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TA3390

Physicall Processing
Separation

Dr ir Rénard Chaigneau

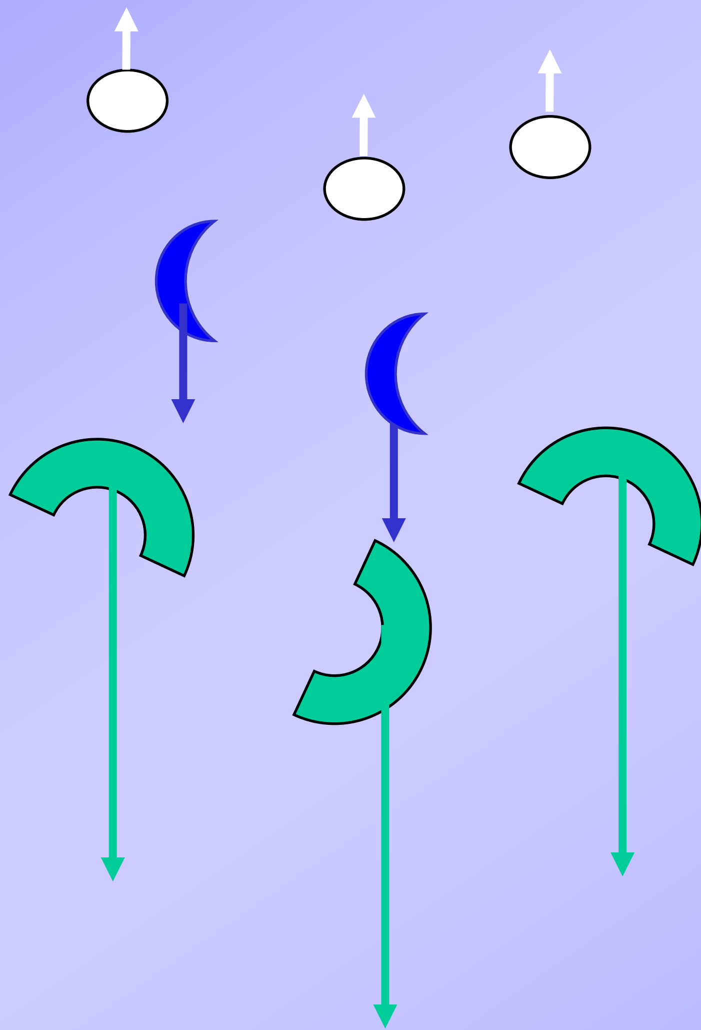
November 2012

Mineral Processing Part B: Separation

Subjects:

- Two-phase systems
- Sedimentation
- Fluidization
- Filtration
- Drying
- Cyclones
- Magnetic separation

Is a bubble different from a solid particle?



Shape: bubbles deform if larger than 2 mm.

"Stick condition" at the interface?

Bubble in a pure liquid: no.

Bubble in a process: yes!

Surface-active materials collect at the interface and create a "solid" interface

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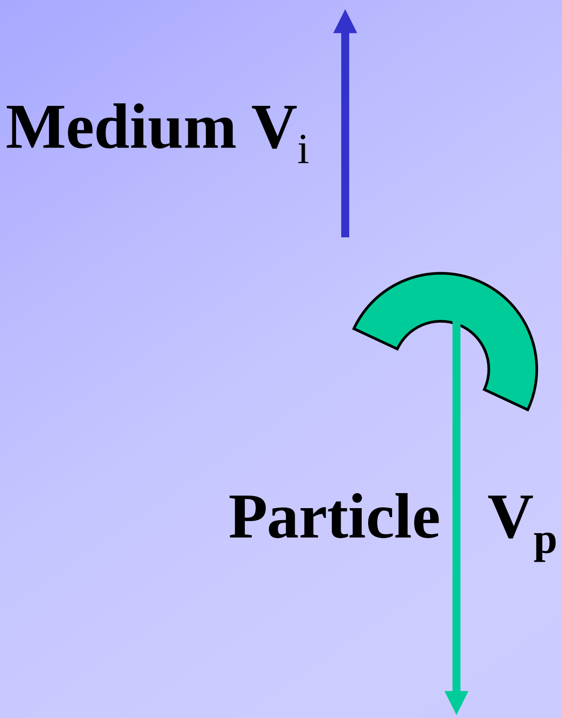
Slip velocity



Slip velocity: $V_s = V_p - V_i$

Depends on:

Particle size d_p

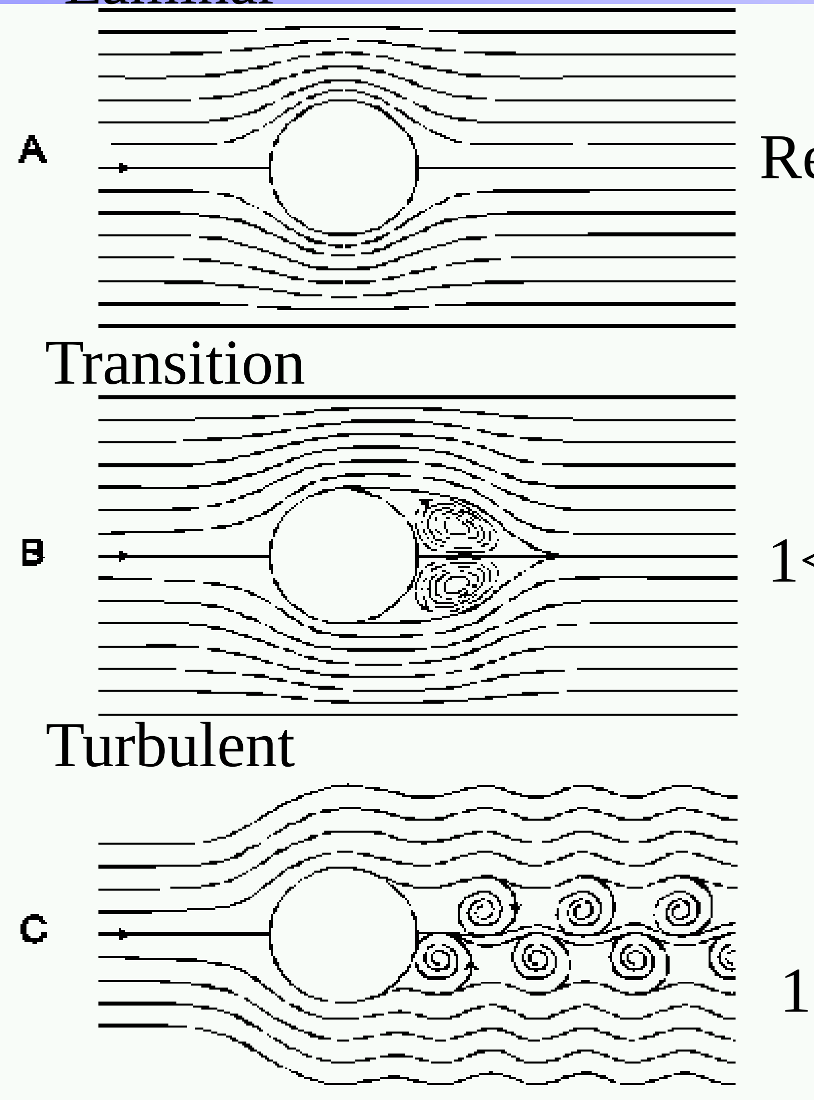
Density difference $\Delta\rho = \gamma(\text{part.}) - \rho(\text{medium})$

Medium properties $\rho(\text{medium}), \eta(\text{medium})$

Particle shape Ψ

Single particle in a fluid

Laminar



Re<1

$1 < Re < 10^3$

$10^3 < Re < 10^5$

$$Re = \frac{v \; d_p \, \rho}{\eta}$$

Reynolds number

$$\text{Drag force} = C_w(\text{Re, shape}) A_\perp \frac{\rho v^2}{2}$$

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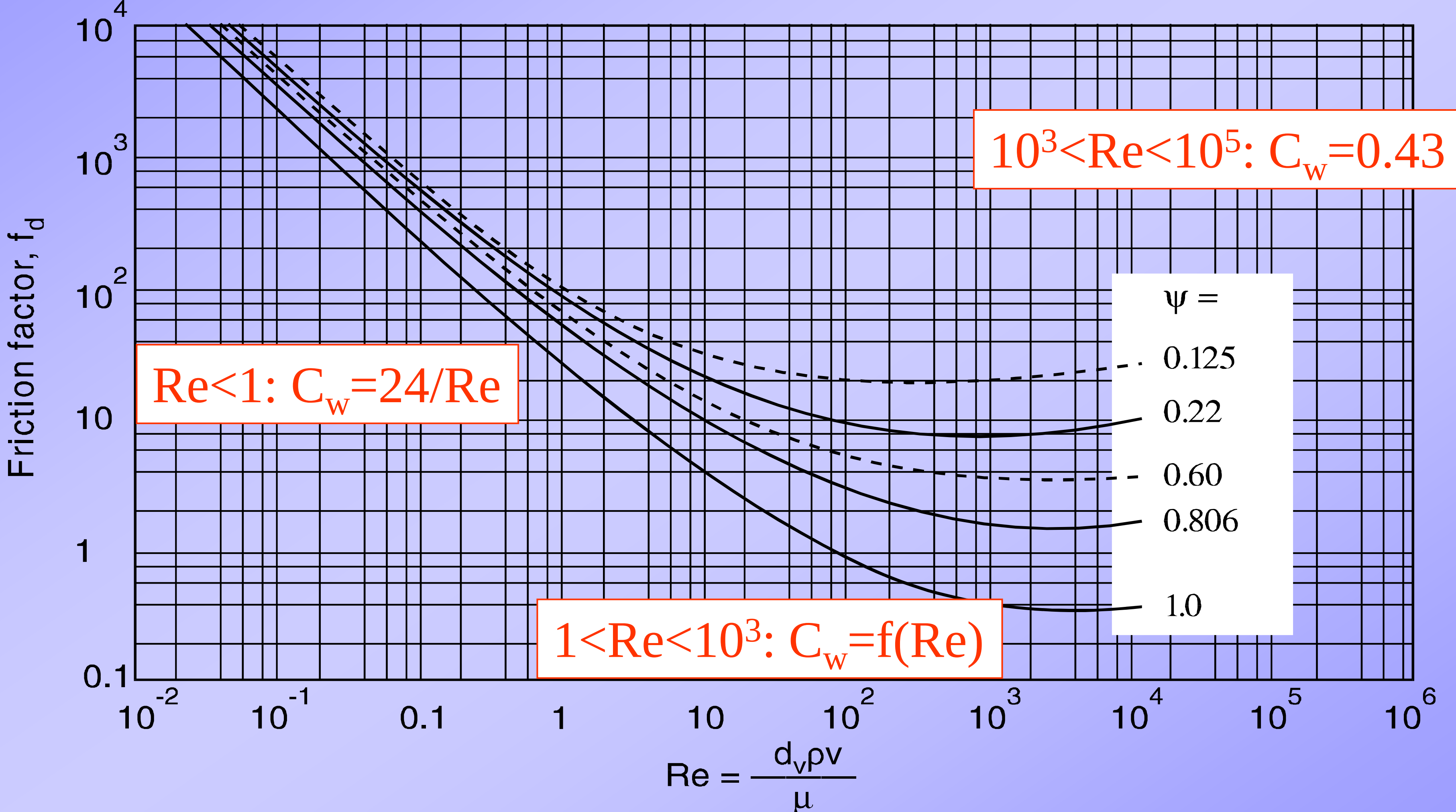
Drag coefficient



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Settling velocity of a single sphere

Force balance:

$$F_G^{/} - F_W = 0$$

$$\frac{\pi}{6} d^3 (\gamma - \rho) g - \frac{\pi}{4} d^2 C_w(\mathrm{Re}) \rho \frac{v_s^2}{2} = 0$$

$$v_s = \sqrt{\frac{4}{3} g \frac{d(\gamma - \rho)}{C_{w(\mathrm{Re})} \rho}}$$

Laminar:

$$v_s = \frac{(\gamma - \rho) d^2 g}{18\eta}$$

Transition:

$$C_W = \frac{18.5}{\mathrm{Re}^{0.6}}$$

Turbulent:

$$v_s = \sqrt{\frac{3gd(\gamma - \rho)}{\rho}}$$

Calculation of Settling velocity

General calculation procedure (single particle):

1. Estimate C_W with graph Re-C_w
2. Calculate v_s with estimated C_W
3. Calculate Re with this v_s
4. Determine new C_W with graph
5. Calculate new v_s with this C_W
6. Repeat 3 t/m 5 until you have sufficient accuracy

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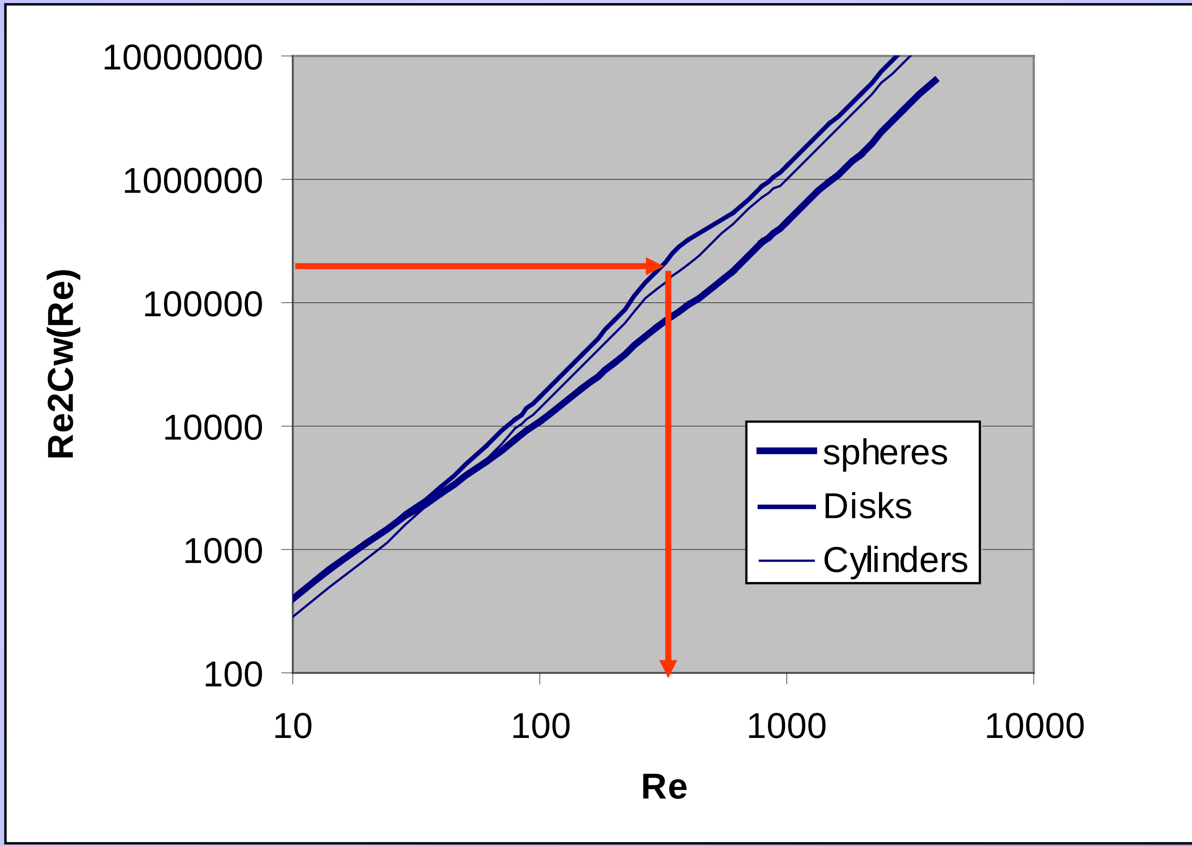
Calculation of Settling velocity

Alternative Procedure:

1. Compute $Re^2 C_W(Re)$

$$(\gamma-\rho)Vg = A_\perp C_w(\mathrm{Re})\rho\frac{v_s^2}{2}$$

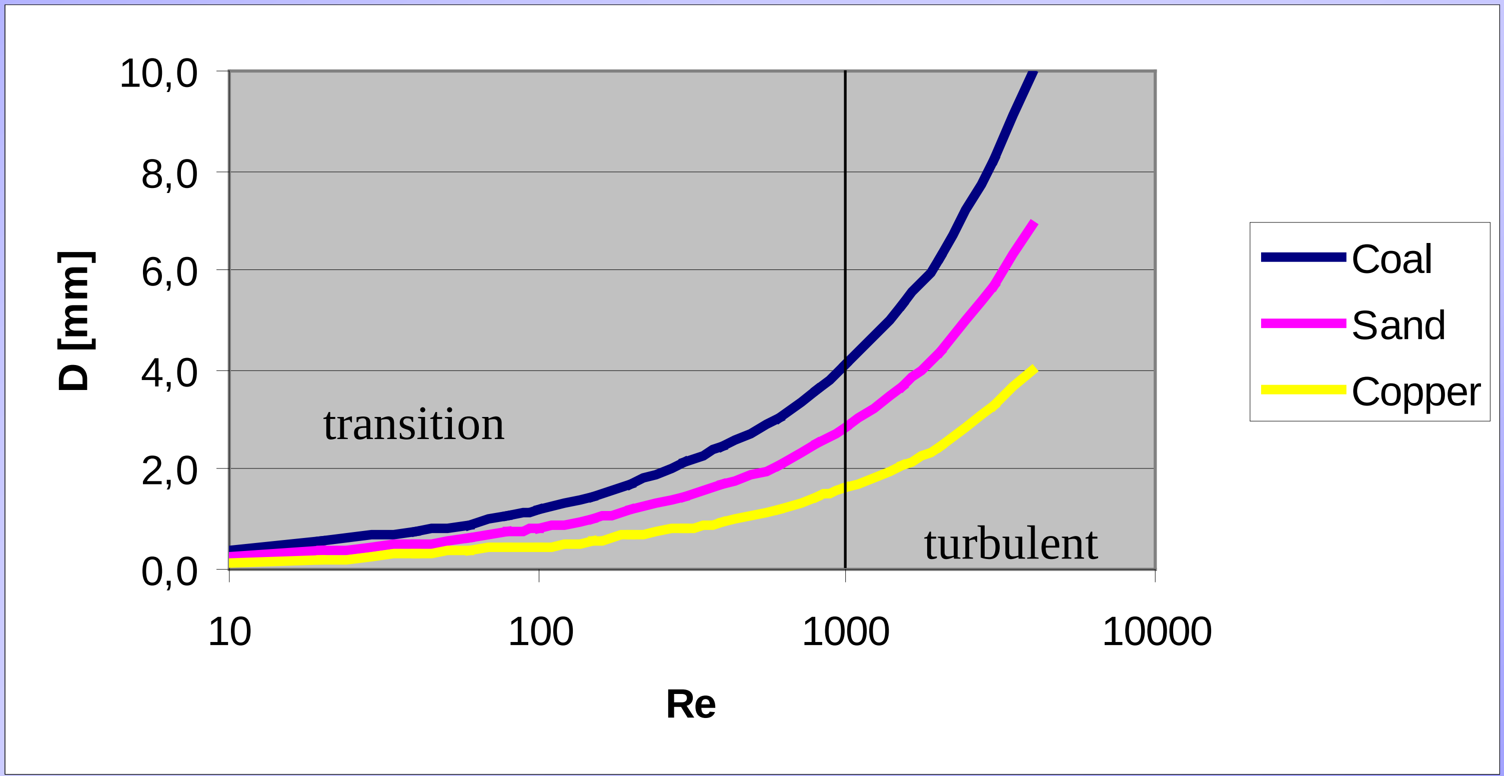
$$\frac{2(\gamma-\rho)\rho V g d_p^2}{A_\perp \eta^2} = \mathrm{Re}^2\, C_w(\mathrm{Re})$$



4. Determine Re with graph
5. Calculate v_s with this Re

Settling velocity: examples

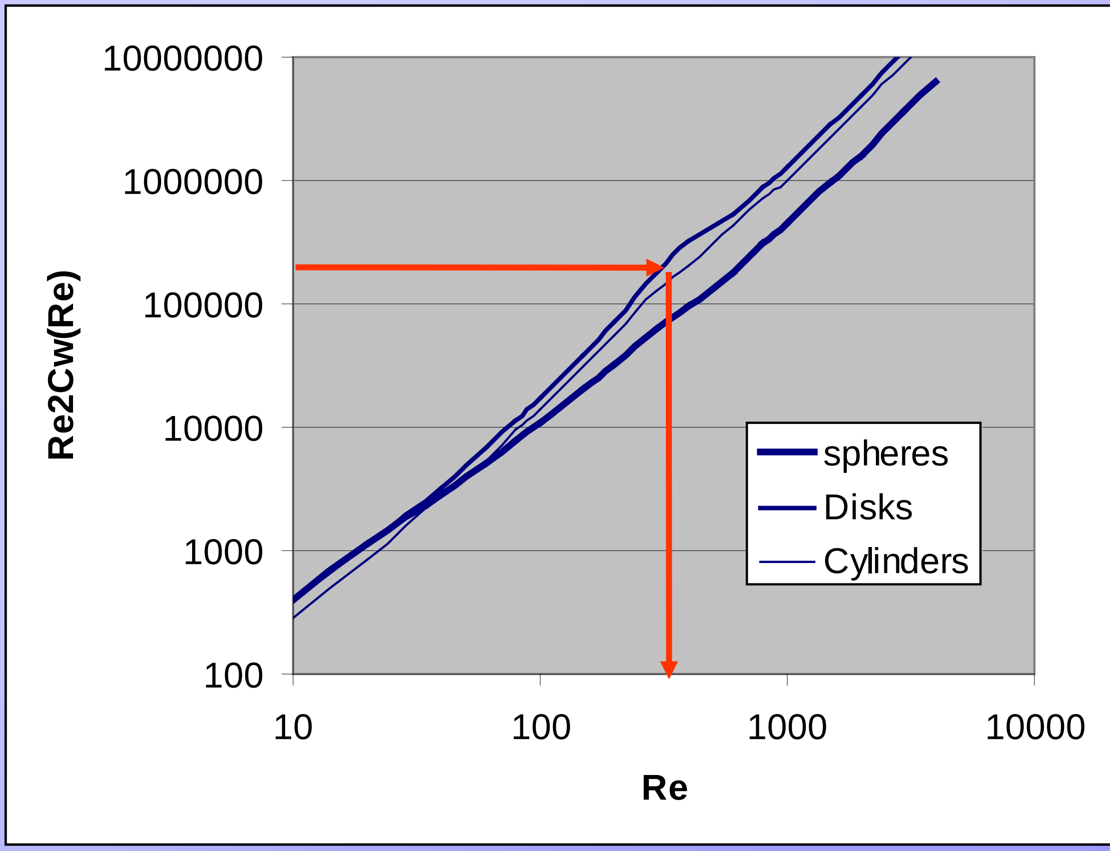
Spherical particles of coal, sand and copper in water



Settling velocity: example

An approximately spherical particle of diameter 0.1 mm and density 2600 kg/m^3 falls in oil with a density of 900 kg/m^3 and viscosity of 0.003 Ns/m^2. Calculate the slip velocity (terminal velocity) of the particle.

$$\frac{2(\gamma-\rho)\rho V g d_p^2}{A_\perp \eta^2} = \mathrm{Re}^2\, C_w(\mathrm{Re})$$



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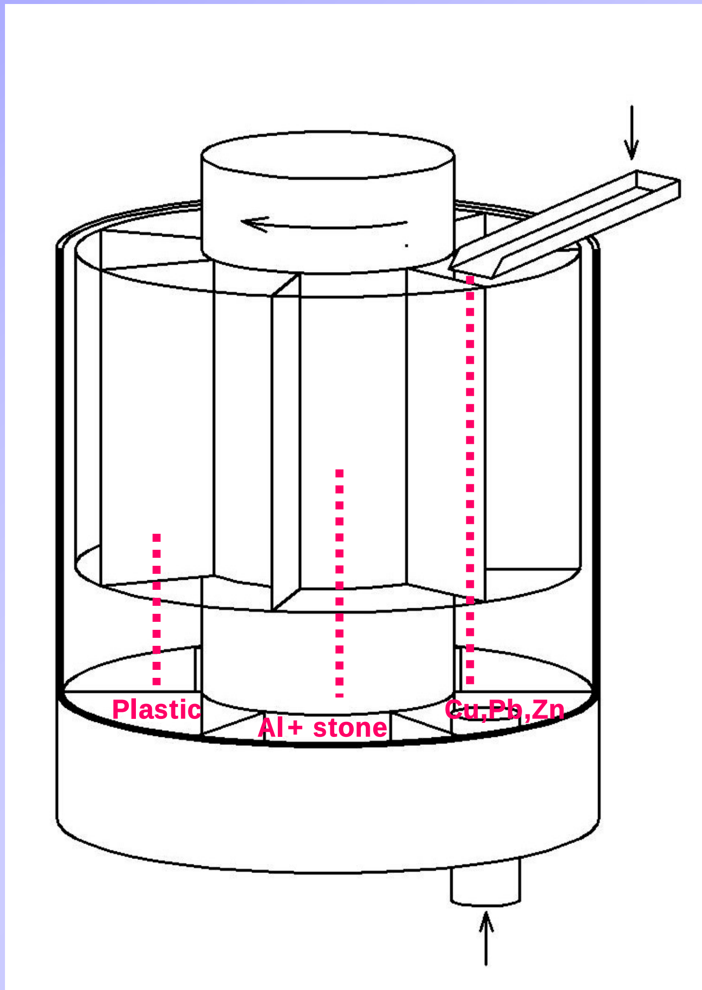
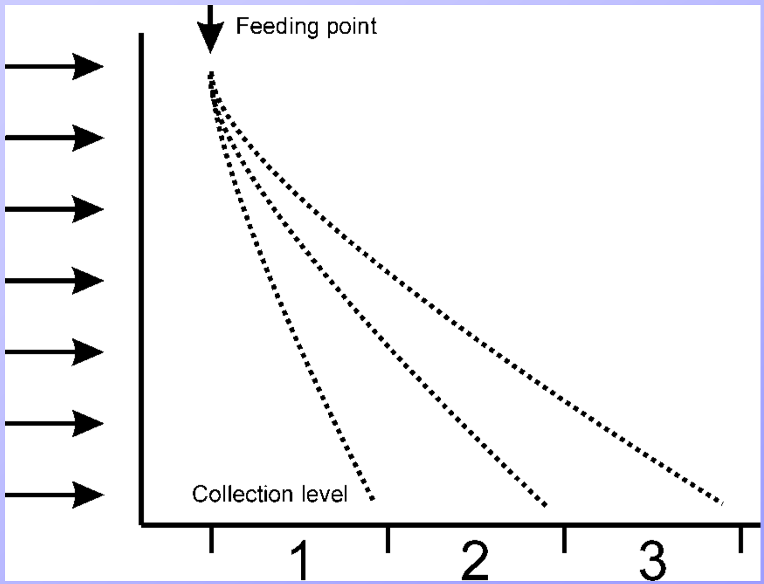
Kinetic gravity separator



Input - stones, sinter, glass, heavy non ferrous metals, light non ferrous metals, organic fraction

Output - organic fraction
- aluminium and stone fraction
- heavy non ferrous metals (Cu, Zn, Pb)



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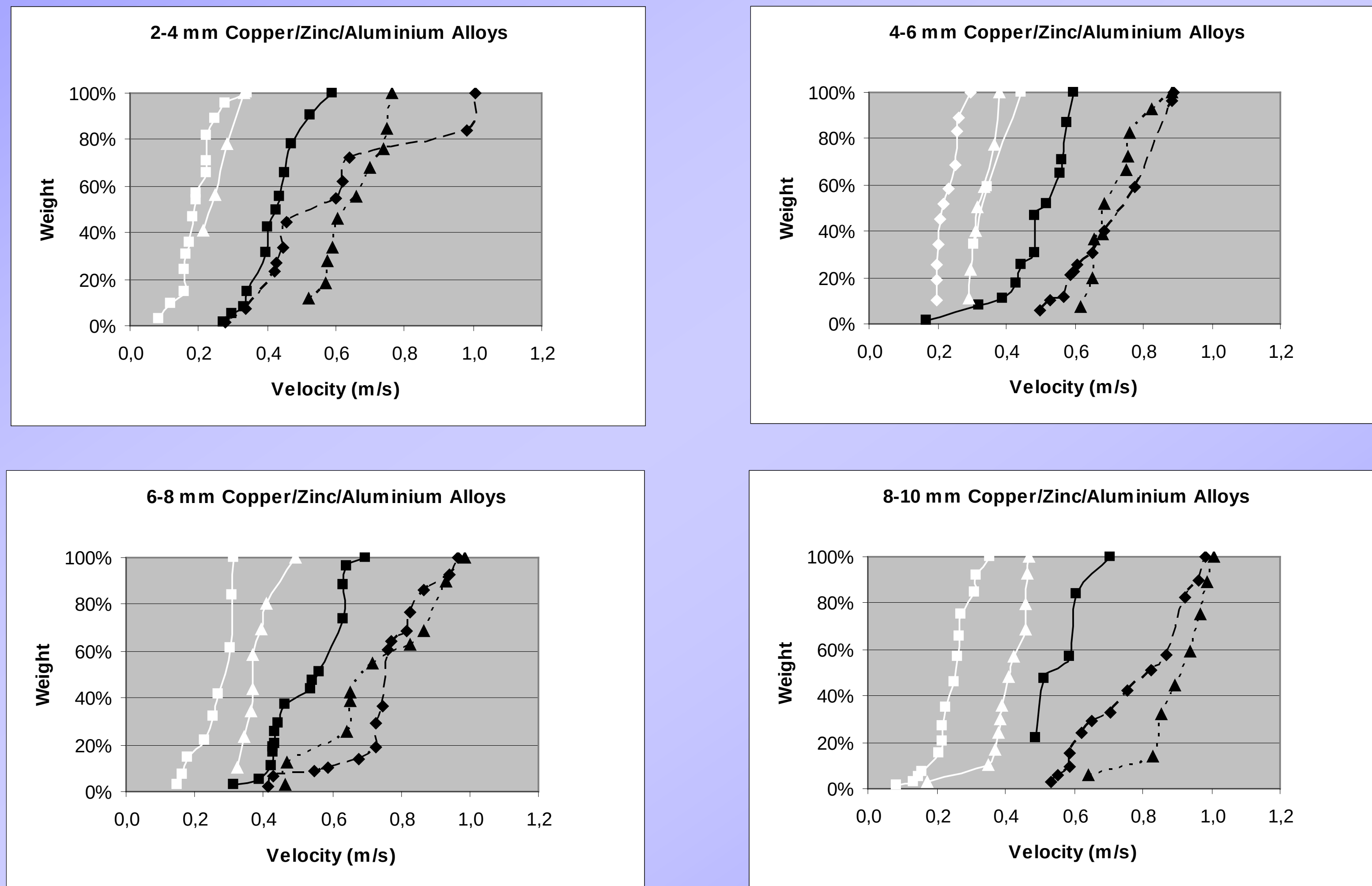
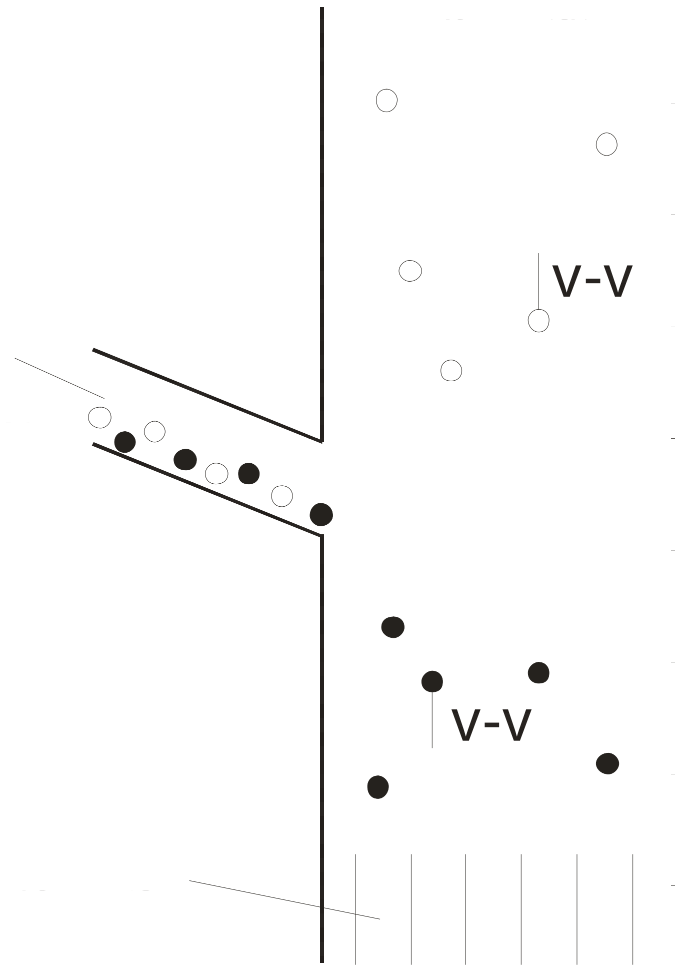
Kinetic gravity separator



Non-ferrous metals 2-10 mm

Rising Current Separation



Principle of separation

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Battery processing
Braubach, Germany

Definition of velocities

ν_p : Particle velocity with respect to the wall of the system.

u : Superficial fluid velocity w.r.t. the wall of the system, i.e., the fluid velocity without particles present.

ν_i : Interstitial fluid velocity w.r.t. the wall of the system, i.e. the average velocity of the fluid between the particles.

$u_\nu(\varepsilon)$: Velocity of a swarm of particles with porosity ε in absence of a superficial fluid velocity -> sedimentation velocity.

$\nu_s = \nu_p - \nu_i$: Slip velocity, i.e., the relative velocity between particle and fluid. ← This is the starting point of calculations.

Note: v_s is often used to indicate the superficial velocity.

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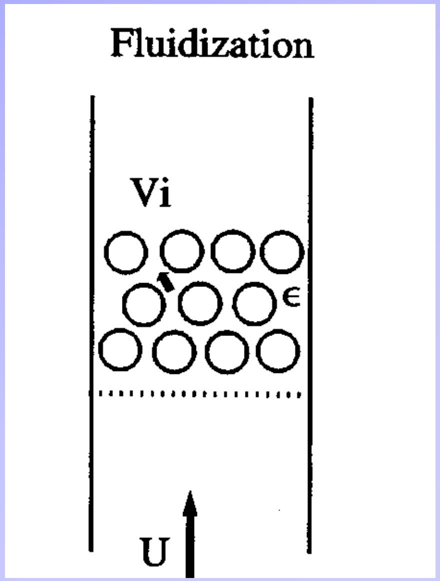
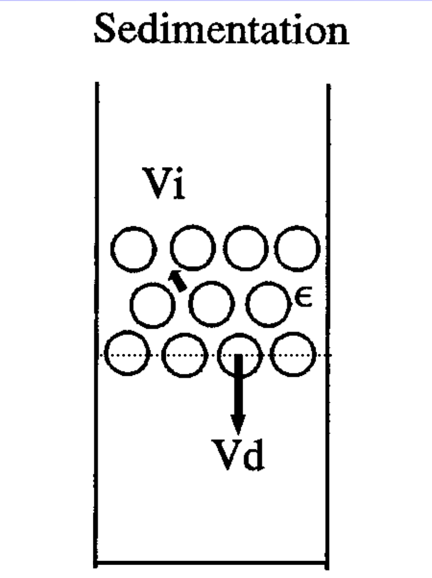
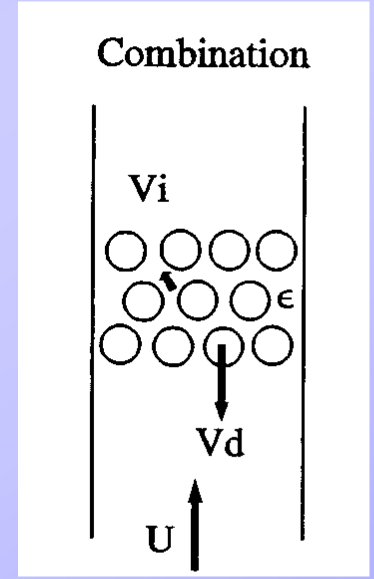
Relation between slip velocity and others

Down velocities are defined as positive!







$$v_p = 0$$

$$v_i = -v_s$$

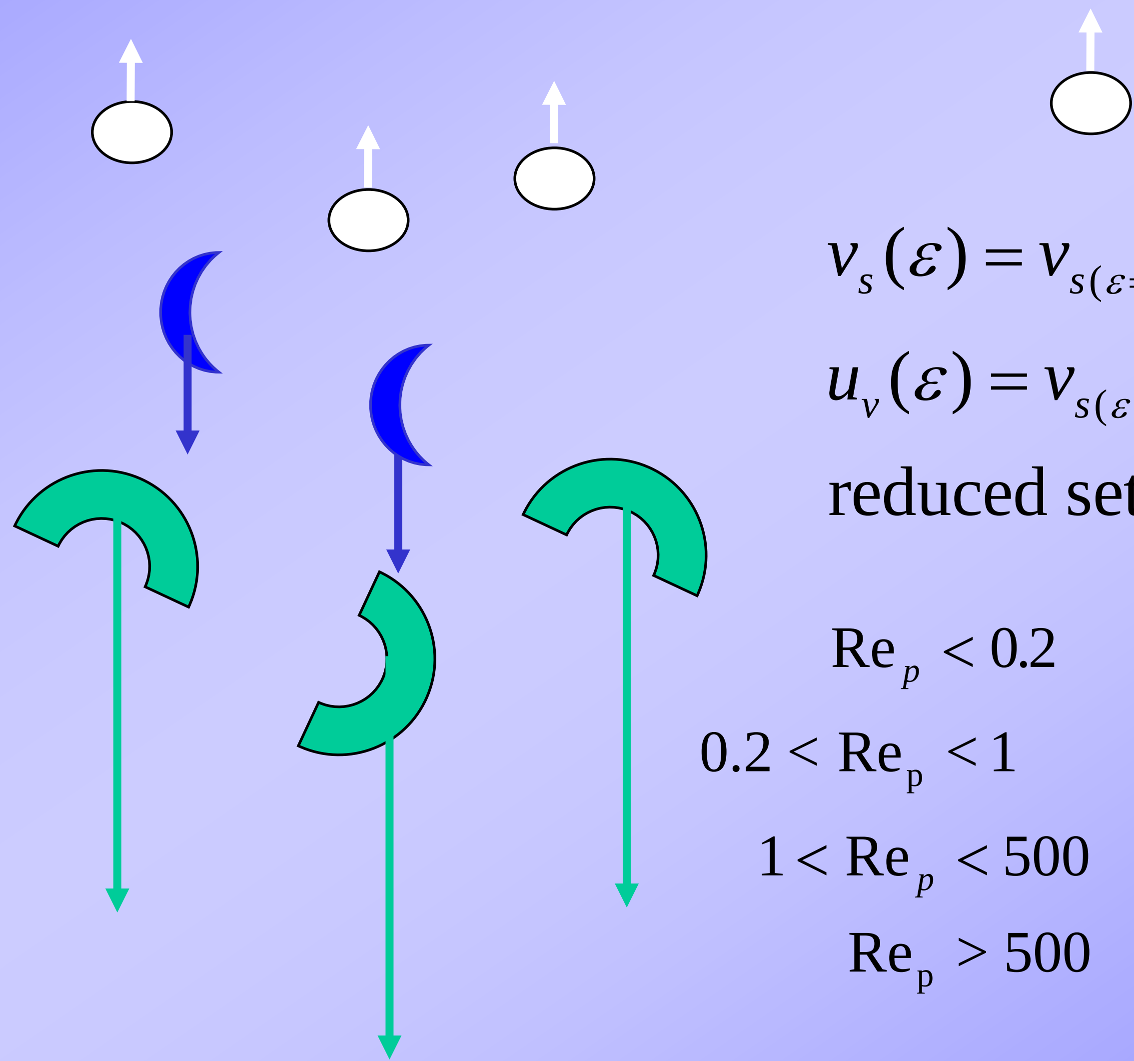
$$u = -\varepsilon v_s$$

$$(1-\varepsilon)v_p + \varepsilon v_i = 0$$

$$v_p = \varepsilon v_s$$

$$u_v = v_p = \varepsilon v_s$$

Many particles: Richardson & Zaki



$$v_s(\varepsilon) = v_{s(\varepsilon=1)} \cdot \varepsilon^{n-1}$$

$$u_v(\varepsilon) = v_{s(\varepsilon=1)} \cdot \varepsilon^{n}$$

reduced settling velocity!!

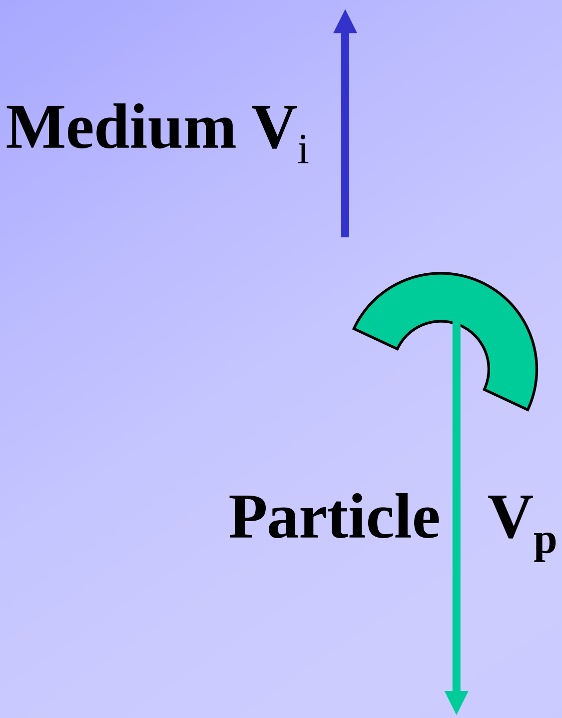
$Re_p < 0.2$	$n = 4.65$
$0.2 < Re_p < 1$	$n = 4.35\ Re_p^{-0.03}$
$1 < Re_p < 500$	$n = 4.45\ Re_p^{-0.1}$
$Re_p > 500$	$n = 2.39$

Settling velocity at high solids concentrations

Explanation of *n* in: $u_v(\varepsilon) = v_{s(\varepsilon=1)} \cdot \varepsilon^n$

1. If solids go down, medium must go up: ε^1
2. Solids contribute to density of medium: $\varepsilon^{(0.5\text{-}1)}$
3. Increased shear: $\varepsilon^{(n-2)}$

Settling velocity and slip velocity



Slip velocity:

$$V_s = V_p - V_i$$

Depends on:

Particle size d_p

Density difference $\Delta\rho = \gamma(\text{part.}) - \rho(\text{medium})$

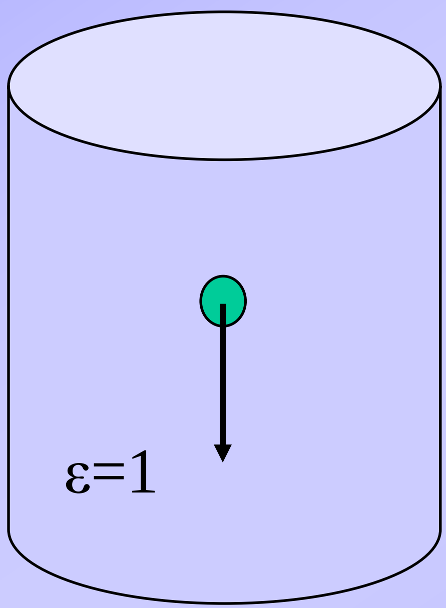
Medium properties $\rho(\text{medium}), \eta(\text{medium})$

Particle shape Ψ

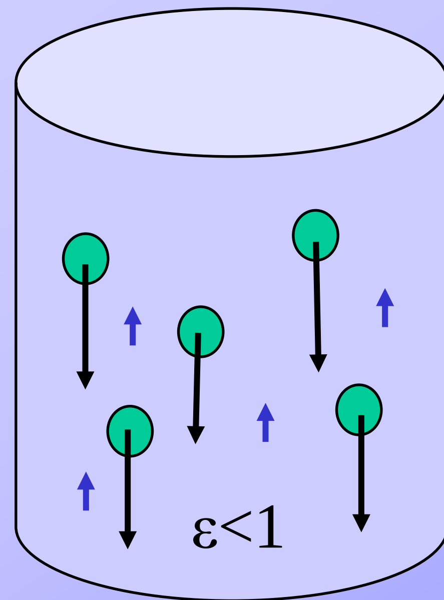
High solids concentrations: Hindered settling

If solids go down, medium must go up: ε^1 :

Slip velocity v_s and particle velocity $u_v = v_p$ are the same



Settling velocity $u_v = \varepsilon v_s(\varepsilon)$: smaller than slip velocity



$$\varepsilon v_i = -(1-\varepsilon) v_p$$

$$v_p = \varepsilon v_p + (1-\varepsilon) v_p$$

$$= \varepsilon(v_p - v_i) = \varepsilon v_s(\varepsilon)$$

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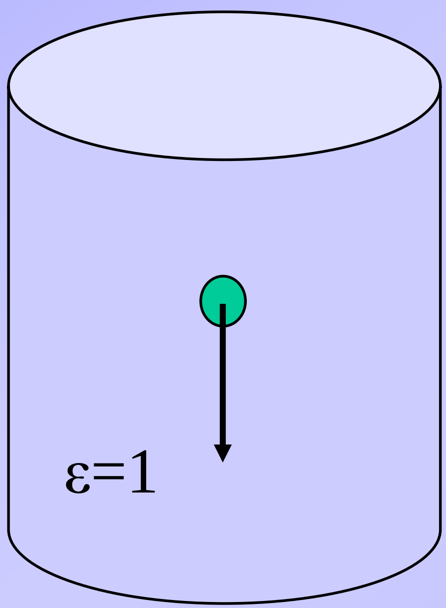
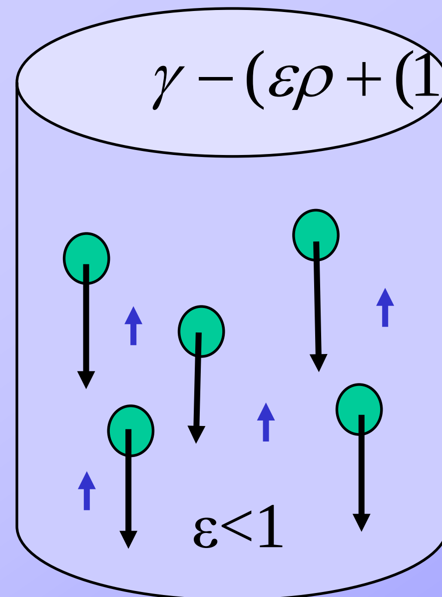
High solids concentrations: Hindered settling

Solids contribute to density of medium: $\varepsilon^{(0.5\text{-}1)}$

Density of medium is ρ, so differential density is (γ-ρ)



Density of medium is ερ+(1-ε)γ, so differential density is ε(γ-ρ)



$$\gamma - (\varepsilon\rho + (1-\varepsilon)\gamma) = \varepsilon(\gamma - \rho)$$

Laminar:

$$v_s = \frac{(\gamma - \rho) d^2 g}{18\eta}$$

Turbulent:

$$v_s = \sqrt{\frac{3gd(\gamma - \rho)}{\rho}}$$

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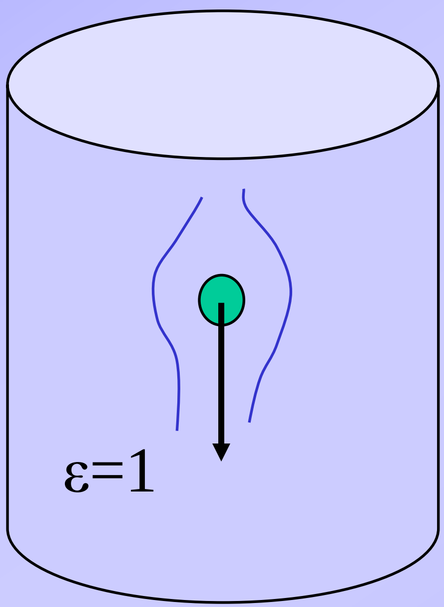
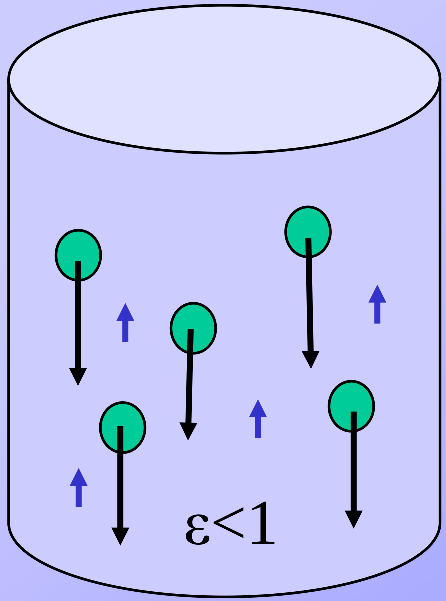
High solids concentrations: Hindered settling

Increased shear: $\varepsilon^{(n-2)}$

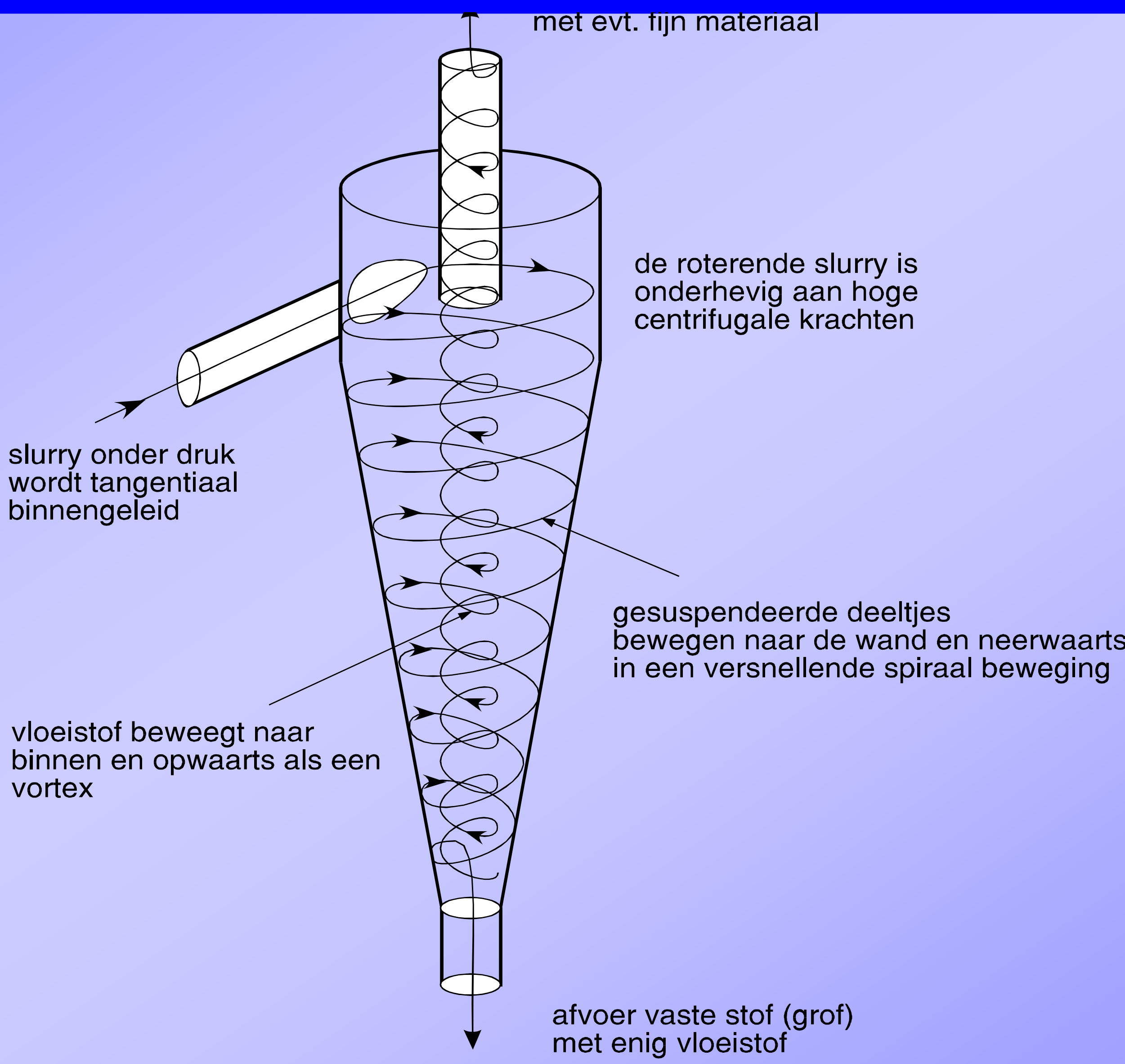
Much room for shear



Room for shear bounded by neighboring particles



Cyclone


met evt. fijn materiaal
de roterende slurry is
onderhevig aan hoge
centrifugale krachten
slurry onder druk
wordt tangentiaal
binnengeleid
gesuspendeerde deeltjes
bewegen naar de wand en neerwaarts
in een versnellende spiraal beweging
vloeistof beweegt naar
binnen en opwaarts als een
vortex
afvoer vaste stof (grof)
met enig vloeistof

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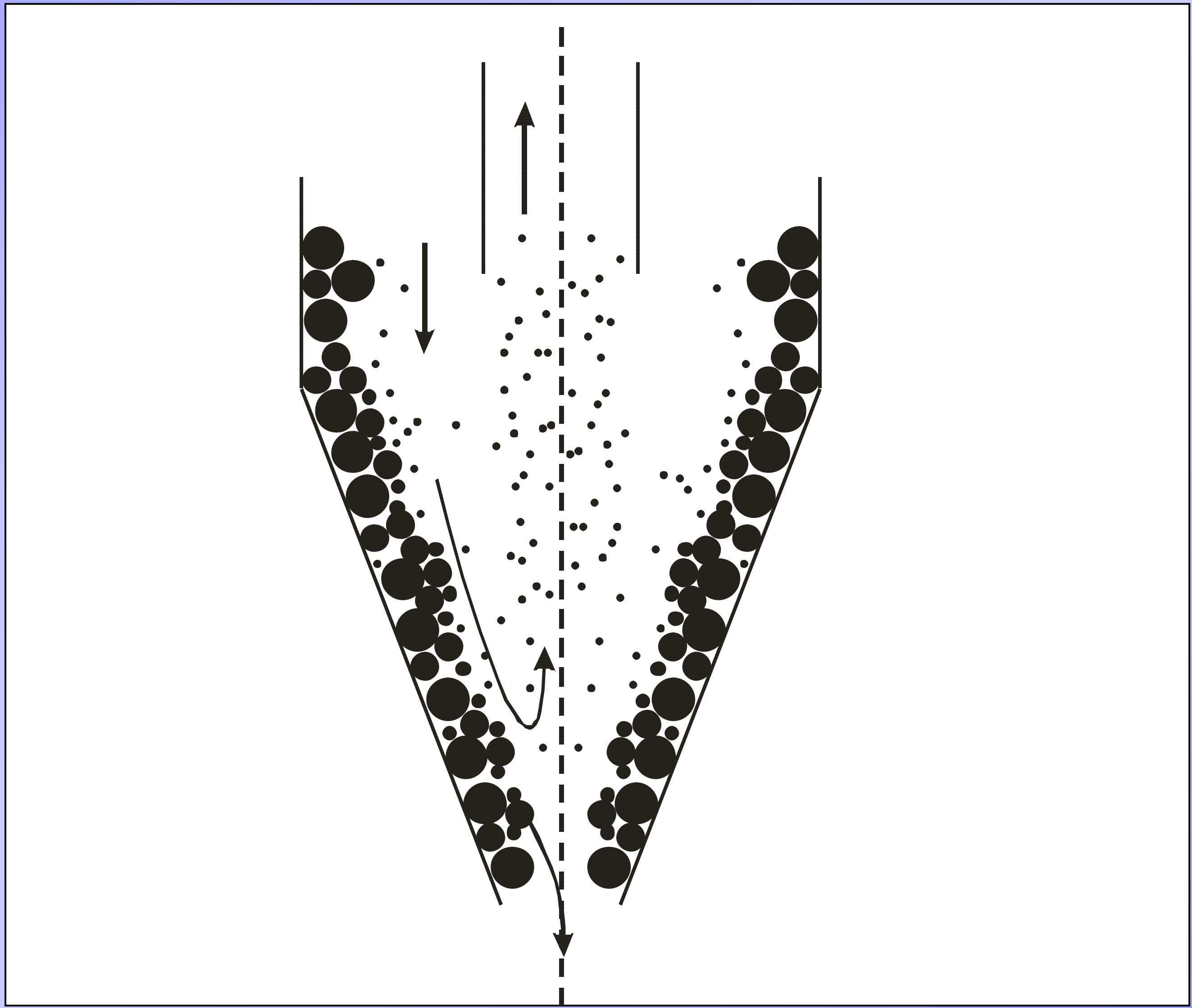
Separation in Hydrocyclone



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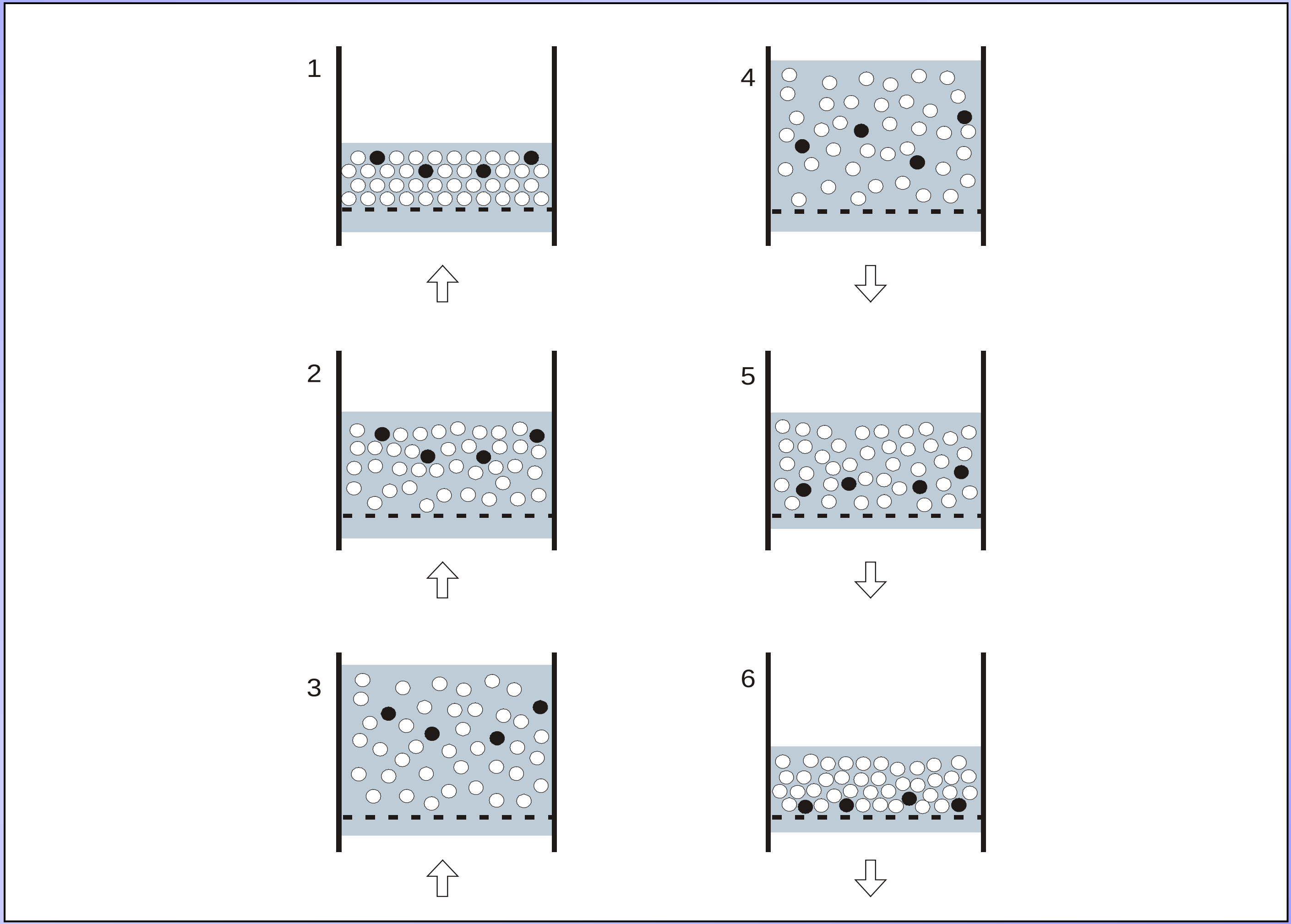
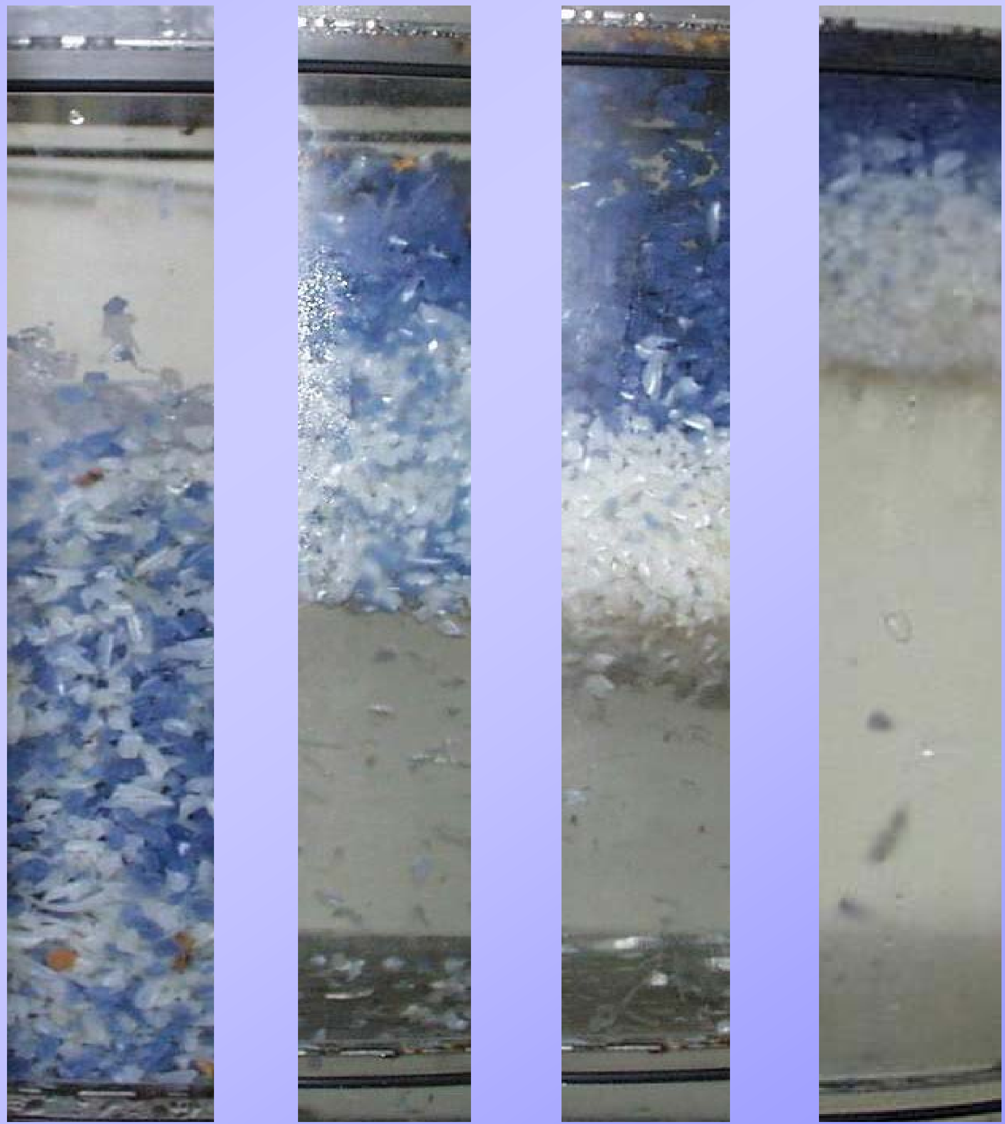
Jigging

Jigging is based on a segregation of particles due to periodical fluidisation (e.g. in an oscillating water flow)



Jigging

Segregation by periodical fluidisation (e.g. in an oscillating water flow)



Jigging of thick PE-PP flakes

Concentration criterion

Taggart, 1956

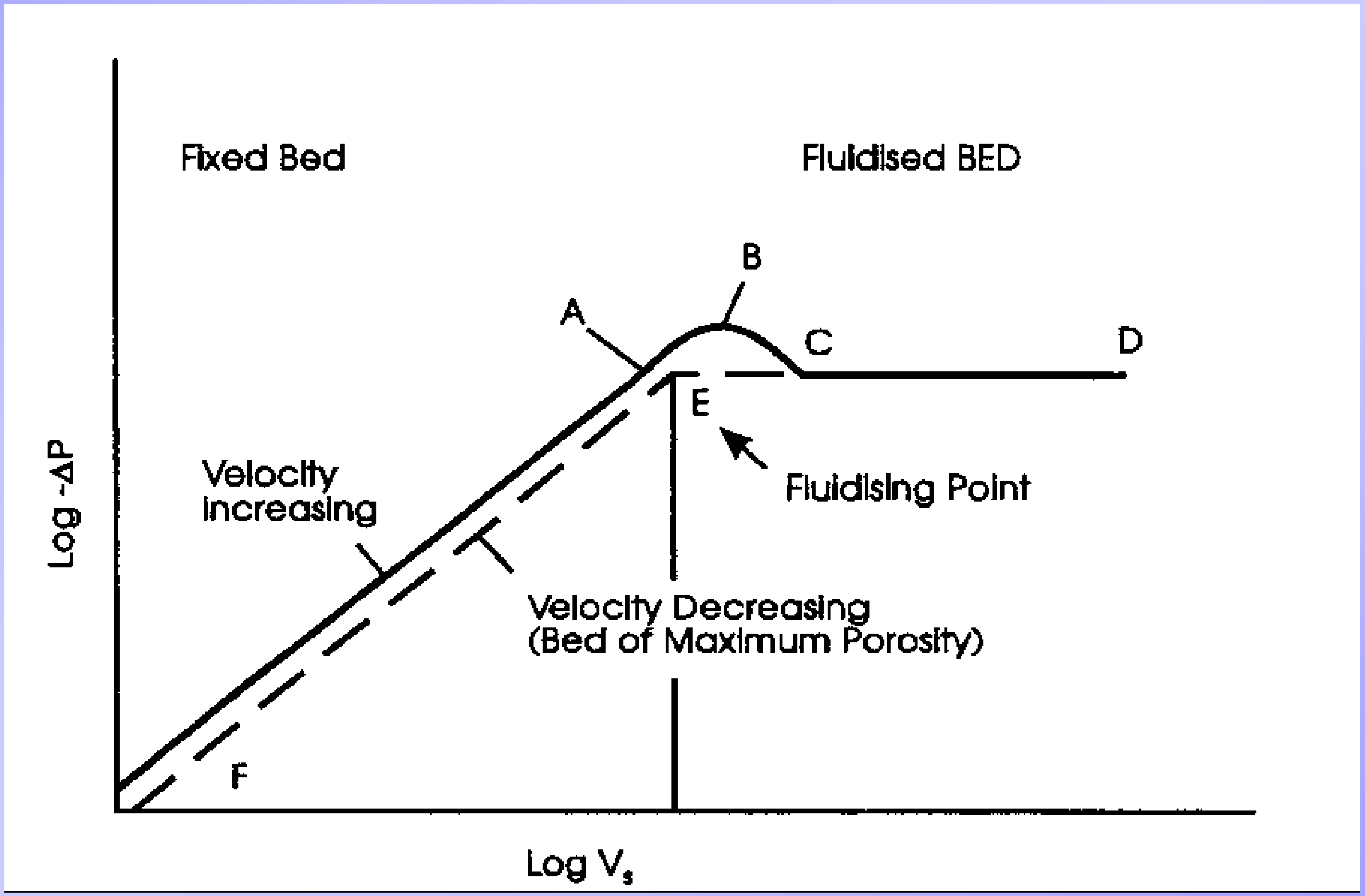
Density heavy — Density light

$$(\Gamma_h - \rho) / (\Gamma_l - \rho)$$

Density medium

If > 2.5: always separation possible, down to finest sands
1.5: sand sizes only
1.25: gravel sizes only
<1.25 very difficult if possible at all

Fluidized beds and fixed beds



Fixed beds: Ergun formula

$$\frac{\Delta P}{L} = \frac{1-\varepsilon}{\varepsilon^3 d_{vs}} \left(\frac{150(1-\varepsilon)\mu u}{d_{vs}} + 1.75 \rho u^2 \right)$$

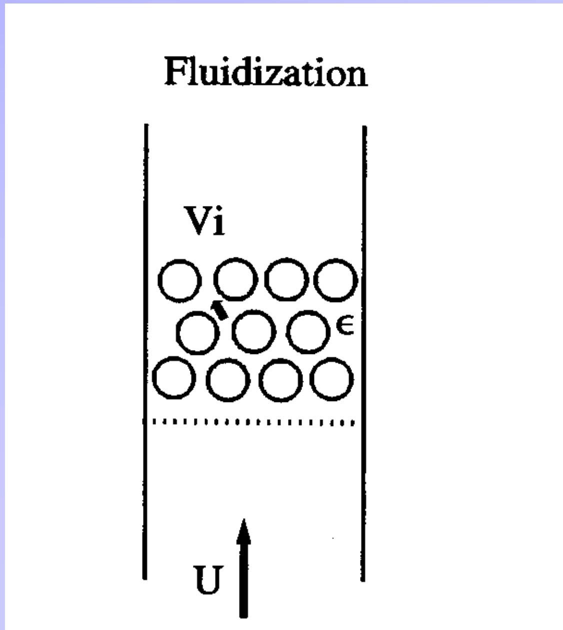
u = superficial velocity
μ = fluid viscosity
d_{vs} = the volume/surface diameter of a particle
L = height of the bed

$$\frac{\Delta P}{L} = \frac{150(1-\varepsilon)^2 \mu u}{\varepsilon^3 d_{vs}^2}$$

Laminar case

(Carman-Kozeny equation)

Fluidization point of a fixed bed



$$\frac{\Delta P}{L} = (1-\varepsilon)(\gamma-\rho)g$$

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Fluidized beds

Two different approaches:

1. Combination of Ergun/Carman Kozeny and $\frac{\Delta P}{L} = (1-\varepsilon)(\gamma-\rho)g$
2. Directly use:

$$u = u_v(\varepsilon) = v_{s(\varepsilon=1)} \cdot \varepsilon^n$$

Example:

Oil with a density of 900 kg/m^3 and viscosity of 0.003 Ns/m^2 passes vertically upward through a bed of catalyst consisting of approx. spherical particles of diameter 0.1 mm and density 2600 kg/m^3. At what mass flow rate per unit area of bed will fluidization occur according to Carman-Kozeny $\frac{\Delta P}{L} = \frac{150(1-\varepsilon)^2 \mu u}{\varepsilon^3 d_{vs}^2}$ versus Richardson and Zaki?

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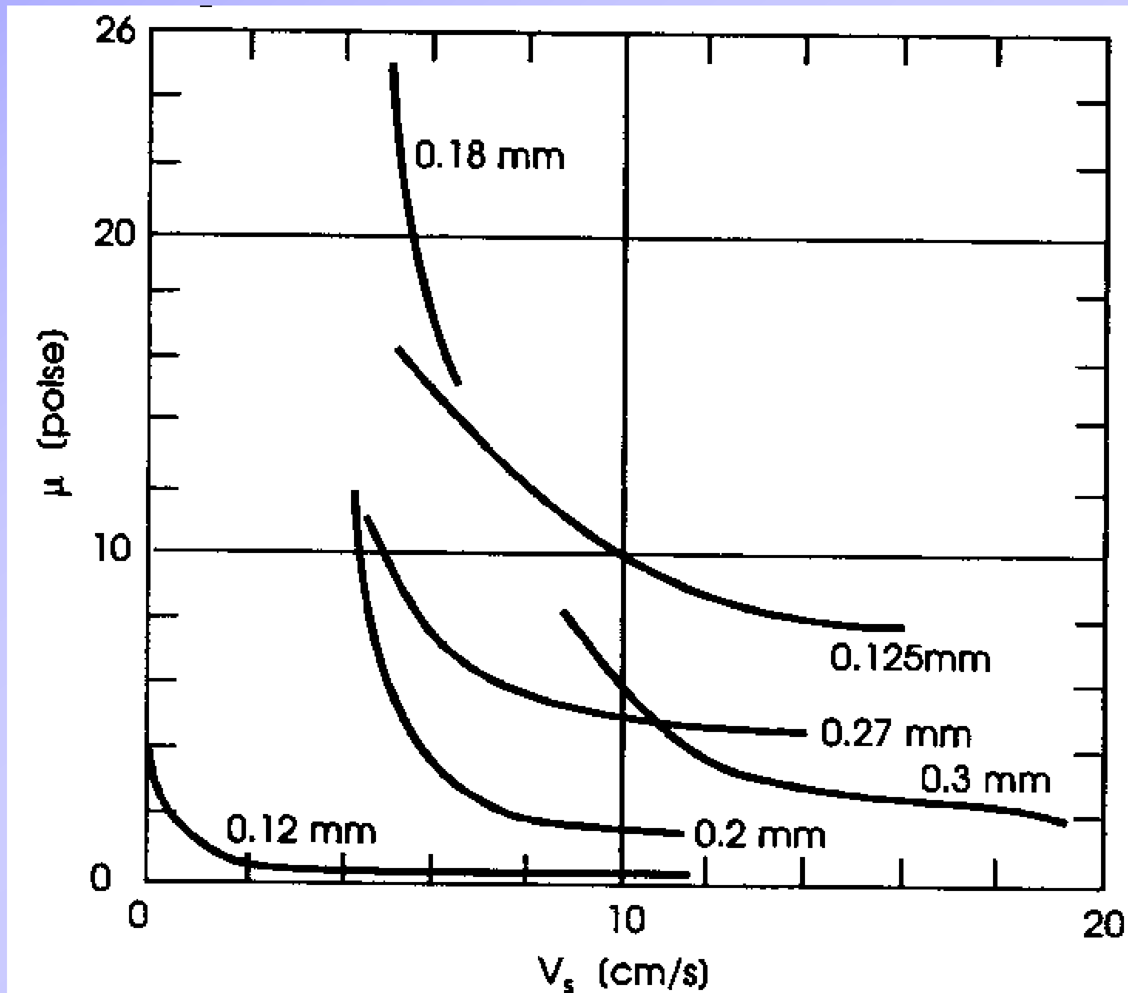
Viscosity of fluidized beds

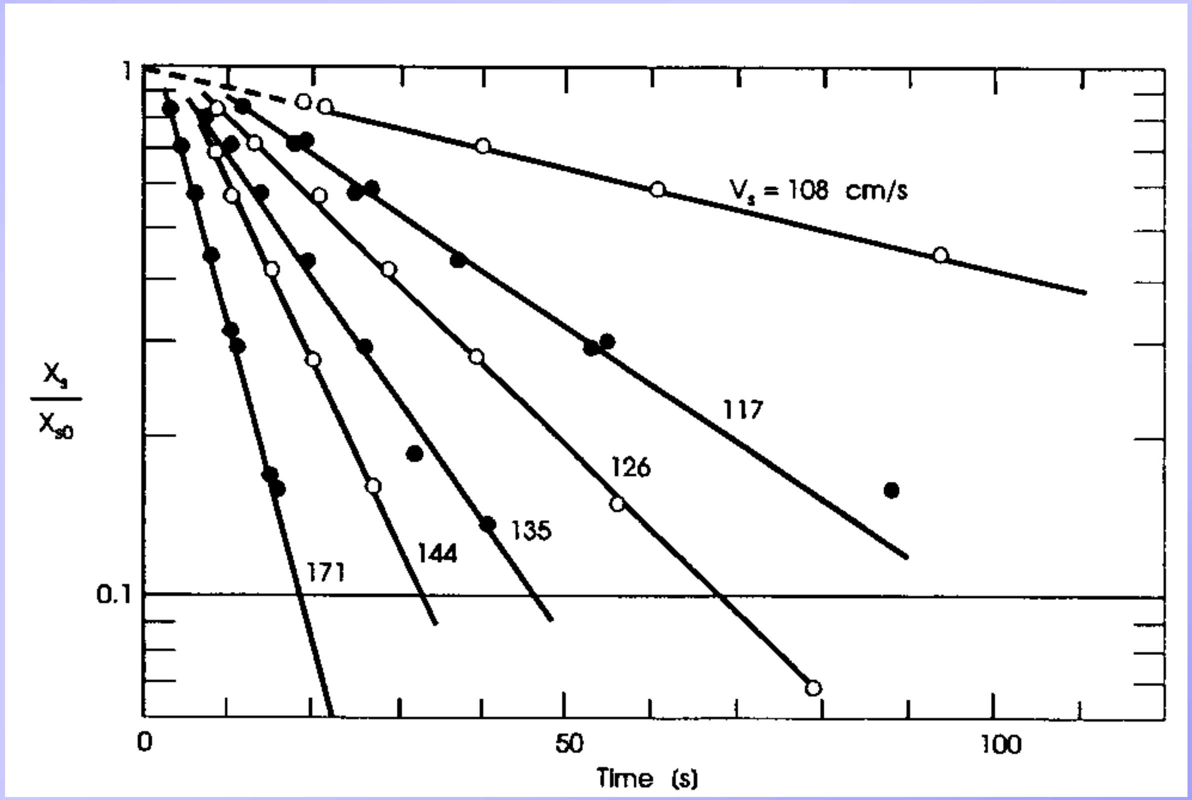
1 Poise = 0.1 Ns/m^2



Elutration in fluidized beds

$$X_s = X_{s0} e^{-k_e t}$$

Amount of fines left in the bed with lower terminal velocity than fluidization velocity



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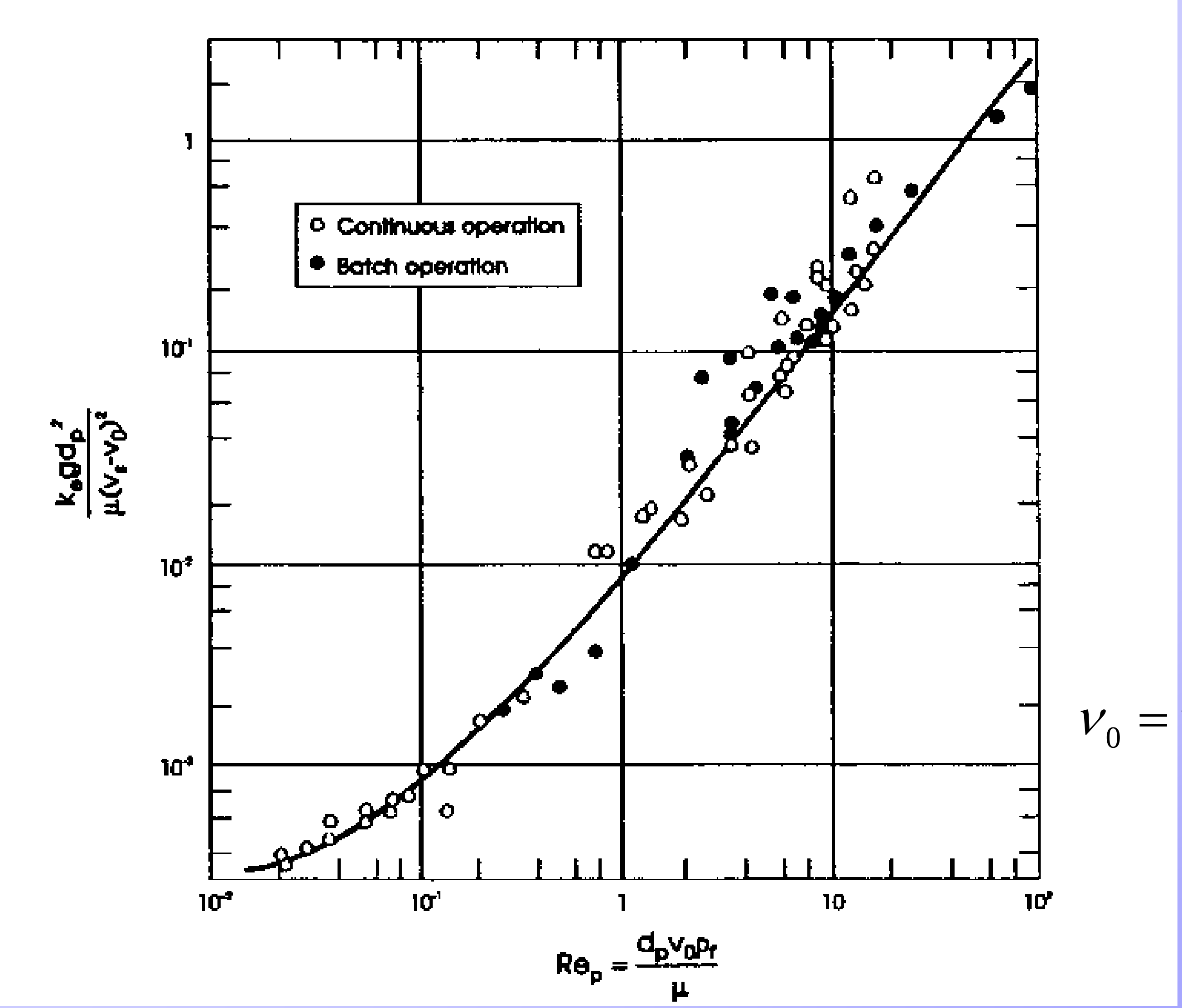
Elutration in fluidized beds



v_0 = terminal velocity of fines

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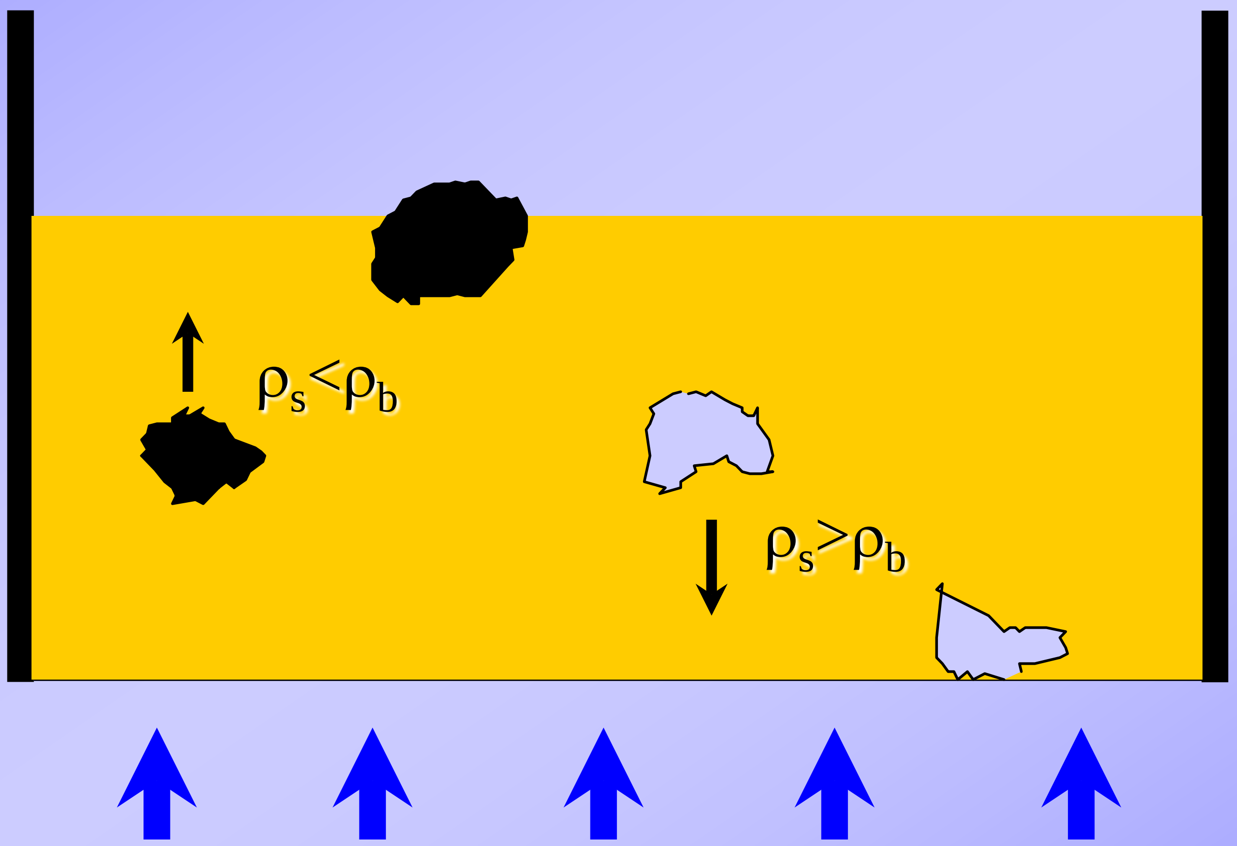
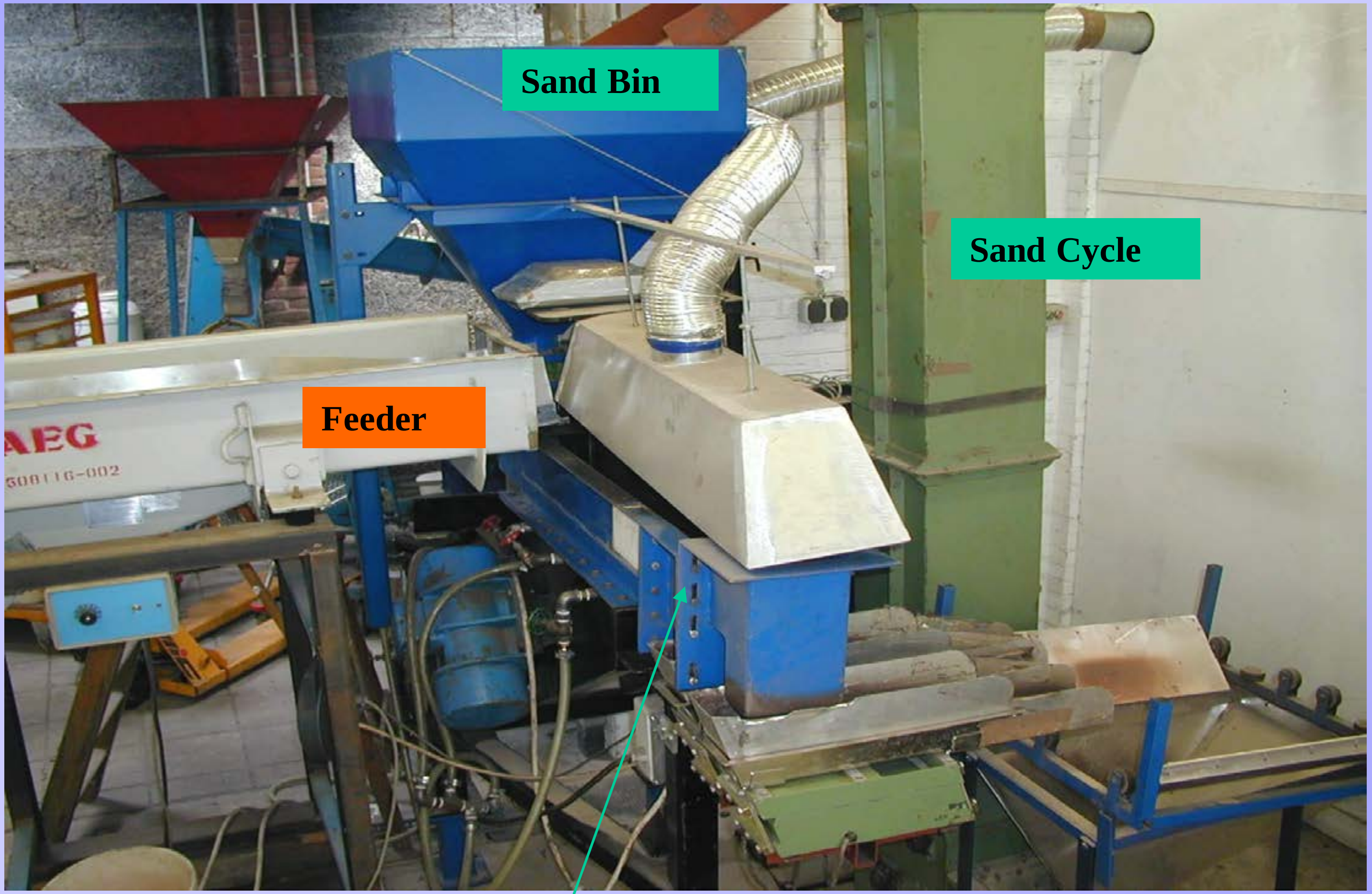
Principle of dry fluidized bed separation



Separation density of medium is $\rho_b \approx \varepsilon\rho+(1-\varepsilon)\gamma$.

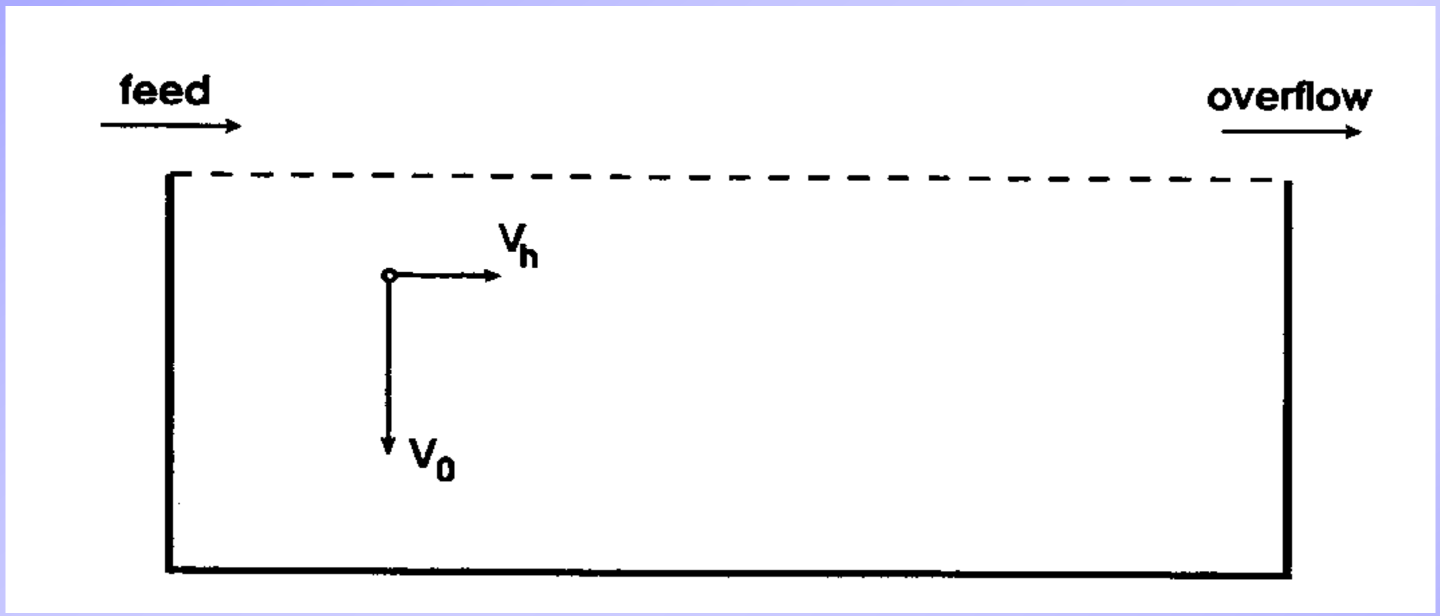
Laboratory Dry Fluidised Bed Separator



Fluidised Bed Separator

Sink and Float

Sedimentation: very dilute streams



Time needed for sedimentation

$$t_0 = \frac{H}{v_0}$$

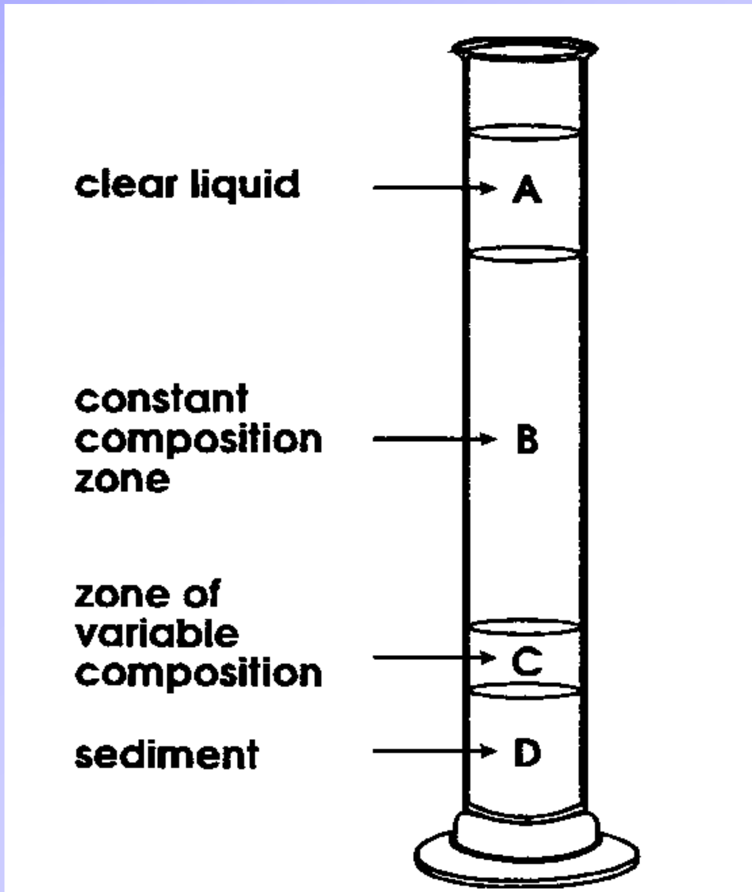
Residence time

$$\tau = \frac{AH}{Q}$$

Maximum capacity

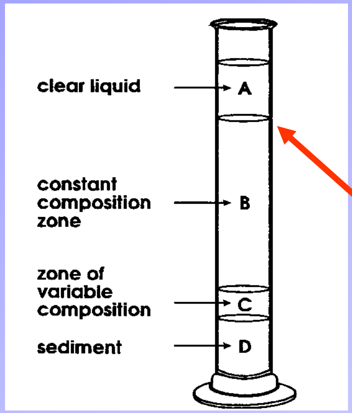
$$Q = Av_0$$

Sedimentation: concentrated streams



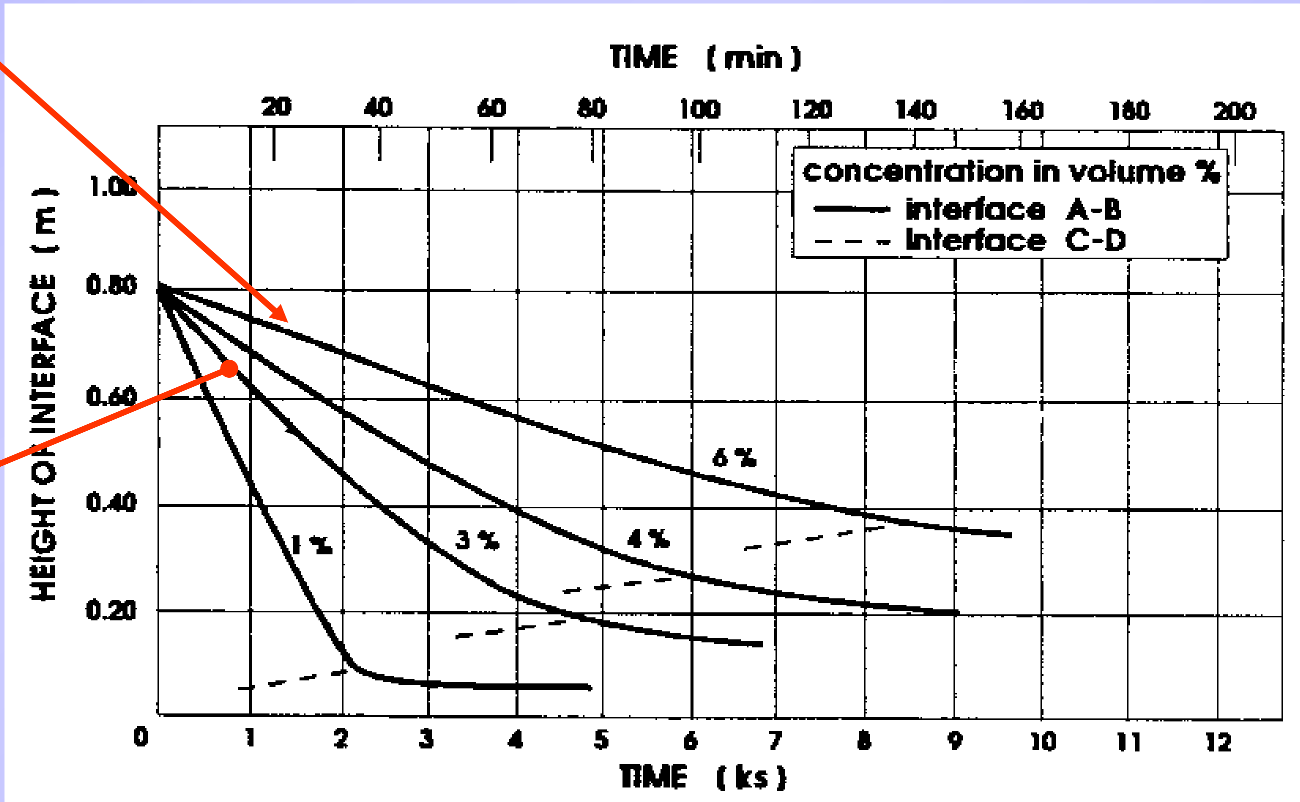
Testing of suspensions

Sedimentation: concentrated streams



Suspensions of several concentrations needed to predict behavior in sedimentation tank.

Settling rate at 3%



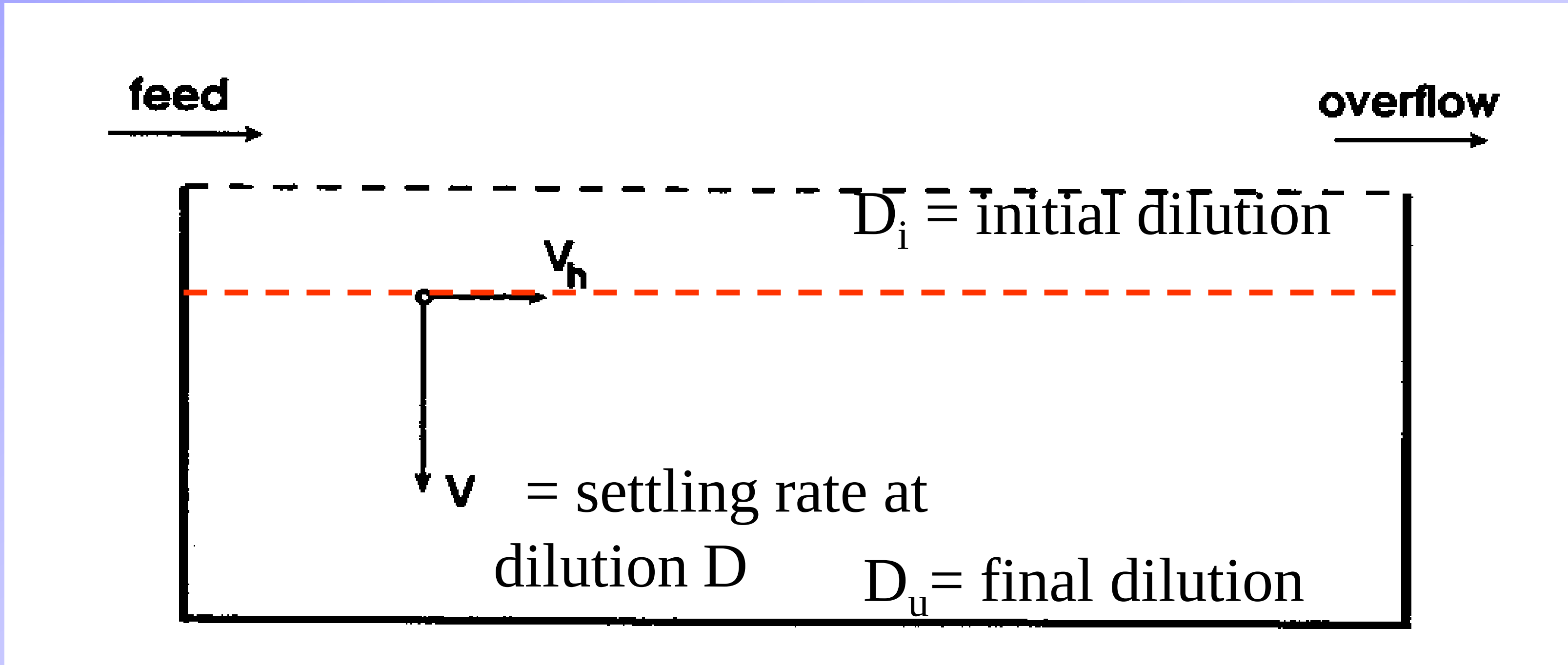
Coe and Clevenger

Estimated area needed for sedimentation tank:

$$A = \max_{D_i > D > D_u} \frac{(D - D_u)}{\rho v(D)} Q C \rho_s$$

D_i = initial dilution, mass of water per mass of solids
D_u= final dilution
ν = settling rate at dilution D
Q = volumetric capacity of the tank
C = volume fraction solid in the feed

Coe and Clevenger



Solids flow in kg/s: $QC\rho_s$

Water going up in kg/s: $(D-D_u)QC\rho_s$

Minimal settling rate in m/s: $v(D) > \frac{(D-D_u)}{\rho A}QC\rho_s$

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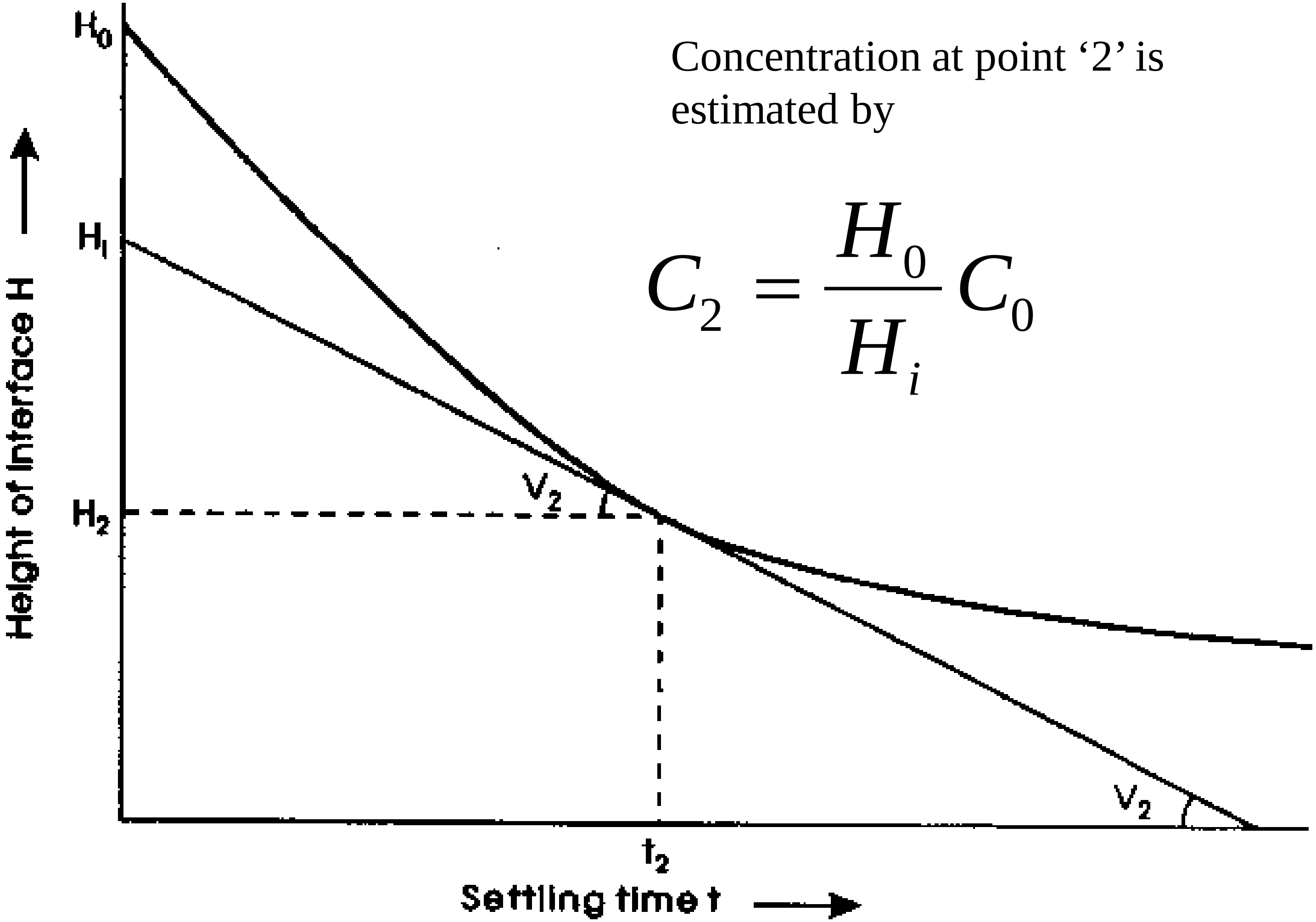
Kynch construction: single column test



Concentration at point ‘2’ is estimated by

$$C_2 = \frac{H_0}{H_i} C_0$$

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Sedimentation: example

A slurry needs to be thickened from 5 kg of water per kg of solids to 1.5 kg of water per kg of solids at a capacity of 1.33 kg/s of solids. What is the required area A?

Dilution D (kg water/kg solid)	5.0	4.2	3.7	3.1	2.5
Meas. rate of sedimentation (mm/sec)	0.20	0.12	0.094	0.070	0.050

D	$D - D_u$ water to overflow	ν sedimentation rate, m/sec	$(D - D_u)/\nu$ (sec/m)
5.0	3.5	$2.00*10^{-4}$	$1.75*10^{4}$
4.2	2.7	$1.20*10^{-4}$	$2.25*10^{4}$
3.7	2.2	$0.94*10^{-4}$	$2.34*10^{4}$
3.1	1.6	$0.70*10^{-4}$	$2.29*10^{4}$
2.5	1.0	$0.50*10^{-4}$	$2.00*10^{4}$

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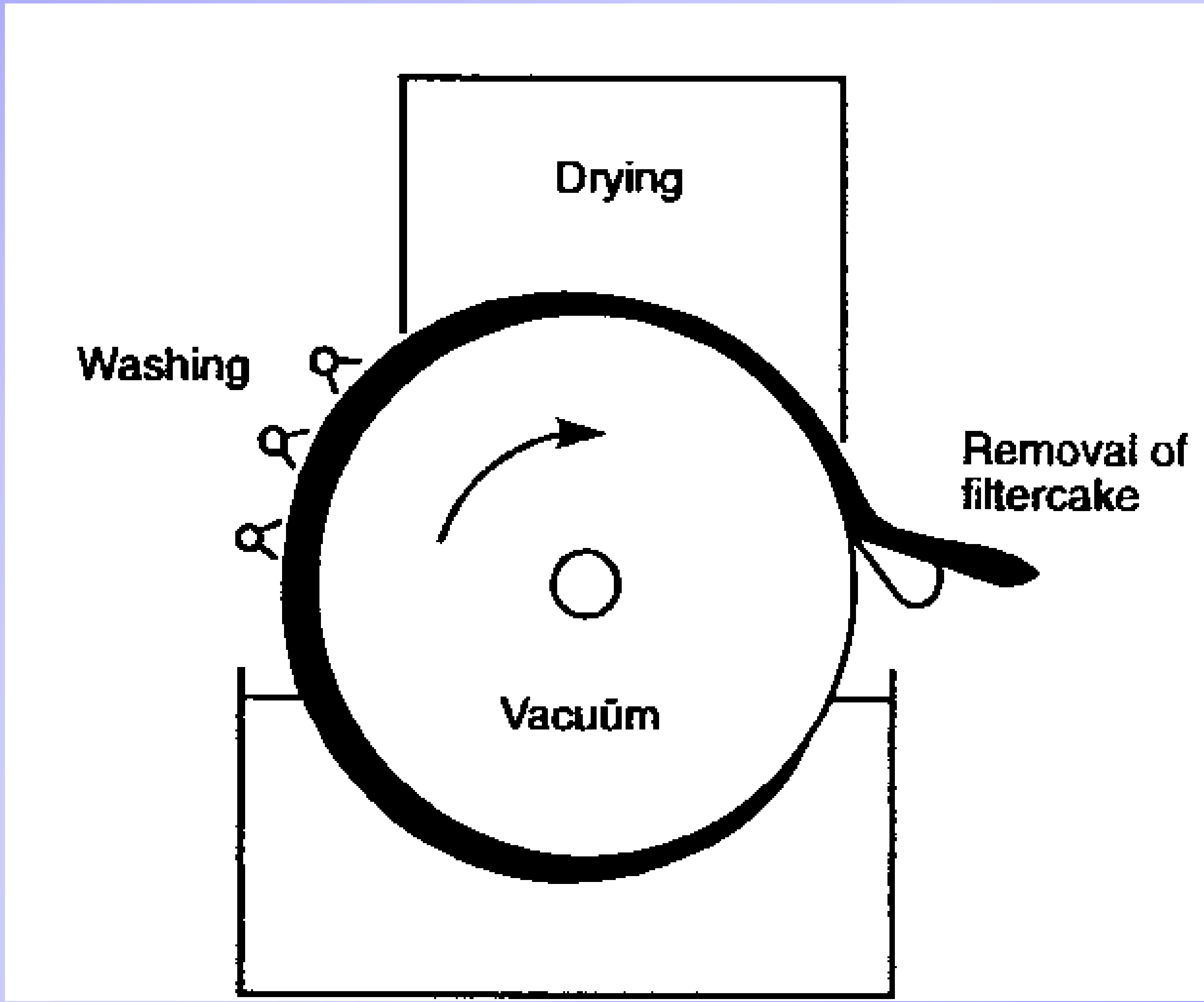
Filtration



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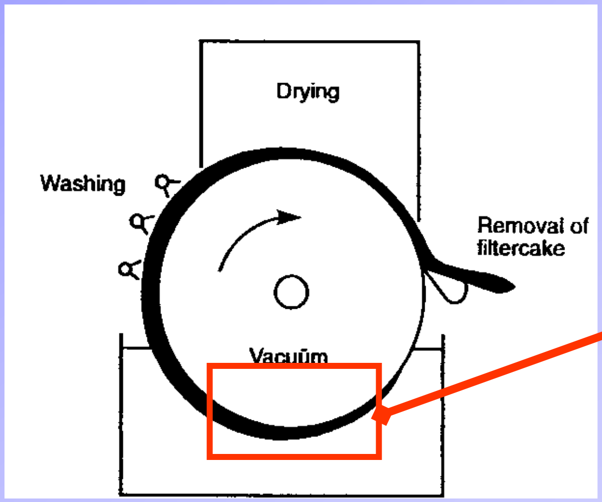
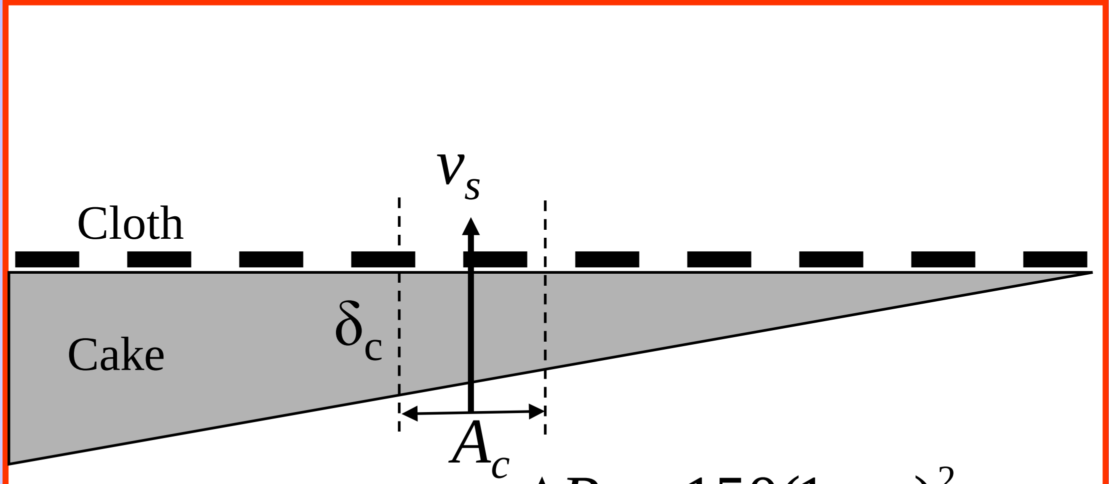
Filtration





$$\frac{\Delta P_c}{\delta_c} = \frac{150(1-\varepsilon)^2 \mu v_s}{\varepsilon^3 d_{vs}^2}$$

ε and d_{vs} unknown $\rightarrow$ $\Delta P_c = \alpha \mu v_s \dfrac{M_c}{A_c}$

Add pressure drop of cloth: $\Delta P_f = \Delta P_m + \Delta P_c = \mu v_s \left[\dfrac{\alpha M_c}{A_c} + R_m \right]$

Filtration

Rewrite the equation in terms of the volume of filtrate V_f that has passed through an area A_c of the cake:

$$M_c = C_c V_f \qquad C_c = \text{ kg solids/m}^3 \text{ filtrate}$$

$$\frac{dV_f}{dt} = A_c v_s$$

$$\rightarrow \qquad \Delta P_f = \mu v_s \left[\frac{\alpha M_c}{A_c} + R_m \right] = \mu \frac{dV_f}{dt} \left[\frac{\alpha C_c}{A_c^2} V_f + R_m \right]$$

Two modes of Filtration

$$\Delta P_f = \mu \frac{dV_f}{dt}\left[\frac{\alpha C_c}{A_c^2} V_f + R_m\right]$$

Constant pressure filtration

$$\Delta P_f t = \mu \left[\frac{\alpha C_c}{2 A_c^2} V_f^2 + R_m V_f\right]$$

If filter cloth is neglected:

$$V_f = A_c \sqrt{\frac{2 \Delta P_f t}{\alpha \mu C_c}}$$

Constant rate filtration (batch)

$$\Delta P_f = \mu Q_f \left[\frac{\alpha C_c}{A_c^2} Q_f t + R_m\right]$$

Pressure increases linearly with time ($Q_f = V_f/t$ is a constant).

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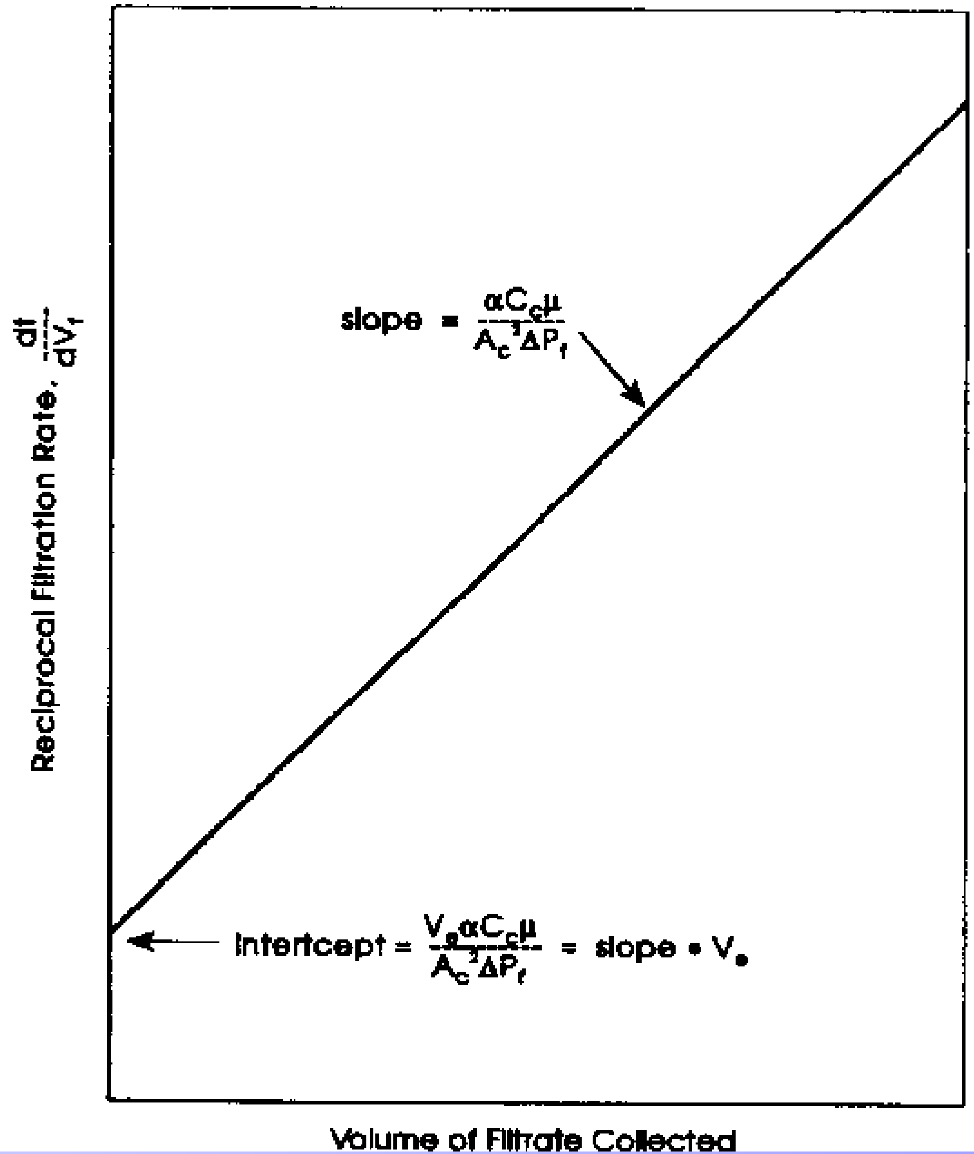
Constant pressure Filtration



Filtration: example

A continuous rotary filter is required for the filtration of a suspension to produce 2 litres/s of filtrate. A sample was tested on a small laboratory filter of area 0.023 m^2 to which it was fed by means of a slurry pump to give filtrate at a constant rate of 12.5 cm^3/s. The pressure difference across the test filter increased from 14 kN/m^2 after 300 s filtration to 28 kN/m^2 after 900 s at which time the cake thickness had reached 38 mm. Calculate the area of a rotary drum filter, assuming that the resistance of the cloth can be neglected, and that the vacuum system is capable of maintaining a constant pressure difference of 70 kN/m^2. The drum will rotate at a speed of 1 rev/min and 20% of the cloth will be submerged.

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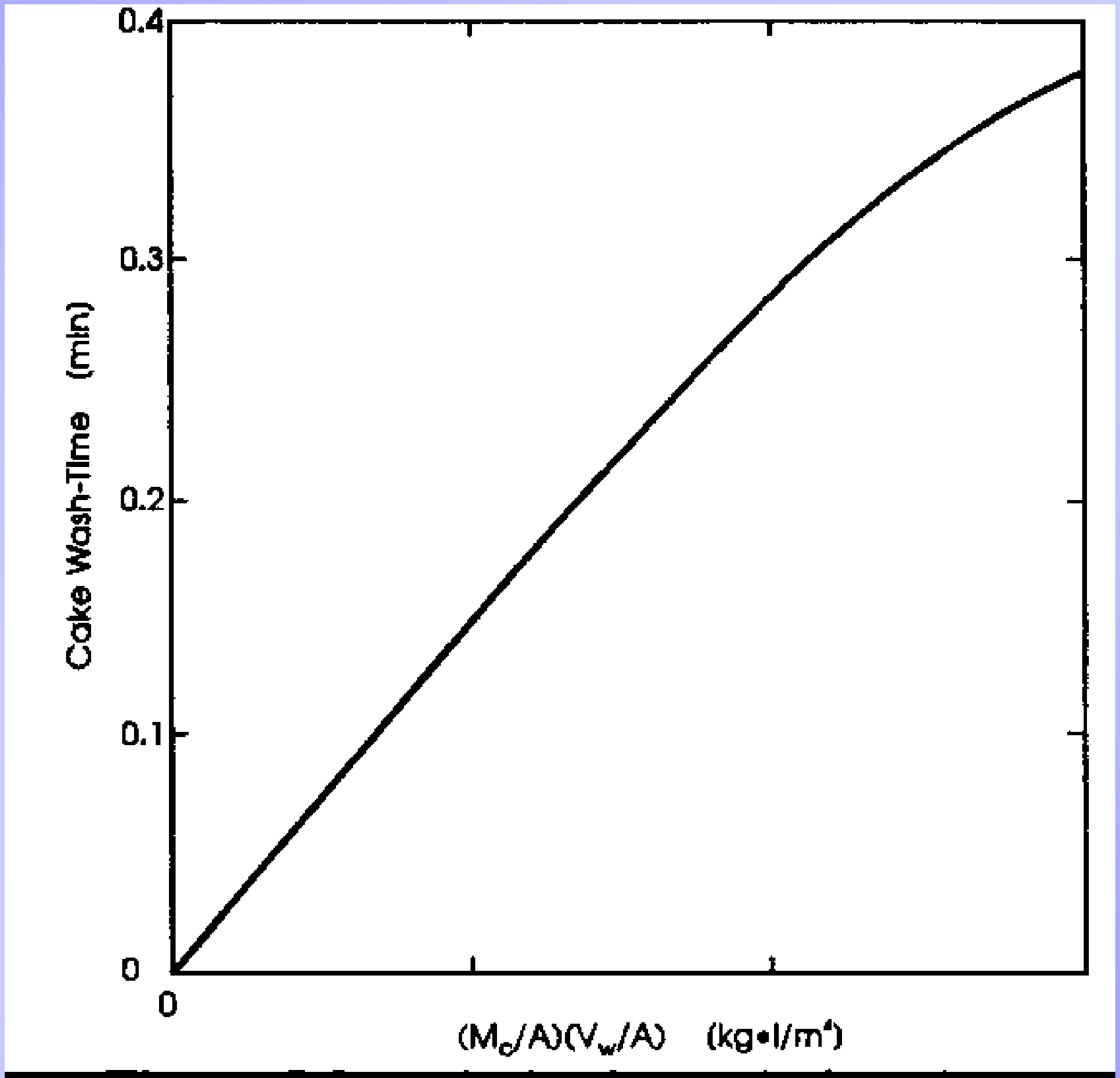
Cake washing



Drying

Definition of moisture content:

- on a wet basis: $$W_w = \frac{W_d}{1+W_d}$$

- on a dry basis: $$W_d = \frac{W_w}{1-W_w}$$

Drying

Drying is:

- Heat of evaporation **in**to contact with water
- Evaporated water **out**

Heat **in**:

- convection
- conduction
- radiation/Ohmic

Water **out:**

- capillary flow
- diffusion
- vapor flow

Supplying the heat

Convection: Turbulent hot air = fast and cheap

Conduction: Heat through “laminar boundary layer” = slow and cheap

Radiation/Ohmic: microwave, radio-frequency, electric currents = instantaneous and expensive process

Efficiency of heat delivered/power used:

- Microwave 50% (very flexible, cost ≈ 0.5 euro/Watt installed)
- Radiofrequency 70% (less flexible: contacting problem)
- Ohmic 100% (not flexible: difficult contacting)

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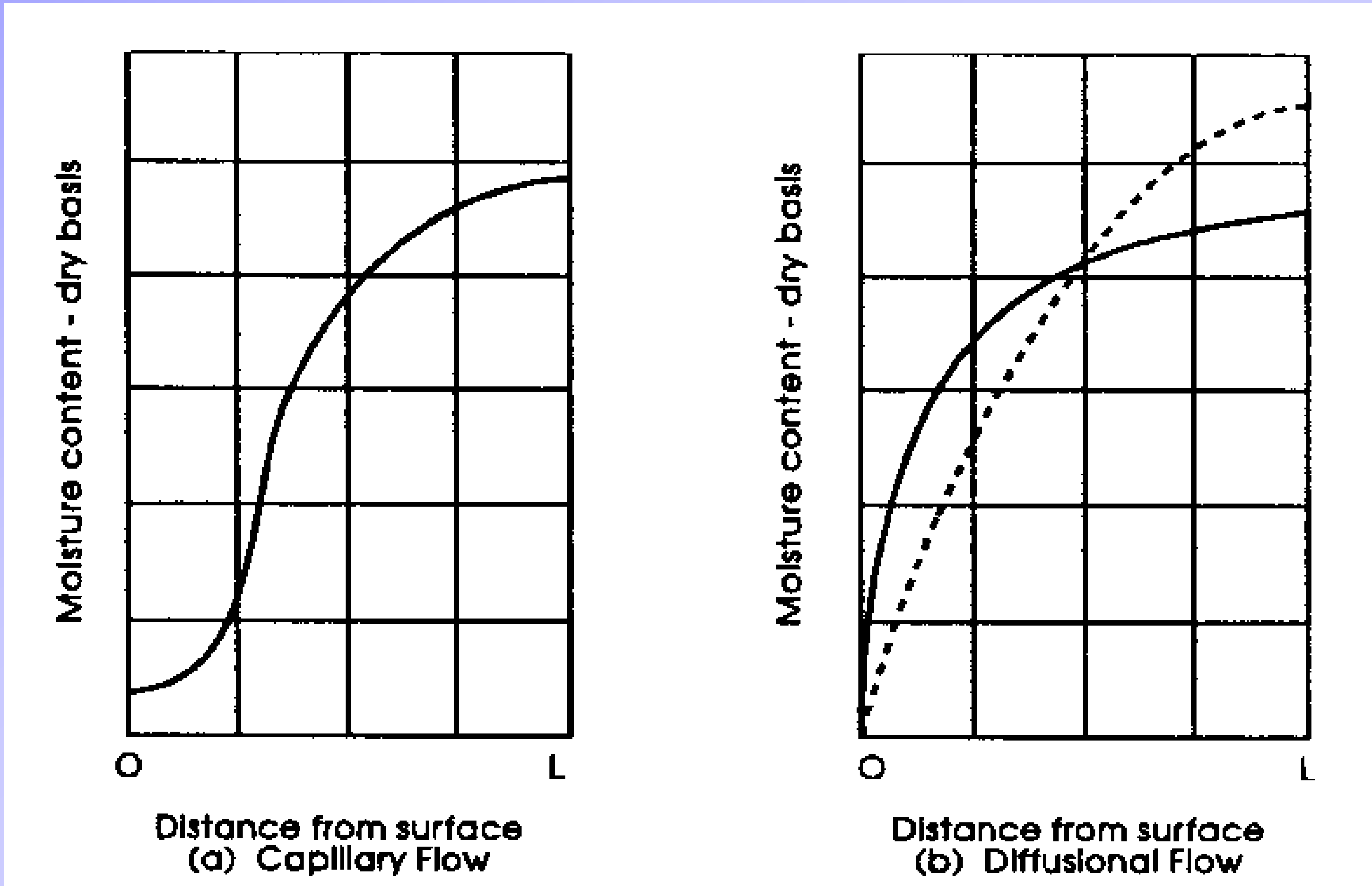
Taking away the water: internal transport



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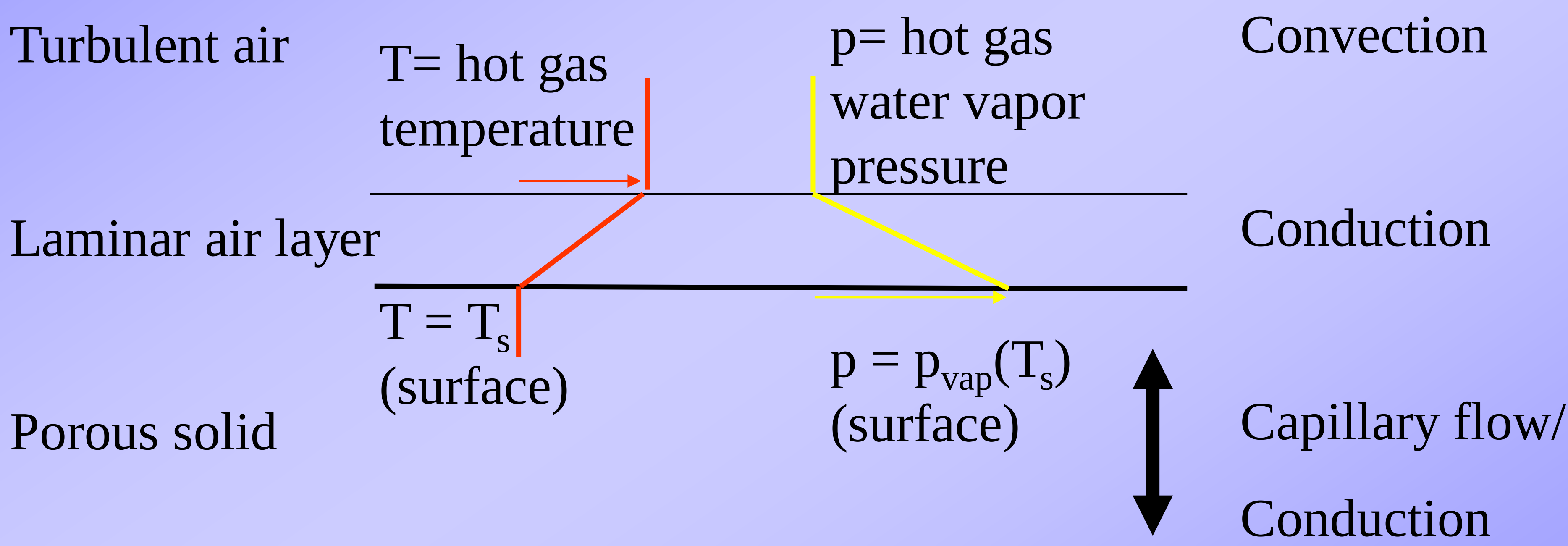
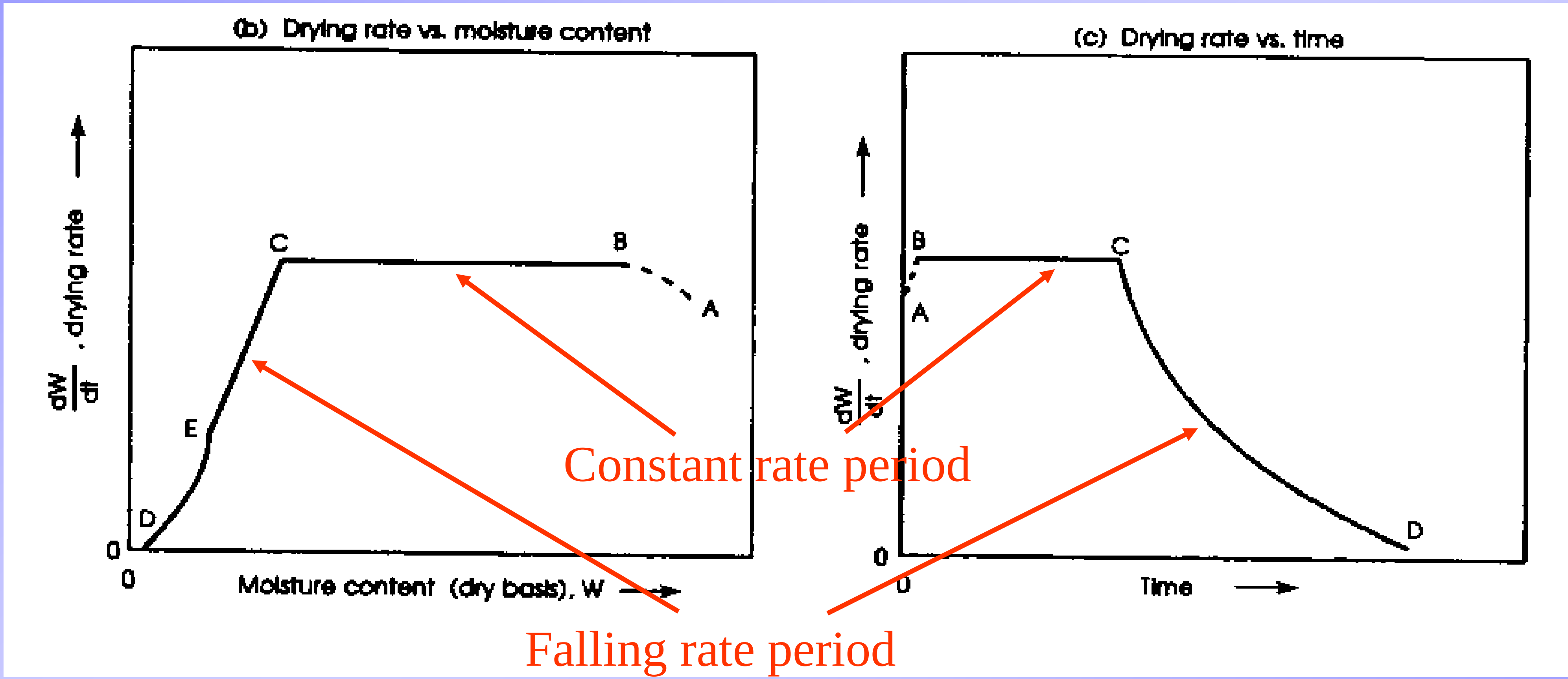
Hot air drying: the “laminar” layer



Balance: $dW / dt = \alpha A \Delta T / \Delta H = k A \Delta P$

Drying with hot air



Constant rate: surface is wet, laminar layer is the bottleneck for heat and mass transfer.

Falling rate: surface is (partially) dry, capillary flow is the bottleneck for mass transfer.

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Drying: fluidized bed

Moisture content single particle:

$$\frac{W}{W_i} = 1 - \frac{t}{\tau}$$

Residence time needed:

$$\tau = \frac{\rho_s d \Delta H W_i}{\alpha (T_s - T)}$$

Actual distribution of residence time:

$$E(t) = \frac{1}{\tau_r} e^{-t/\tau_r}$$

Average final moisture content:

$$\overline{W} = \int_{t=0}^{t=\tau} W E(t) dt \qquad \frac{W}{W_i} = 1 - \frac{1 - e^{-\tau/\tau_r}}{\tau / \tau_r}$$

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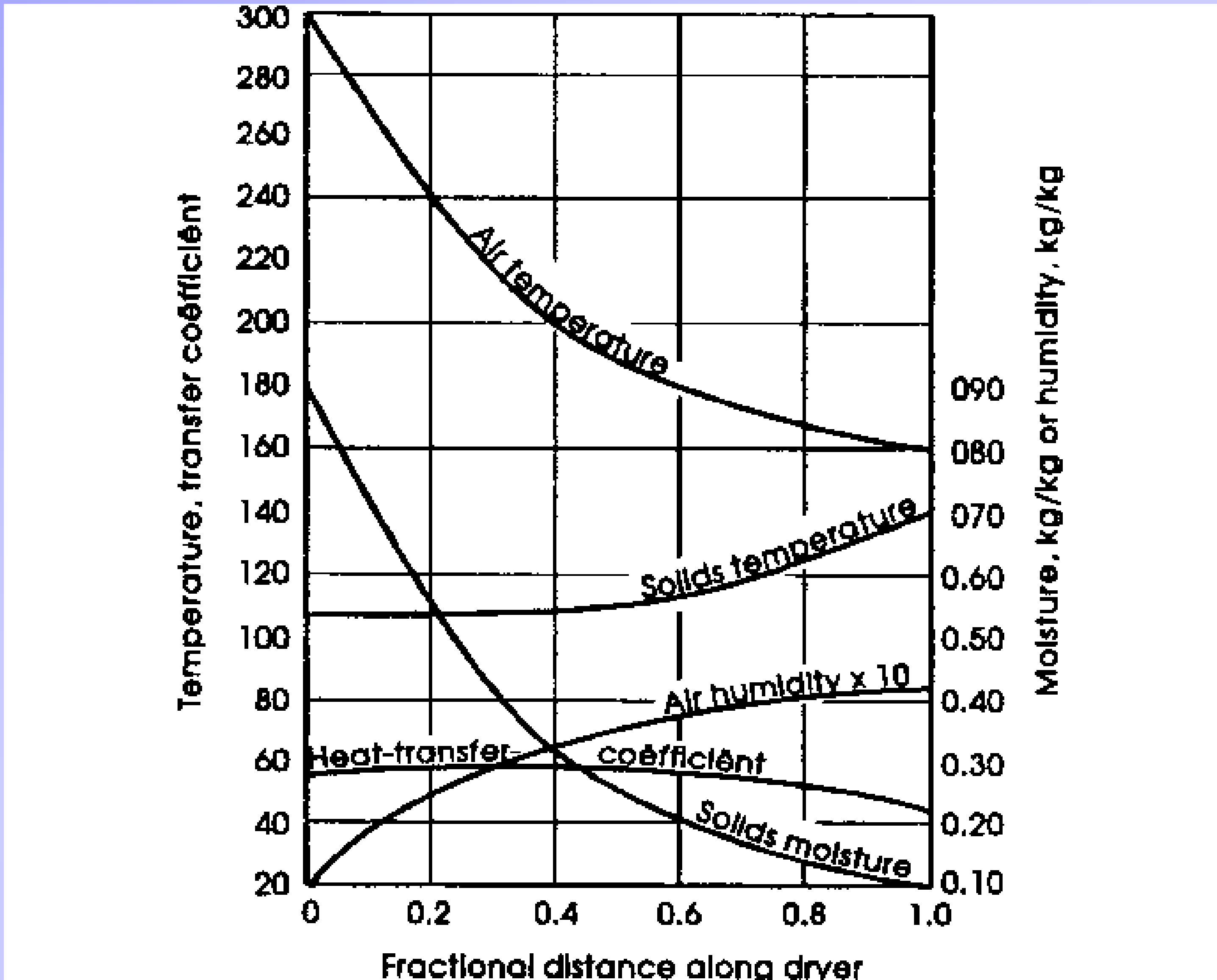
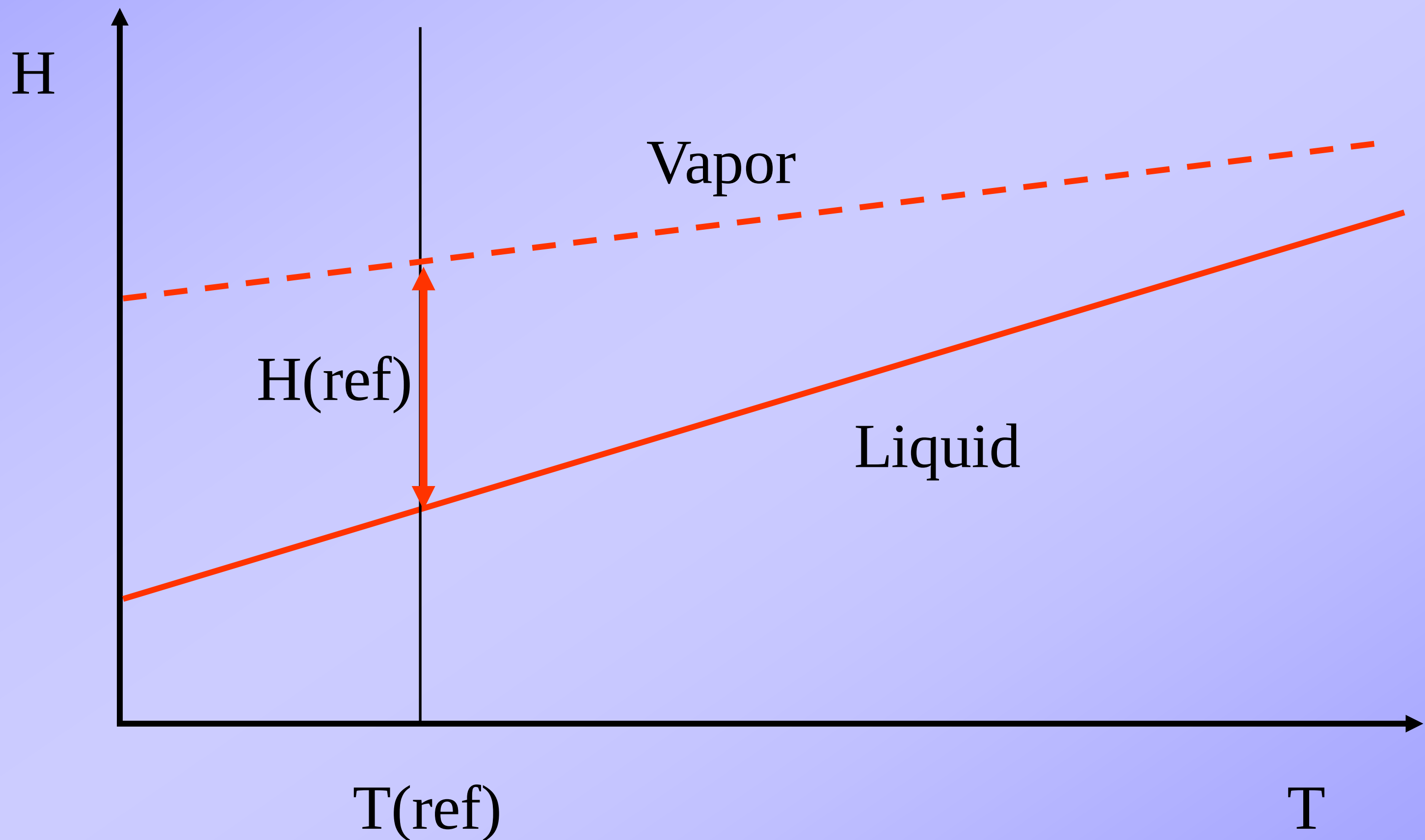
Drying: measurement and heat and mass balance



Heat of water (liq and vap)



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Drying: measurement and heat and mass balance

Heat and Mass Balance:

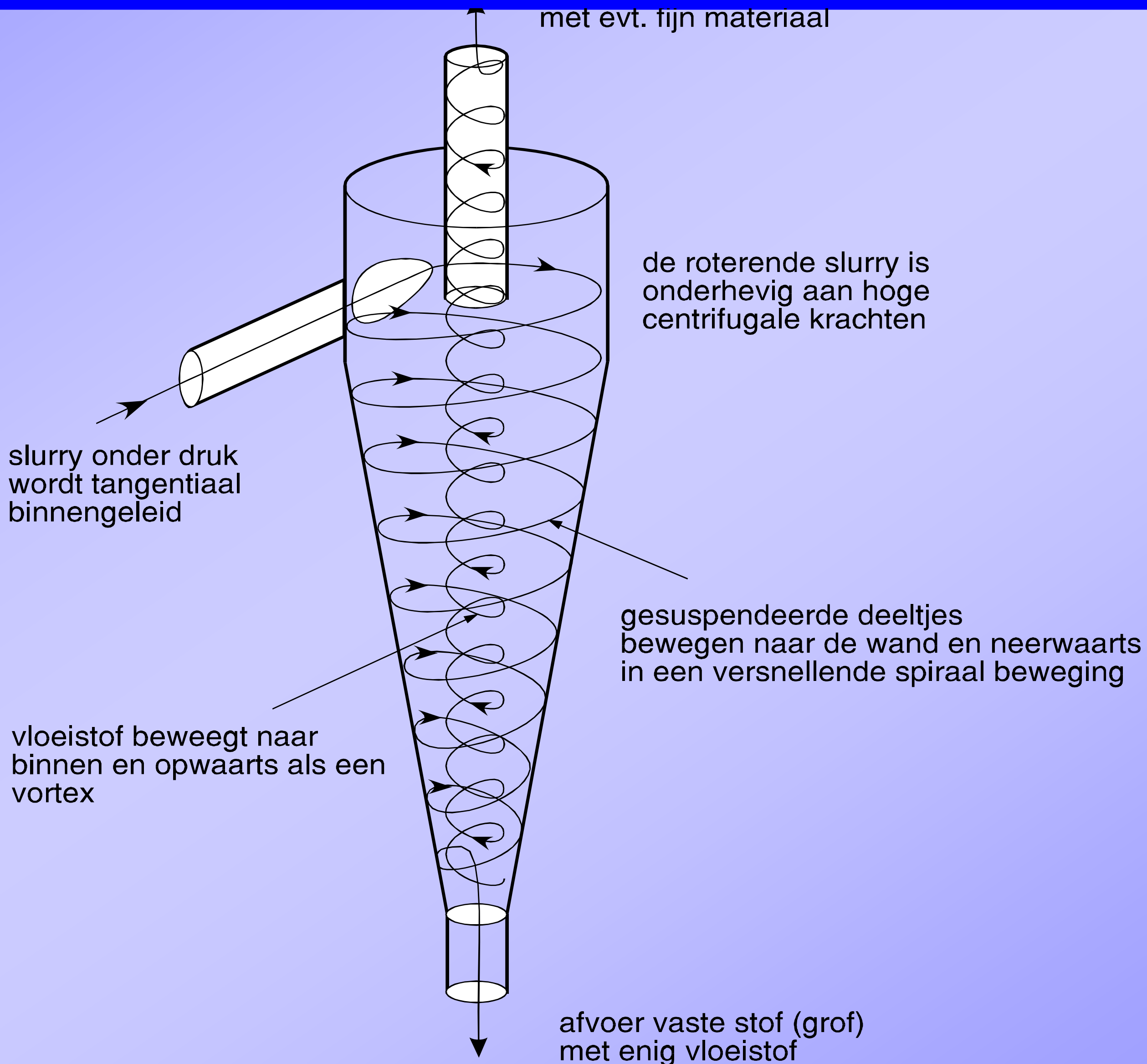
Material	Mass in kg/s	Mass out kg/s	Heat in MJ/s	Heat out MJ/s
Solids	M_{solid}	M_{solid}	$M_{solid}C_{p,sol}T_{solid,in}$	$M_{solid}C_{p,sol}T_{solid,out}$
Air	M_{air}	M_{air}	$M_{air}C_{p,air}T_{air,in}$	$M_{air}C_{p,air}T_{air,out}$
Liquid Water	M_{liq}	M'_{liq}	$M_{liq}C_{p,liq}(T_{liq,in}-T_{ref})$	$M'_{liq}C_{p,liq}(T_{liq,out}-T_{ref})$
Water Vapor	M_{vap}	$M'_{vap}=M_{liq} - M'_{liq} + M_{vap}$	$M_{vap}C_{p,vap}(T_{vap,in}-T_{ref}) + M_{vap}H_{ref}$	$M'_{vap}C_{p,vap}(T_{vap,out}-T_{ref}) + M_{vap}H_{ref}$

Heat and mass balance: example

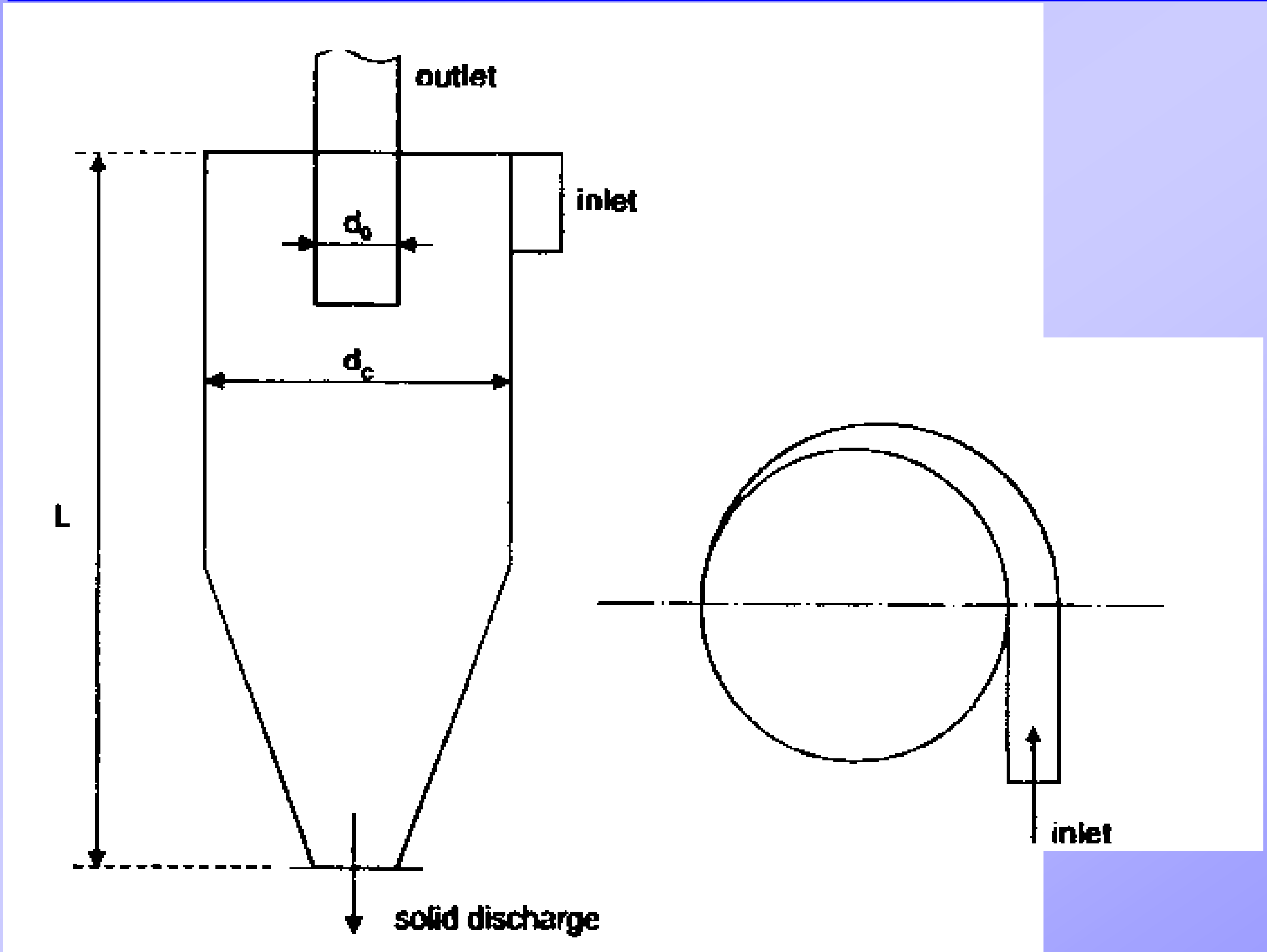
Granular material with 40% moisture (wet basis) is fed to a countercurrent rotary dryer at a temperature of 295 K and is withdrawn at 305 K containing 5% moisture. The hot air contains 0.006 kg water vapor per kg of dry air, enters at 385 K and leaves at 310 K. The dryer handles 0.125 kg/s wet stock. Assuming that radiation losses amount to 20 kJ/kg dry air used, determine the weight of dry air supplied to the dryer per second and the humidity of the air leaving it.

Latent heat of water vapor H at 295 K = 2449 kJ/kg
Specific heat of dried material = 0.88 kJ/kg K
Specific heat of dry air = 1.00 kJ/kg K
Specific heat of water vapor = 2.01 kJ/kg K
Specific heat of liquid water = 4.18 kJ/kg K

Cyclones


met evt. fijn materiaal
de roterende slurry is
onderhevig aan hoge
centrifugale krachten
slurry onder druk
wordt tangentiaal
binnengeleid
gesuspendeerde deeltjes
bewegen naar de wand en neerwaarts
in een versnellende spiraal beweging
vloeistof beweegt naar
binnen en opwaarts als een
vortex
afvoer vaste stof (grof)
met enig vloeistof

Cyclones



Cyclones: dimensions

$$\frac{l}{d_c} = 0.4$$

$$\frac{L}{d_c} = 5$$

$$\frac{d_i}{d_c} = 0.28$$

$$\frac{d_0}{d_c} = 0.34$$

d_c = cyclone diameter
l = length of vortex finder
L = length of the cyclone
d_i = inlet diameter
d_0 = overflow outlet diameter

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Cyclones: cutpoint

Centrifugal force on particle:

$$\frac{\pi d^3}{6}(\rho_s - \rho)\frac{u_t^2}{r}$$

Equilibrium with drag of inflowing fluid:

$$\frac{\pi d^3(\rho_s - \rho)}{6}\frac{u_t^2}{r} = 3\pi\mu d u_r$$

Virtually all fluid is drawn into the vortex:

$$u_r = \frac{Q}{2\pi r L}$$

$$d^2 = \frac{9}{\pi}\frac{\mu Q}{(\rho_s - \rho)u_t^2 L}$$

Cyclones: cutpoint

$$d^2 = \frac{9}{\pi}\frac{\mu Q}{(\rho_s - \rho)u_t^2 L}$$

Velocity profile of cyclone: $u_t = u_{t0}\sqrt{\frac{d_c}{2r}}$

Tangential velocity at inlet: $Q = A_i\, u_{t0}$

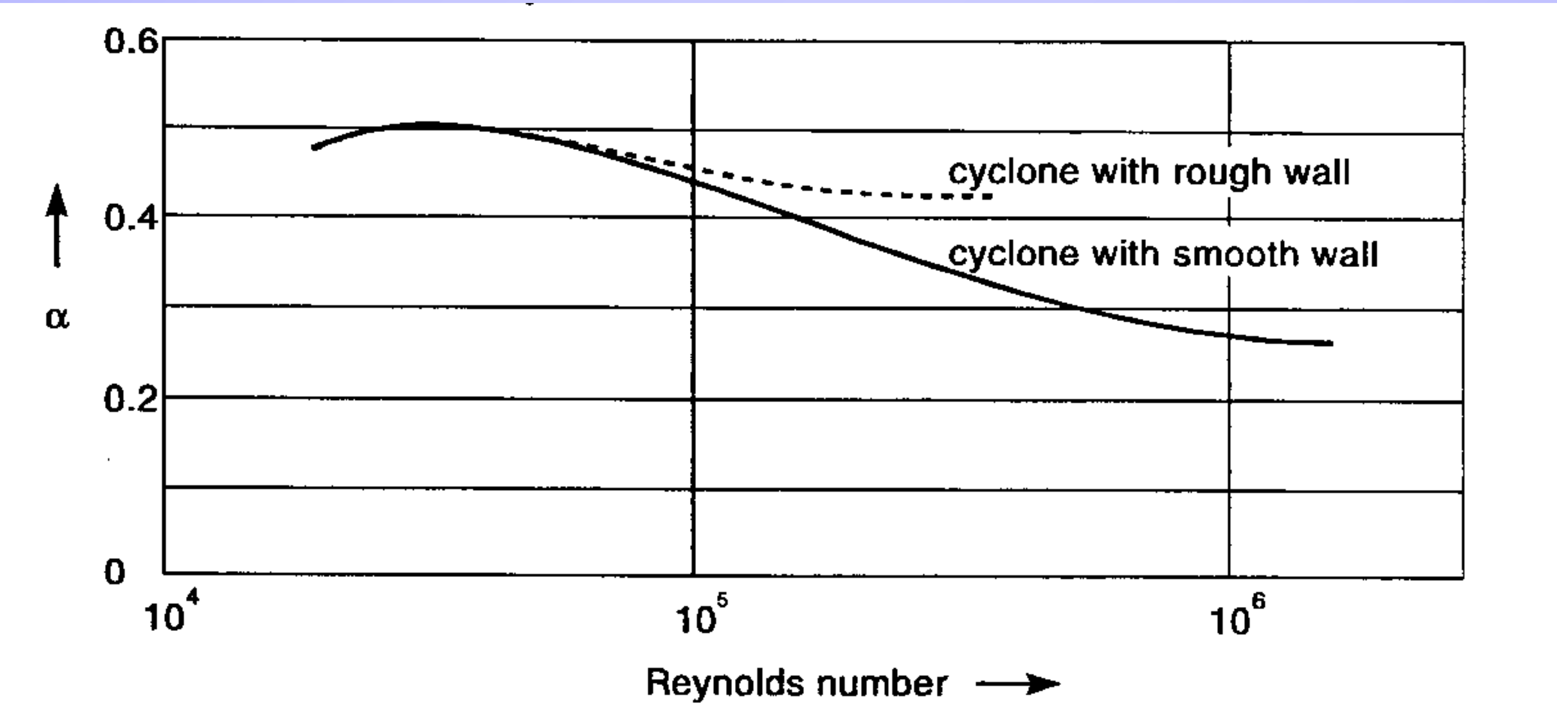
Vortex starts at r = 0.2d_0:

$$d^2 = \frac{3.6}{\pi}\frac{\mu A_i^2 d_0}{(\rho_s - \rho)d_c L Q}$$

Cyclones: pressure drop

$$\alpha = \frac{Q}{A_f \sqrt{\frac{2\Delta p}{\rho}}}$$

Q = cyclone capacity (m^3/s)
A_f = area of the feed opening (m^2)
ρ = density of the fluid (kg/m^3)
Δp = pressure drop across the cyclone (N/m^2)



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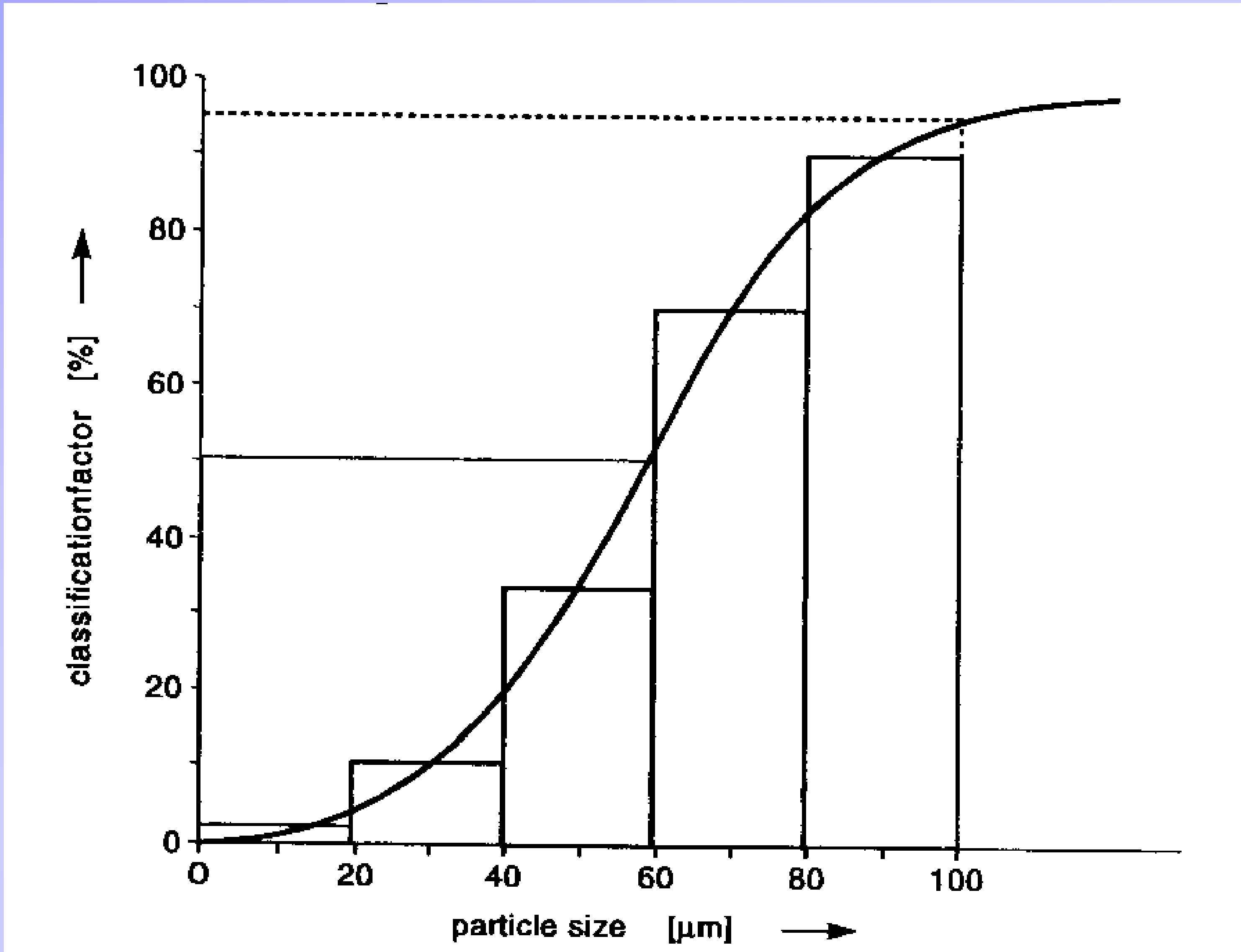
Cyclones: classification curve



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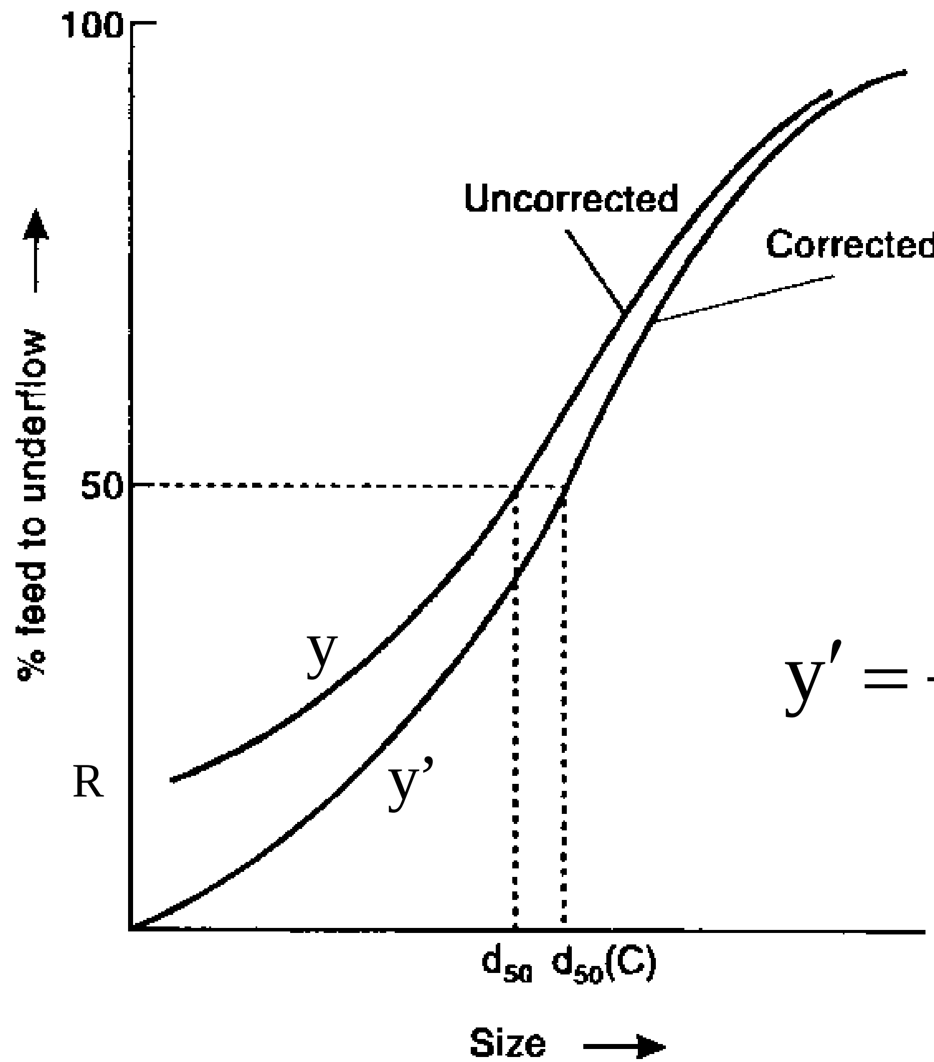
Cyclones: classification curve



$$y' = \frac{y - R}{1 - R}$$

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Cyclones: cutpoint

$$d_{50} = 13.7 \frac{(d_0 d_i)^{0.68}}{Q^{0.53}(\rho_s - \rho)^{0.5}}$$

$$d_{50} = 14.8 \frac{d_c^{\ 0.46} d_i^{\ 0.6} d_0^{\ 1.21} e^{0.063V}}{d_u^{\ 0.71} h^{0.38} Q^{0.45} (\rho_s - \rho)^{0.5}}$$

Dahlstrom:

d_{50}	= cut point (micron)
d_0	= overflow diameter (cm)
d_i	= inlet diameter (cm)
Q	= flow rate (m3/h)
ρ_s	= specific gravity of solids
ρ	= specific gravity of fluid

Plitt:

d_c	= cyclone diameter (cm)
d_u	= apex diameter (cm)
V	= vol. perc. solids
h	= height of vortex (cm)

Cyclones: cutpoint

$$d_{50} = 14.8 \frac{d_c^{0.46} d_i^{0.6} d_0^{1.21} e^{0.063V}}{d_u^{0.71} h^{0.38} Q^{0.45} (\rho_s - \rho)^{0.5}}$$

V = vol. perc. solids

Replace V in Plitt's formula by the correct expression in ε. Do you expect this behavior with V on the basis of Richardson and Zaki (Note that $\varepsilon^{4.65} \approx e^{-4.65(1-\varepsilon)}$):

$$u_r = \frac{(\gamma - \rho) d_{50}^2}{18\eta} \frac{u_t^2}{r} = \frac{(\gamma - \rho) d_{50}^{'2}}{18\eta} \frac{u_t^2}{r} \varepsilon^{4.65}$$

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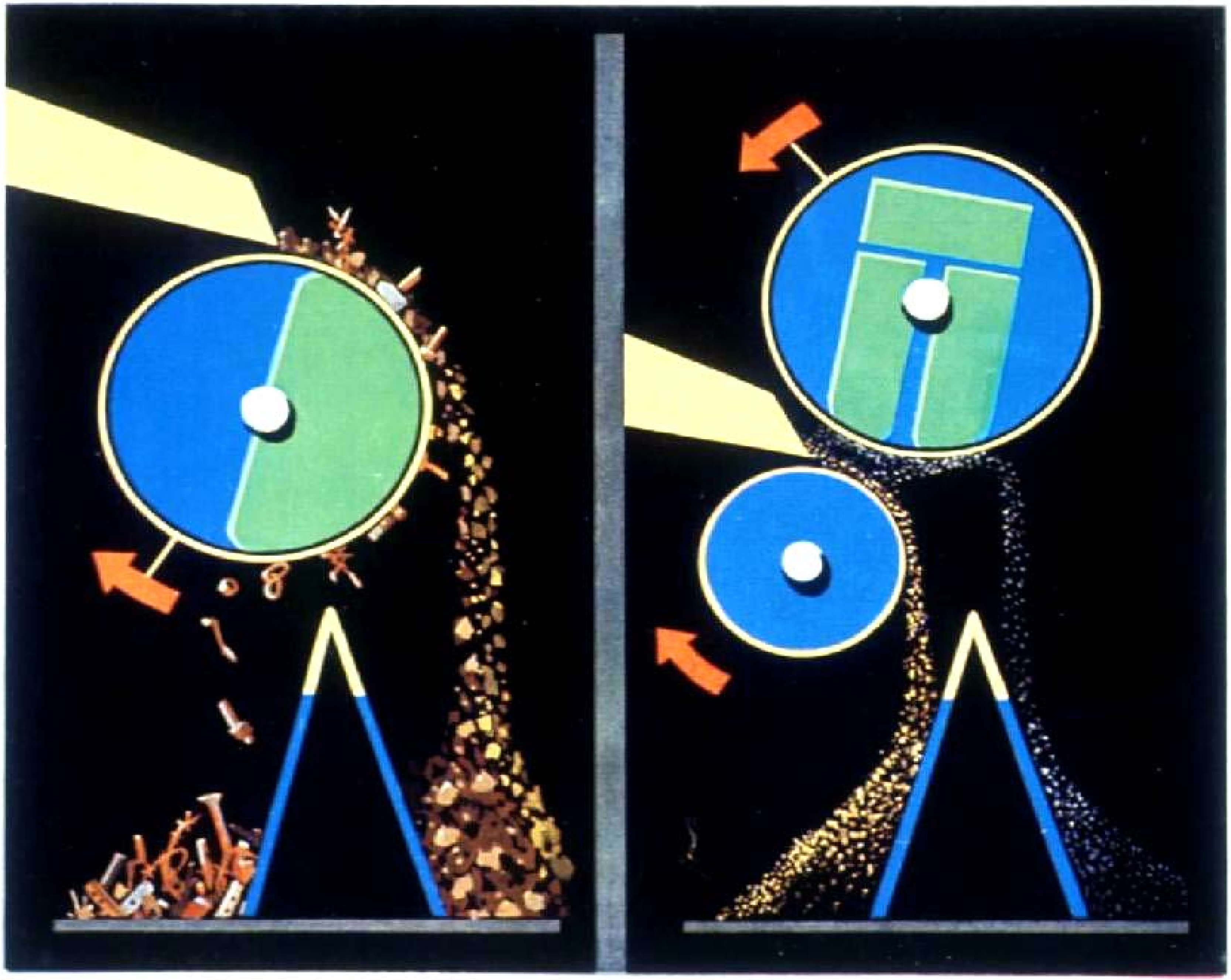
Magnetic separation



Holding Method

Pick-Up Method

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Magnetic separation

Magnets (B in Tesla):

- Permanent magnets 0.3-0.6 Tesla
- Electromagnets 1.5 Tesla
- Superconducting magnets 5 Tesla

Magnetic materials (M in A/m):

- Ferromagnetic 100,000-2,000,000 A/m
- Paramagnetic 1000-10,000 A/m
- "Non-magnetic" <100 A/m

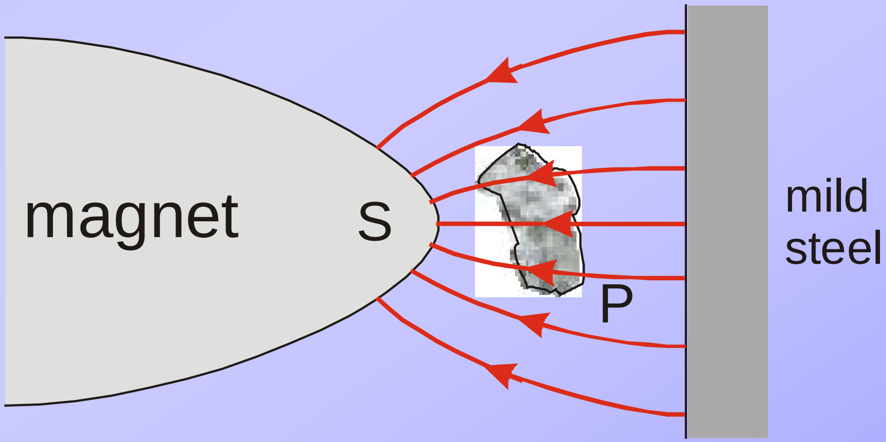
A magnet's **reach** is roughly the width *W* of its poles and the **force** on a volume *V* of magnetic material with magnetization *M* is roughly:

$$F = MV\frac{B}{W}$$

Magnetism

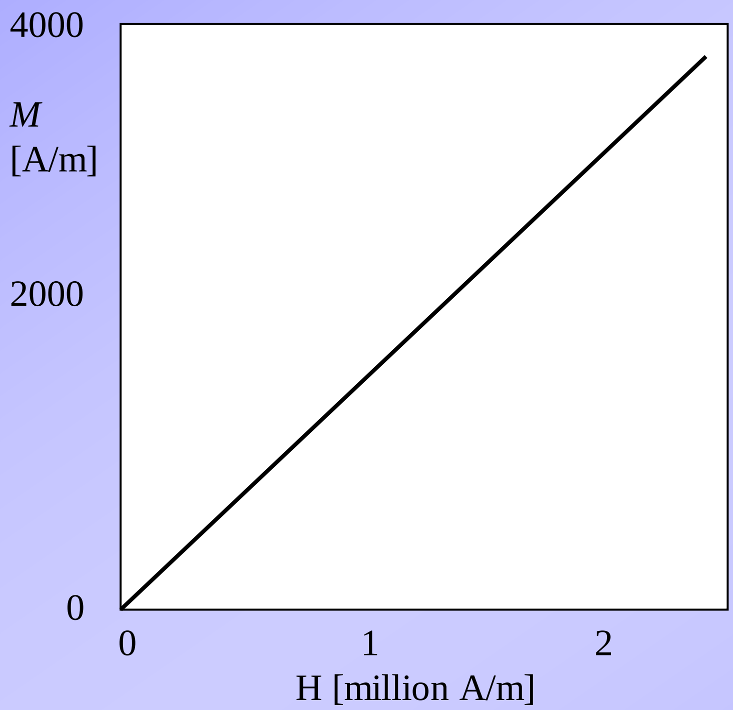
Principle:

1. Magnet creates field
2. Field magnetizes particle
3. Particle is attracted towards increasing field strength



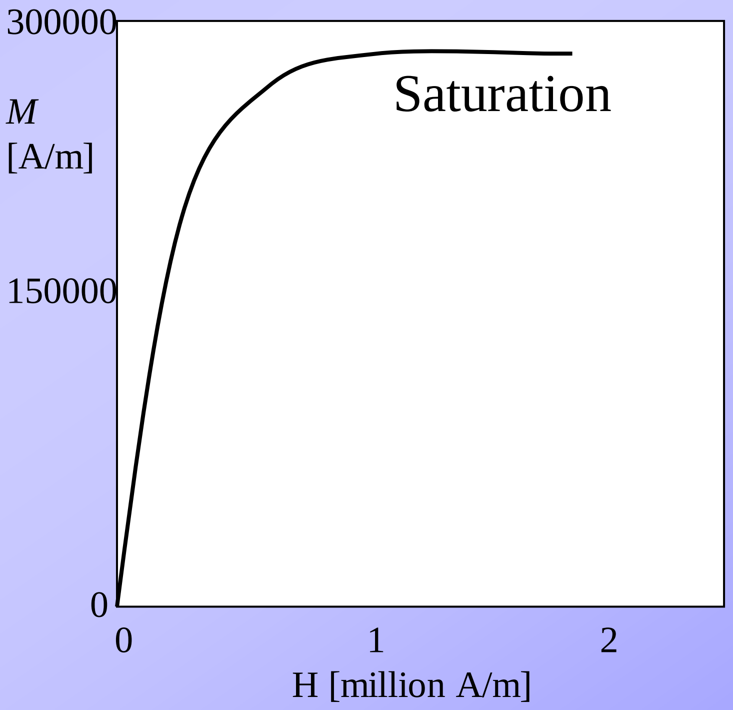
Magnetic materials

Goethite FeOOH



Paramagnetic

Magnetite Fe_3O_4



Ferromagnetic

Magnetic materials

Magnetic materials:

- Ferromagnetic: Steel, Magnetite, Hematite, Ilmenite

  Main parameter is saturation magnetization M = 100,000-2,000,000 A/m

- Paramagnetic: Goethite, Chromite M =1000-10,000 A/m

  Main parameter is magnetic susceptibility χ = 0.001 – 0.01

- "Non-magnetic" <100 A/m

$$M = \chi H$$

χ (volume) magnetic susceptibility

$\chi_s = \dfrac{\chi}{\rho}$ specific magnetic susceptibility [m^3/kg]

Magnetic separation: Conclusion

Assume: H varies in coordinate *z*: gradient *d*H/*dz*

$$\vec{F} = \mu_0 \frac{d(\vec{P}_m \cdot \vec{H})}{d\vec{r}}$$

$$= \mu_0 \chi H V \frac{dH}{dz}$$ paramagnetic particle

$$= \mu_0 M_{sat} V \frac{dH}{dz}$$ saturated ferromagnetic particle

$$= \frac{\mu_0 H V}{N} \frac{dH}{dz}$$ non-saturated ferromagnetic particle

Steel scrap:

H > 300,000 A/m

H < 300,000 A/m

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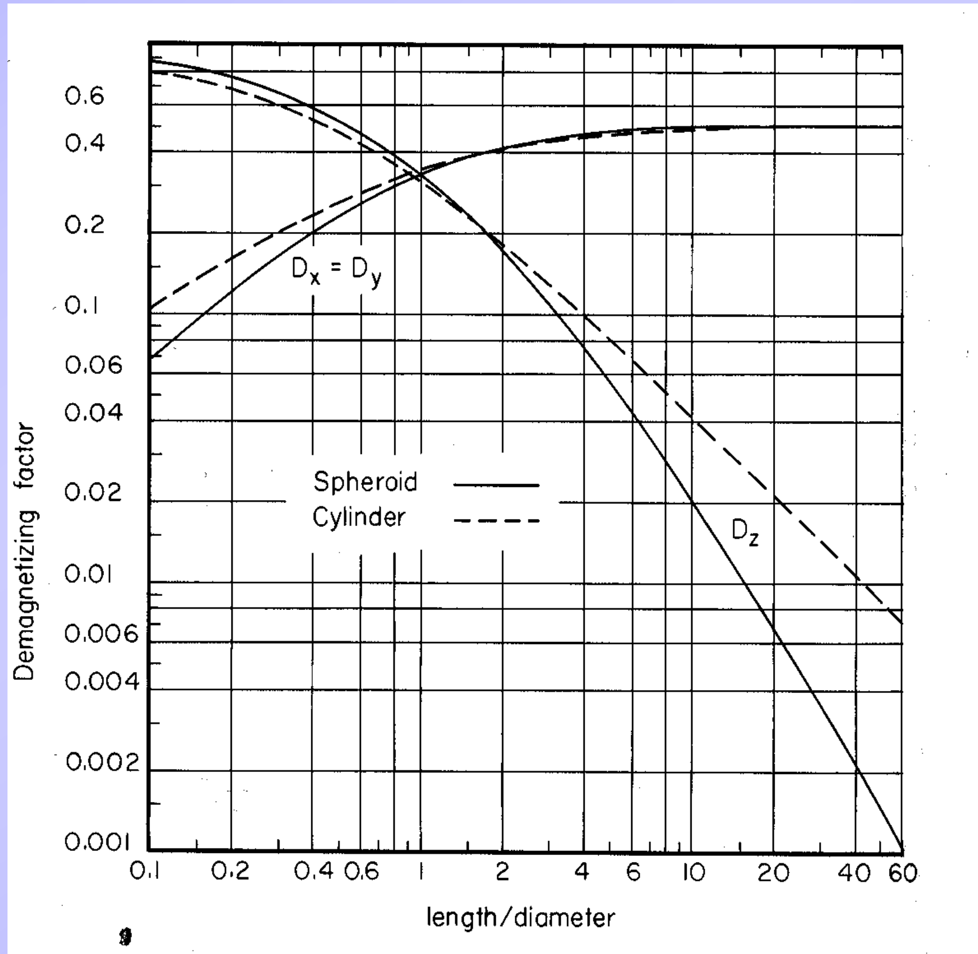
Demagnetizing factor N

Particle shapes:

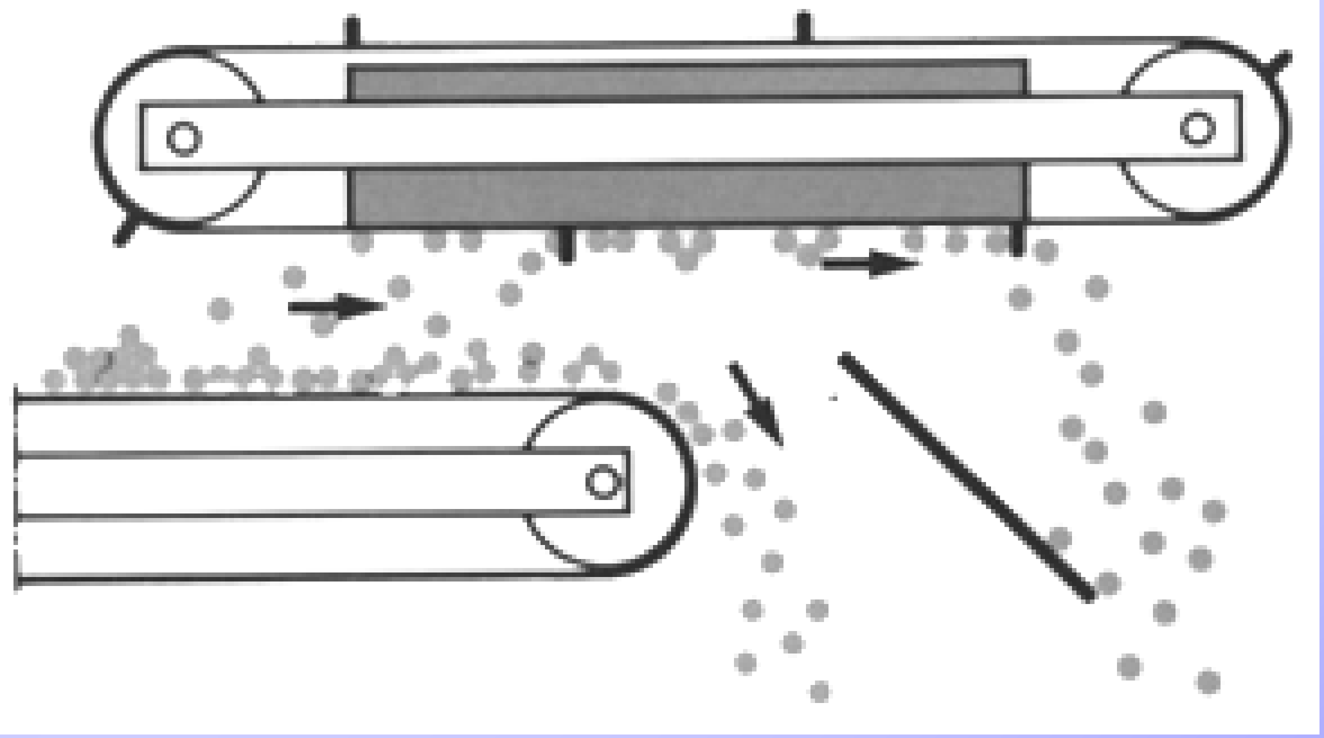
- Granular N=1/3
- L/D=4 Cylinder

  N=0.1
- D/d=4 Disk

  N=0.13
- Scrap: N=0.05-0.2



Magnetic separation



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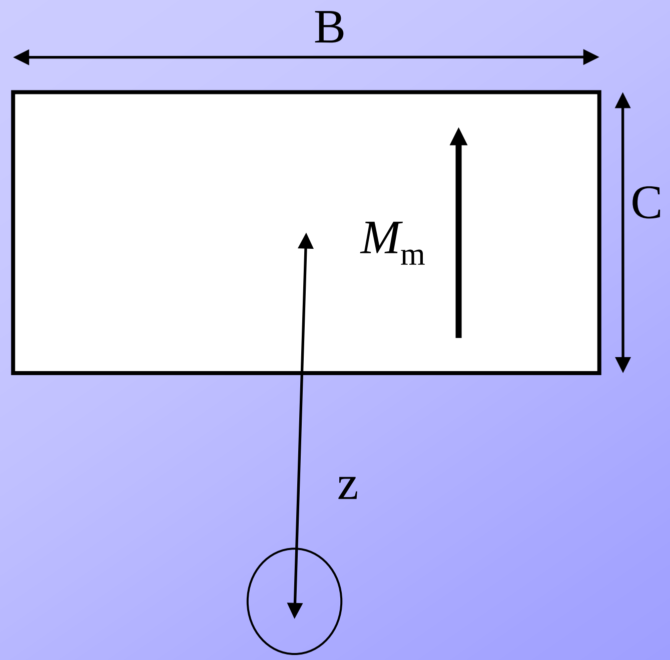
Magnetic separation: Dipole magnet

Dipole magnet: recovery of steel from a large distance (z = 0.5 m)

VANGVELDDIEPTEN

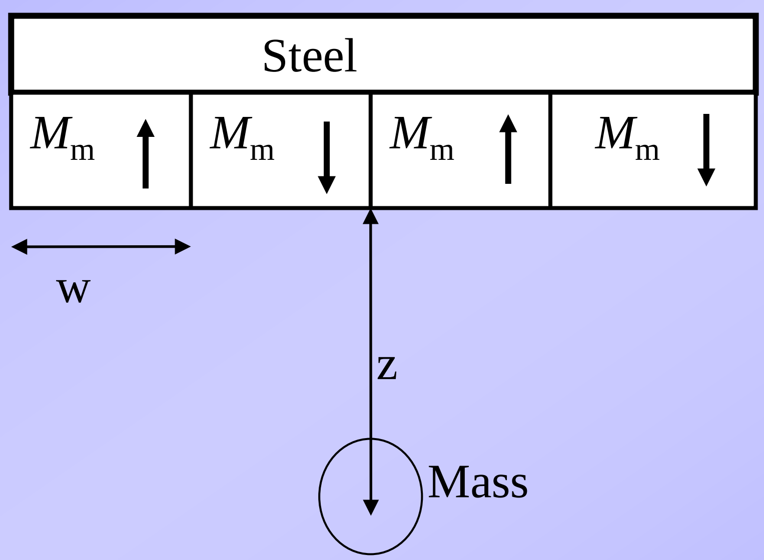
Art.no.	staaf ø 5x25	staaf ø 5x75	moer M16
28.190 t/m 28.192	165	225	130
28.200 t/m 28.210	255	370	180
28.230 t/m 28.240	260	380	195
28.310 t/m 28.318	295	430	225
28.320 t/m 28.328	315	460	240
28.330 t/m 28.340	335	480	250
28.353 t/m 28.367	360	500	275

Art.nr.	A	B	C
BM 28.330	835	810	350
BM 28.332	1040	810	350
BM 28.334	1250	810	350
BM 28.336	1450	810	350
BM 28.338	1650	810	350
BM 28.340	1850	810	350
BM 28.353	835	900	410
BM 28.355	1040	900	410
BM 28.357	1250	900	410
BM 28.359	1450	900	410
BM 28.361	1650	900	410
BM 28.363	1850	900	410
BM 28.365	2050	900	410
BM 28.367	2250	900	410



Magnetic separation: Multi-pole magnet

Multi-pole magnet: recovery of (partly: e.g. 20%) steel objects from a short distance (z = 0.1-0.3 m)



$$F = \frac{\mu_0 H V_{steel}}{N}\frac{dH}{dz}$$

$$= mg = \rho_{steel} V_{steel}\, g / 0.2$$

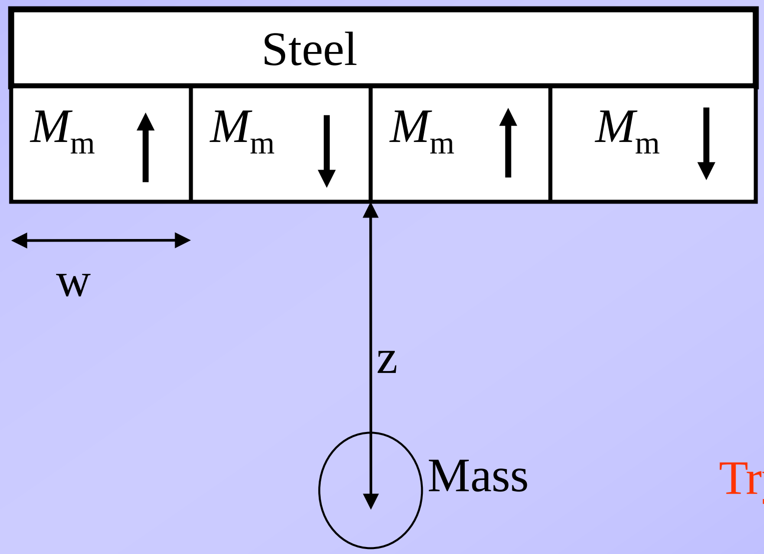
$$\Rightarrow \quad H\frac{dH}{dz} = \frac{\rho_{steel}\, g N}{0.2\mu_0} = \frac{8000 \times 10 \times 0.2}{0.2 \times 1.2\ 10^{-6}}$$

$$= 6\ 10^{10}\ \ A^2/m^3$$

Magnetic separation: Multi-pole magnet

Multi-pole magnet: recovery of (partly: 20%) steel objects from a short distance (z = 0.1-0.3 m)

$$\Rightarrow \quad H\frac{dH}{dz} = 6\ 10^{10}\ \ A^2/m^3$$



$$H = \frac{2M_m}{\pi} e^{-\pi z/w} \quad [A/m]$$

$$\frac{dH}{dz} = \frac{2M_m}{w} e^{-\pi z/w} \quad [A/m^2]$$

Try e.g. z=0.1m; w=0.12m; M_m=1,000,000 A/m

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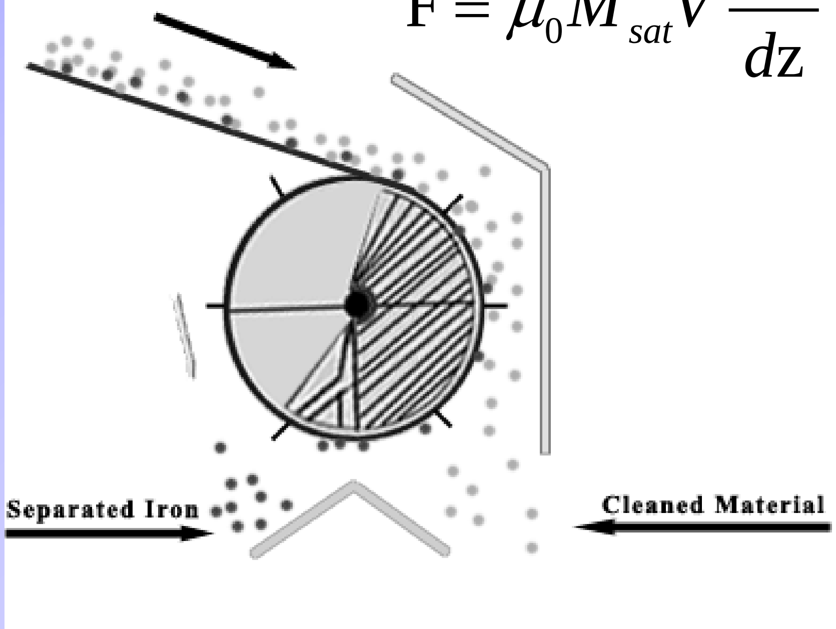
Magnetic separation: Drum magnet

Drum magnet: recovery of objects containing steel in contact with a multipole magnet (z = 0.02-0.03 m)



$$\vec{F} = \mu_0 M_{sat} V \frac{dH}{dz}$$

saturated ferromagnetic particle

M_{sat}=2,000,000 A/m; w=0.06 m

$$\frac{dH}{dz} = \frac{2M_m}{w} e^{-\pi z/w} \quad [A/m^2]$$

What is min. % steel for recovery??

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Magnetic separation: paramagnetic materials

$$\vec{F} = \mu_0 M V \frac{dH}{dz} \quad ; \qquad M = \chi H$$

$$= \mu_0 \chi H V \frac{dH}{dz} \qquad \text{paramagnetic particle}$$

H≈1,000,0000 A/m

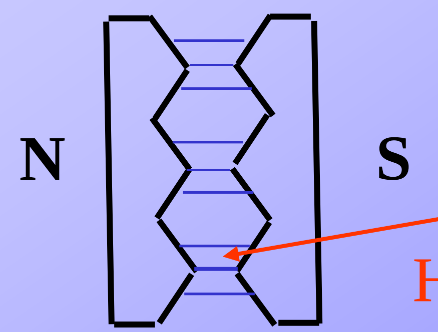
Example:

$$\chi \approx 0.001 \; ; \quad \rho = 5000 \text{ kg/m}^3$$

dz is mm-size *and so is the particle!!!*

$$mg = \rho g V = \mu_0 \chi H V \frac{dH}{dz}$$

$$\Rightarrow H \frac{dH}{dz} = \frac{\rho g}{\mu_0 \chi} = 4\ 10^{13} \text{ A}^2/\text{m}^3$$



Measurement of χ_s

Frantz Isodynamic separator: balance of gravity and known magnetic force

$$F_{magn} = c \cdot \chi V I^2$$

$$F_{grav} = \rho V g \sin \alpha$$

$$\chi_s = \frac{\chi}{\rho} = f \cdot \frac{\sin(\alpha)}{I^2}$$

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Eddy current separation

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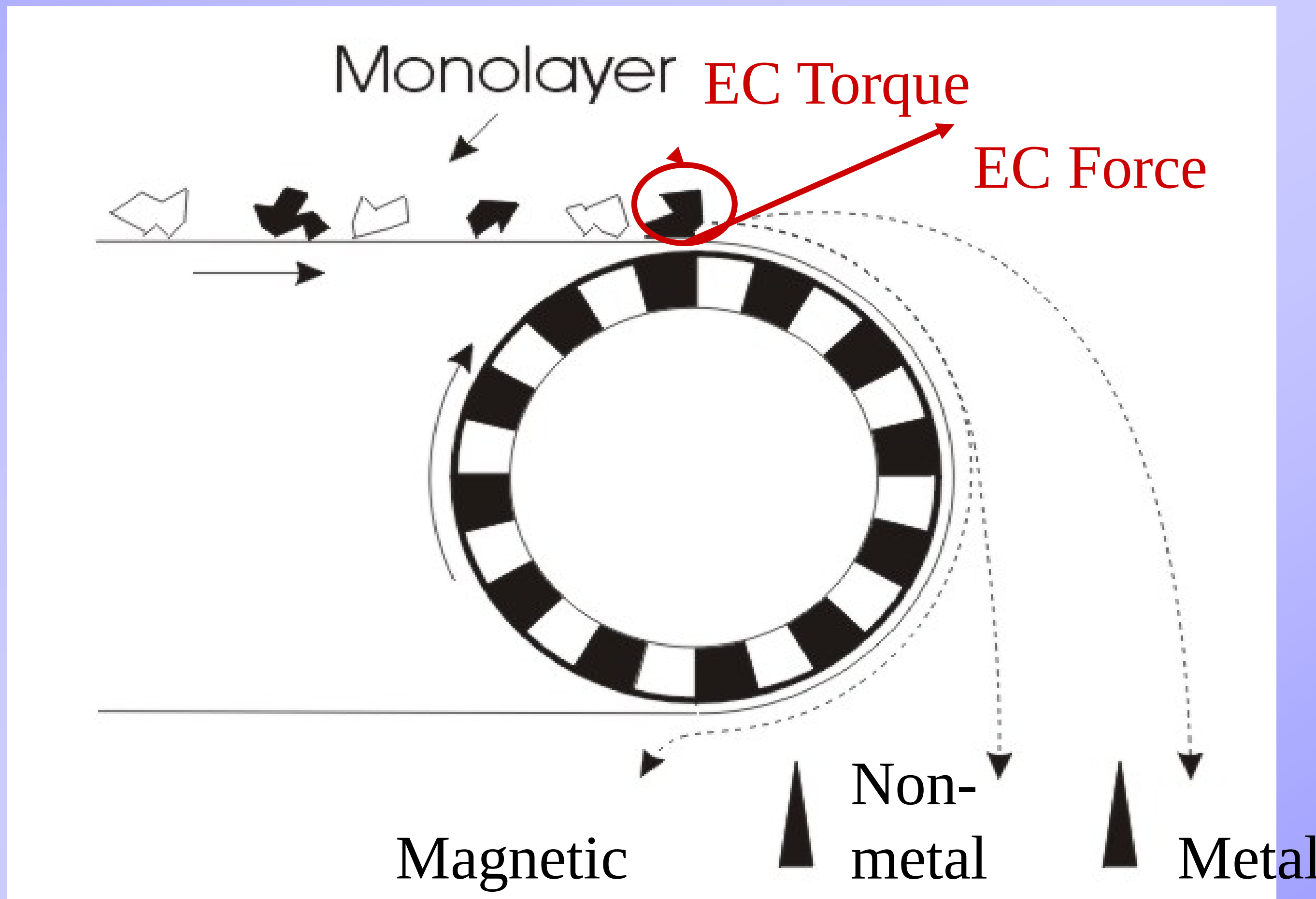
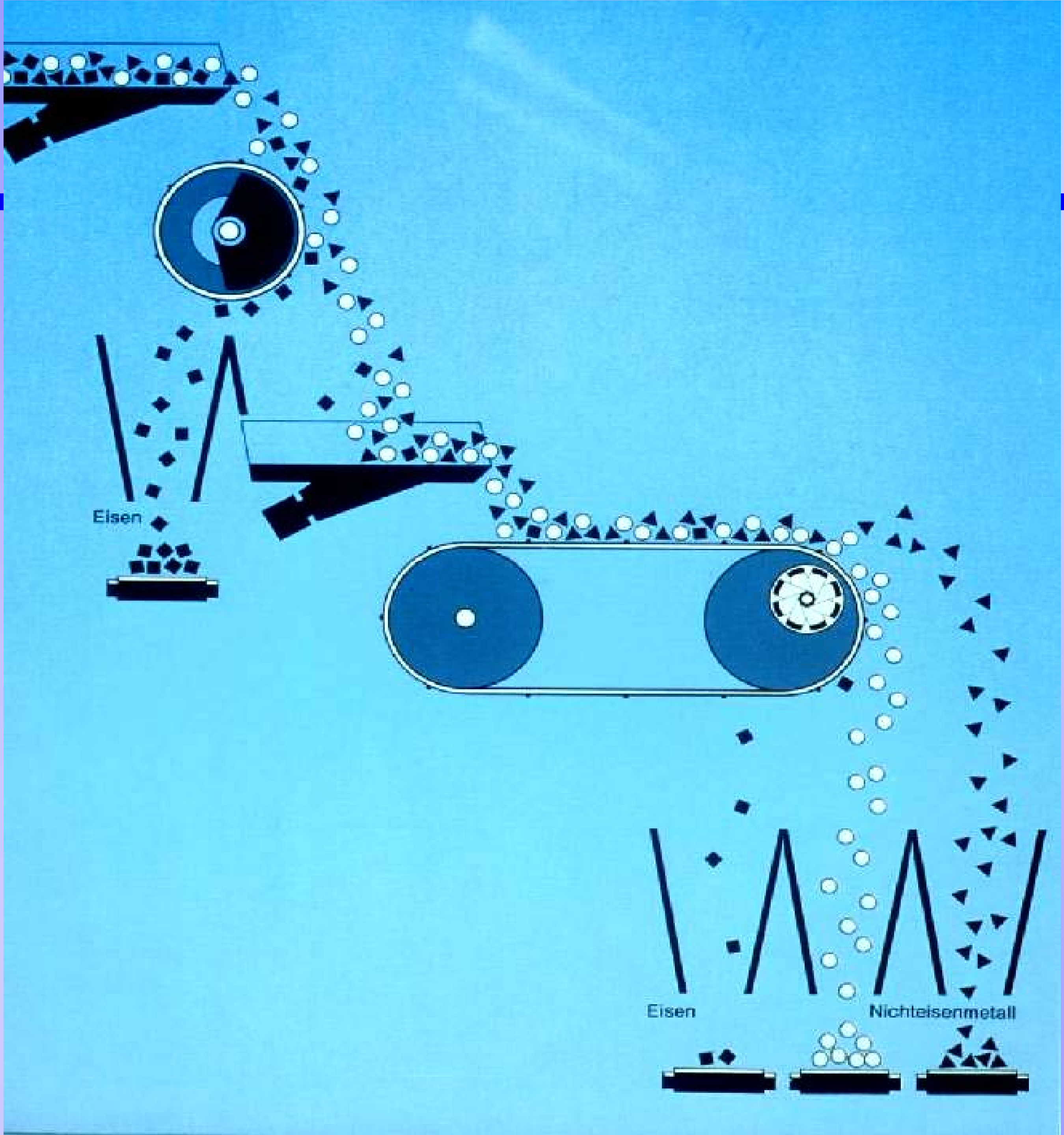
Eddy Current Separation (ECS)



CONDUCTIVITY / DENSITY RATIO

MATERIALS	$(\sigma / \rho) \times 10^3$ m^2/ohm kg
ALUMINIUM	13.1
COPPER	6.6
ZINC	2.4
BRASS	1.7
LEAD	0.4
GLASS/ PLASTIC	0


Eisen
Eisen
Nichteisenmetall