Partial examination

31 October 1997

2:00 - 5:00 p.m.

room: mijnb./227

mp4780

‘Flow and Transport in Fractured Rock Masses’

The questions may be answered in any language

(dus ook in het Nederlands)

Numerical models

1. Describe the basic characteristics of two popular numerical methods, and explain why these techniques are called *approximation* methods.

Fractured rock models

In the context of flow and transport through *fractured* porous rock masses, the models fall into the following three main classes:

(i) equivalent continuum models,
(ii) discrete network models, and
(iii) hybrid models.

2. Suppose, you have to perform a large scale regional groundwater modeling study in a geological setting with a great number of closely spaced small fractures and only a few large fractures (faults) at relatively large distances. What would be the best modeling approach when choosing from the main classes? Give arguments.

Scale up

Consider a periodic porous and fractured medium in two dimensions. Each periodicity cell is a square with dimensions of 4 $cm \times 4$ cm consisting of $M = 16$ square subcells of 1 $cm \times 1$ cm; 8 subcells have an isotropic hydraulic conductivity of $k_1 = 1$ *m/day* (the 'fractures') and the other 8 subcells have an isotropic conductivity of $k_2 = 0.0001$ *m/day* (the 'intact rock').
Many algebraic expressions can be found in the literature to calculate up scaled (composite) conductivities K from the fine scale conductivities k_i.
Well known are the arithmetic mean value $K = (k_1 + k_2 + k_3 + ... + k_M) / M$, the harmonic mean value $K = M / (1/k_1 + 1/k_2 + 1/k_3 + ... + 1/k_M)$ and the geometric mean value $K = (k_1 \times k_2 \times k_3 \times .... \times k_M)^{1/M}$.

3a. Give an arrangement of subcells in the periodicity cell for which the arithmetic mean value 0.50005 *m/day* and the harmonic mean value 0.00019 *m/day* apply. [Make a picture.]

3b. Give an arrangement for which the geometric mean value 0.01 *m/day* applies.

Parameters and their context

In many text books the parameters *transmissivity*, T [m^2/day], and ‘integrated’ (non specific) *storage*, s [–], are introduced for aquifers. This way the originally three dimensional (3D) equation for the potential (hydraulic head) φ in the aquifer is approximated by to a two dimensional (2D) equation.

4a. What is the underlying schematization of the subsurface when using this 2D approximation ? Mention regions in the world where such types of schematization apply.

4b. Let us assume that the aquifer is isotropic. How are the three components of Darcy’s 3D law $\underline{q} = -k\, \underline{\nabla}\varphi$ approximated when using this 2D approximation (the Dupuit approximation) ? Why is it not allowed to assume that this approximation describes a purely horizontal flux $\underline{q}$?

Flow in fractured porous rocks

A consolidated sandstone aquifer with connected vertically oriented fractures has a very anisotropic composite permeability, in such a way that the groundwater flow may be considered as essentially one dimensional (1D) in the vertical direction. In this case the equation describing the propagation of a pressure front is given by

$$s\frac{\partial\varphi}{\partial t} - K\frac{\partial^2\varphi}{\partial z^2} + \tau\frac{\partial}{\partial t}\left(s\frac{\partial\varphi}{\partial t} - k\frac{\partial^2\varphi}{\partial z^2}\right) = 0$$

where $s = 10^{-6}\ m^{-1}$ is the specific elastic storage coefficient, φ is the potential (hydraulic head) in the fractures, $n = 0.1$ is the porosity of the intact rock (the primary porosity) and of the material filling the fractures, $N = 0.001$ is the flow effective porosity (the secondary porosity), $k = 0.5\ m/day$ is the vertical hydraulic conductivity of the fractures. K is the vertical composite conductivity of the fractured rock, $\tau = 0.01\ day$ is the pressure relaxation coefficient, t is the time, and z is the vertical spatial coordinate (positive in downward direction).

5. What is the magnitude of the hydraulic conductivity of the intact rock when the composite (upscaled) conductivity is given by $K = N\,k\,/\,n = 0.005\ m/day$?

The characteristic time scale over which we are interested in the pressure propagation phenomenon is denoted by Θ and the related characteristic length is $L = \sqrt{(k\Theta/s)}$. Using the above presented characteristic time and length we can define a dimensionless time $T = t/\Theta$ and a dimensionless vertical coordinate $Z = z/L$.

6. Write the above pressure propagation equation in the dimensionless space and time coordinates *Z* and *T*.

First we are interested in wave propagation phenomena on the relatively short time scale Θ of seconds to minutes after the sudden start of flow (the sudden start of pumping).

7a. What is then the order of magnitude of the dimensionless number Θ/τ ?

7b. Suppose now that this number is sufficiently small to assume $\Theta/\tau \to 0$ in the pressure propagation equation. This way the pressure equations simplifies considerably. Make a graph of the 'early time' pressure as a function of *z* for three different short times.
[Hint: use the fact that equations of the type

$$\frac{\partial f}{\partial t} = d\frac{\partial^2 f}{\partial z^2}$$

have an error function type solution with penetration depth $\delta(t) = 2\sqrt{(d\,t)}$.]

Now we are interested in wave propagation phenomena on relatively long time scales Θ of many hours to days after the start of pumping.

8a. What is then the order of magnitude of the dimensionless number τ/Θ ?

8b. Suppose that this number is sufficiently small to assume $\tau/\Theta \to 0$ in the pressure propagation equation. How relates the penetration velocity of this 'late time' pressure wave to the 'early time' pressure wave velocity ? (How much slower or faster does it penetrate?)

9. Describe in words the physics that explains why the pressure propagation velocity slows down after some time $t = \tau \approx 15$ *min*.

10. In books on flow through *porous* (non fractured) formations the one dimensional pressure propagation equation is generally given as

$$s\frac{\partial \varphi}{\partial t} - K\frac{\partial^2 \varphi}{\partial z^2} = 0$$

Do you think this is correct for non fractured porous media ? Why ?

Transport in fractured porous rocks

In the same fractured rock mass as above, transport may be considered as essentially one dimensional in the vertical direction. Initially a miscible toxic solute is dissolved in the water that fully saturates the rock's pores. One tries to remove the toxic solute from the shallow part of the rock by 'pushing' it deeper. This is done by injecting clean water from the top of the rock mass. The dissolved matter is conservative (no sorption, no decay). In this case the equation describing the propagation of a concentration front is given by the advection dispersion equation

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial z} \pm \frac{\alpha_L}{\eta w}\frac{\partial}{\partial t}\left(\frac{\partial c}{\partial t} + w\frac{\partial c}{\partial z}\right) = D\frac{\partial^2 c}{\partial z^2}$$

The plus sign holds if $w > 0$ and the minus sign holds if $w < 0$, where $w = q / N$ is the vertical flow velocity in the fractures; $v = q / n$ is the volume averaged flow velocity; n is the porosity of the intact rock and of the granular material in the fractures; and N is the flow effective porosity, which is located in the fluid conducting fractures; $\eta = (n - N) / n$. In this example the vertical flux $q = 0.01$ *m/day*; $n = 0.1$; $N = 0.001$; the diffusion coefficient $D \rightarrow 0$ (*i.e.*, $D << \alpha_L v$); the longitudinal dispersion length $\alpha_L = 10$ *m*. There is negligible flow in the intact rock.

First we consider the short time scale Θ where we may assume $\eta w \Theta / \alpha_L \rightarrow 0$. In this situation the advection dispersion equation simplifies to a simple advection equation.

11a. Calculate the velocity of the moving solute front under this early time condition.

11b. Prove that $c(z,t) = F(z - w t)$ is a solution of the early time advection equation, where $F(\xi)$ may be any differentiable function of ξ.

Let us now look at a large time scale Θ where we may assume $\alpha_L / (\eta w \Theta) \rightarrow 0$. Also in this situation the advection dispersion equation simplifies to a simple advection equation.

12a. Calculate the velocity of this late time moving solute front.

12b. Prove that $c(z,t) = F(z - v t)$ is a solution of the late time advection equation.

13. Describe in words the physics that explains why the early time front propagation velocity decreases after some time $t = \alpha_L / (\eta w) \approx 1$ *day*.

Let us now limit the discussion to the *late time behavior*.

Until now we have neglected dispersion. Therefore, we want to investigate where and when it is allowed to neglect dispersion and where and when it is essential to take dispersion into account.
Assume that $F(\xi) = C_0\, U(\xi)$, where $U(\xi)$ is the unit step function, *i.e.*,
$U(\xi) = 0$ for $\xi < 0$, and $U(\xi) = 1$ for $\xi > 0$; $C_0 = 1.0\ \mu g/L$ is the initial concentration. (Note that the vertical coordinate z is positive in downward direction.)

14a. Show that dispersion may be neglected almost everywhere, except in the neighborhood of the moving front $z = v\,t$.
[Hint: substitute the solution $c(z,t) = C_0\, U\,(z - v\,t)$ into the dispersion term,]

14b. What do you expect the dispersion will do near $z = v\,t$? [No calculation, just a qualitative answer.] Make a picture of the concentration profile near $z = v\,t$ without and with dispersion.

In text books on geohydrology in *porous* (non fractured) formations the one dimensional advection dispersion equation is generally given in the following form

$$\frac{\partial c}{\partial t} + v\frac{\partial c}{\partial z} = (D \pm \alpha_L v)\frac{\partial^2 c}{\partial z^2}$$

where the plus sign holds if $v > 0$ and the minus sign holds if $v < 0$.

15a. Show that when the solution $c(z,t) = F(z - v\,t)$ is a good approximation (*i.e.*, if $c(z,t) = F(z - v\,t)$ + a negligibly small function $\varepsilon(z,t)$), the two dispersion terms (the one for transport in *porous* media and the one for transport in *fractured* porous media) are approximately equal.

15b. Mention a condition under which you may expect large differences between the solutions of the two equations, and describe a practical application where this condition applies. Which of the two solutions will give the most realistic results ?