1d. In the momentum balance (Darcy’s law), nine parameters (coefficients) occur. These parameters are generally not considered as unknowns in the mathematical sense. Nevertheless, these parameters are quite often unknown. Describe these nine coefficients and relate the unit in which they are expressed to the units of the other quantities that occur in Darcy’s law.

1e. The mathematician’s task is to solve the continuity equation and Darcy’s law under the condition that the conductivity distribution is known. What is the main problem of the engineering geologist or geohydrologist? Mention briefly some ways to solve this problem.

1f. What is a conductivity *distribution*?
What is the difference between heterogeneity and homogeneity?
What is the difference between anisotropy and isotropy?

1g. Explain in words and/or pictures the meaning of the following five mechanisms that play a role in the mass balance of dissolved mass: ‘storage,’ ‘advection,’ ‘molecular diffusion,’ ‘longitudinal mechanical (hydrodynamical) dispersion’ and ‘transversal (lateral) mechanical dispersion.’

1h. What is the difference between microdispersion and macrodispersion?

Question 2

The equations governing groundwater flow and transport are *partial differential equations* in *continuous* space and time.

2a. Why do we need numerical approximation methods?

2b. Into what type of equations do numerical methods transform the partial differential equations?

2c. What are direct methods, what are iterative methods, and why do we need them?

2d. What is the basic idea behind Finite Difference Methods?

2e. What is the basic idea behind Finite Element Methods?

2f. Make a picture of a finite element mesh and of a finite difference mesh, and show the refinement around a well in the two pictures.

> For a ‘mathematician’ a numerical method is an *approximation* method, since such a method converges to the exact solution only in the limit of vanishingly small discretization intervals. This is the case even for partial differential equations with constant coefficients (for instance, the equation $\nabla^2\phi=0$).
> However, for an engineering geologist or a geohydrologist, there is another reason why numerical methods are considered as *approximation* methods. This reason has to do with the parameters that occur in the equations.

2g. Explain the kind of engineering geological approximations that have to be introduced. Use the *scale* concept in your explanation.

Question 3

> A *perfectly layered* porous and/or fractured medium is a hypothetical medium that resembles really existing porous and/or fractured media in some cases. Since it is relatively easy to perform exact calculations for perfectly layered media, they are quite often studied to obtain insight.

> A *periodic* porous and/or fractured medium is a hypothetical medium that does not exist in reality, but has nevertheless a high resemblance to real porous and/or media. It is quite difficult to perform calculations for periodic media, but it is possible, especially with the aid of numerical methods.

3a. What is a perfectly layered fractured medium. Explain why in such a medium the coarse-scale conductivity is always anisotropic, even if the fine-scale conductivity is isotropic. Make a picture.

3b. What is a periodic porous medium? Make a picture.

> Both perfectly layered porous media and periodic porous media have always a symmetric conductivity matrix.

3c. Explain what a symmetric conductivity matrix is, and explain why it is advantageous to consider symmetric conductivity matrices. What is a principal coordinate system for the conductivity?

> Consider a periodic medium in two dimensions. Each periodicity cell is a square of 4 cm $\times$ 4 cm consisting of 16 square subcells of 1 cm $\times$ 1 cm. Eight (8) subcells have a high conductivity k_1 = 1 m/day and the other 8 subcells have a low conductivity k_2 = 0.0001 m/day.
> There are many algebraic expressions described in the literature to calculate the coarse-scale conductivity K from the fine-scale permeability distribution k_i. Well-known are the arithmetic mean value $K = (k_1 + k_2 + k_3 + ... + k_N)/N$, the harmonic mean value $K = N/(1/k_1 + 1/k_2 + 1/k_3 + ... + 1/k_N)$ and the geometric mean value $K = (k_1 \times k_2 \times k_3 \times .... \times k_N)^{1/N}$.

3d. Make a picture of a distribution of fine-scale conductivities in the subcells in the periodicity cell for which the harmonic and the arithmetic mean value apply.

3e. Make a picture of a distribution for which the geometric mean value applies.

The fact that coarse-scale permeability values are distribution dependent (configuration dependent) makes that there exist no universal algebraic upscaling equations. Therefore, numerical upscaling is sometimes applied.

3f. Explain the basic principle of numerical upscaling.

Question 4

The conductivity $\underline{\underline{k}}$ is a tensor in Darcy's law relating the flux vector $\underline{q}$ to the potential gradient vector $\underline{grad}\ \Phi$ in the following way

$$\underline{q} = -\underline{\underline{k}} \bullet \underline{grad}\ \Phi$$

Here Darcy's law is written in vector-tensor notation, which is a notation that is *independent* of the choice of a coordinate system.
Let us now limit the discussion to two-dimensional flow. In an arbitrary Cartesian *x y* coordinate system this vector-tensor equation is then given by

$$\begin{pmatrix} q_x \\ q_y \end{pmatrix} = -\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} \bullet \begin{pmatrix} \dfrac{\partial \phi}{\partial x} \\ \dfrac{\partial \phi}{\partial y} \end{pmatrix}$$

(In a coordinate system, a column of components represents a vector, and a tensor is represented by a matrix of components.)
Under the assumption that the permeability matrix is symmetric, i.e., $k_{xy} = k_{yx}$, there exists a *u v* coordinate system in which the matrix of permeability components has a diagonal form

$$\begin{pmatrix} q_u \\ q_v \end{pmatrix} = -\begin{pmatrix} k_u & 0 \\ 0 & k_v \end{pmatrix} \bullet \begin{pmatrix} \dfrac{\partial \phi}{\partial u} \\ \dfrac{\partial \phi}{\partial v} \end{pmatrix}$$

This coordinate system is called the *principal coordinate* system. The following expression holds

$$\begin{pmatrix} k_{xx} & k_{xy} \\ k_{yx} & k_{yy} \end{pmatrix} = \begin{pmatrix} k_1 c^2 + k_2 s^2 & (k_1 - k_2)cs \\ (k_1 - k_2)cs & k_1 s^2 + k_2 c^2 \end{pmatrix}$$

where $c = \cos\omega$ and $s = \sin\omega$, in which ω is the angle over which the principal *u v* coordinate system has been rotated to obtain the general *x y* coordinate system.

4a. Consider a fractured block of rock with plane parallel fractures that make an angle of –45° with the horizontal *x* direction. The composite (coarse-scale) conductivity of the fractured block is 1 m/day in the direction parallel with the fractures and is 0 (negligibly small) in the direction normal to the fractures. The fine-scale conductivity is isotropic. What is the value of the conductivity of the intact rock?

4b. Write the coarse-scale Darcy's law in the 'horizontal-vertical' $x\,y$ coordinate system. (Hint: write Darcy's law first in the principal coordinate system and perform then the transformation to the $x\,y$ coordinate system.)

4c. Many commercially available groundwater flow model codes cannot handle off-diagonal terms in the conductivity matrix (especially codes based on the finite difference method cannot). In order to be able to use these codes, the off-diagonal terms in the conductivity matrix are neglected (Parsons's approximation). Write Darcy's law in the 'horizontal-vertical' $x\,y$ coordinate system using Parsons's approximation.

4d. Suppose there is a potential gradient over the block with components $\begin{pmatrix} \frac{\partial \phi}{\partial x} & \frac{\partial \phi}{\partial y} \end{pmatrix} = \begin{pmatrix} 1 & 1 \end{pmatrix}$, calculate the flux components in x and y direction, first using the exact equations and then using Parsons's approximation.

4e. Give your opinion about the applicability of Parsons's approximation when dealing with flow through fractured rocks? Under what conditions Parsons's approximation will yield reasonable results?