

3. Size Reduction

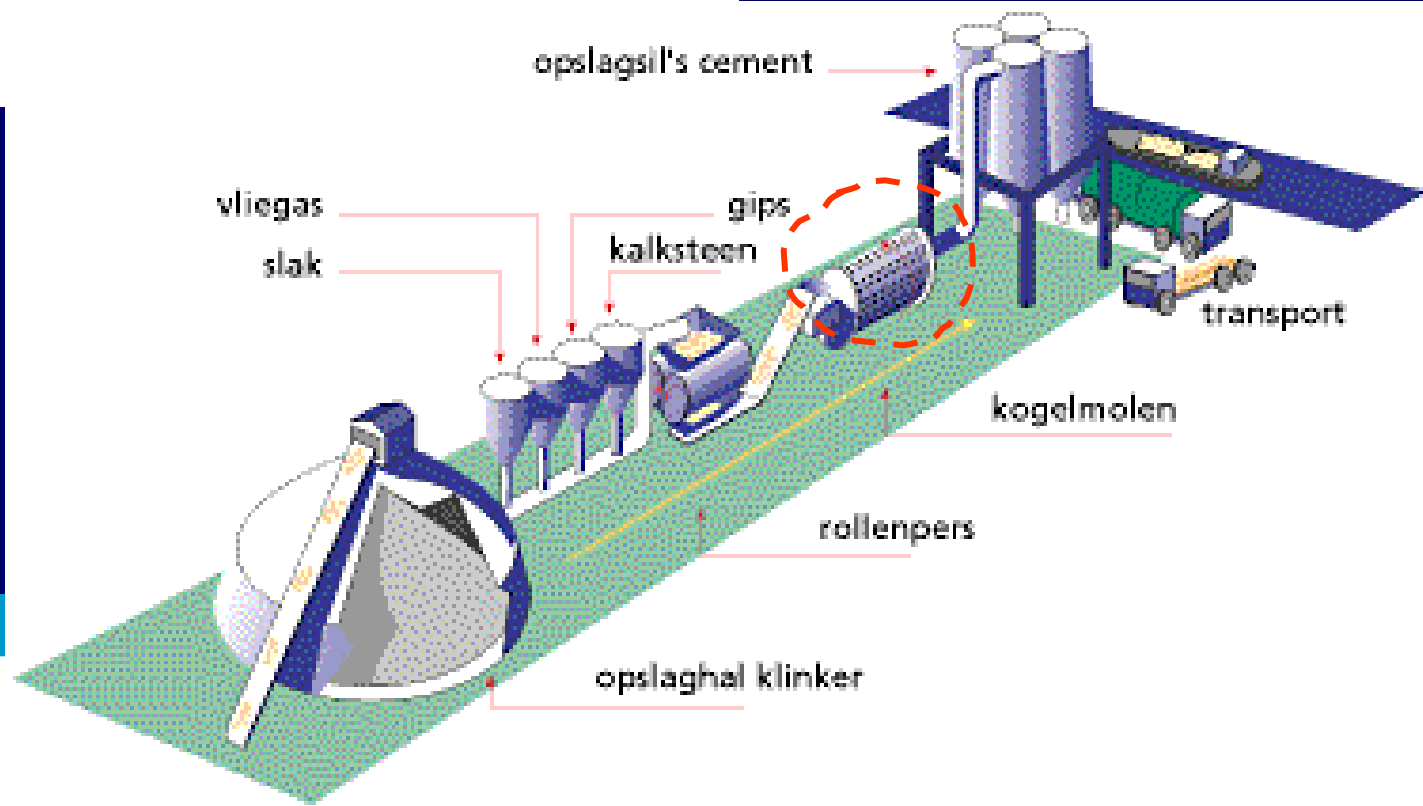
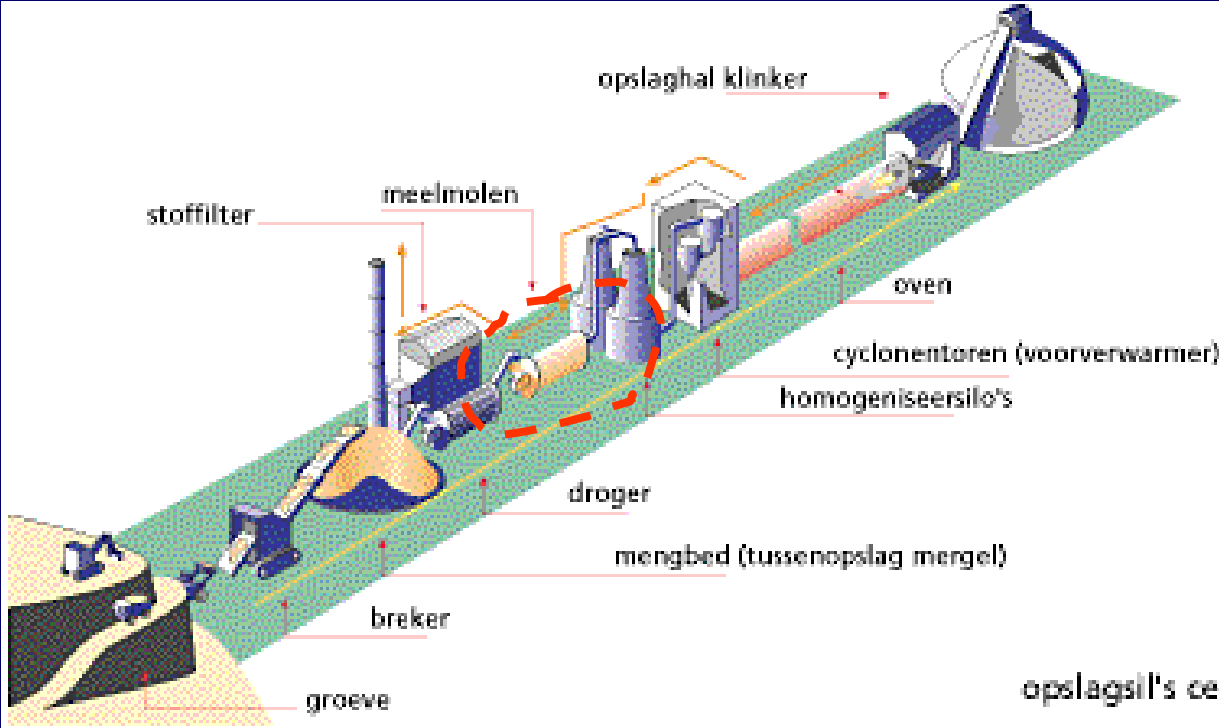


Applications

- Cement, limestone
 - ENCI / Heidelberger (Maastricht, IJmuiden)
- Industrial minerals
 - Ankerpoort, Maastricht, Geertruidenberg, Winterswijk
- Iron ore
 - Tata/Corus, IJmuiden pellet plant
- Lead/zinc ore
 - Lisheen, Galmoy, Tara (Ireland)
- Pulverised coal combustion (power plants, metallurgy)
- & Many, many others !!!!!

Let alone remainder of the world

Cement



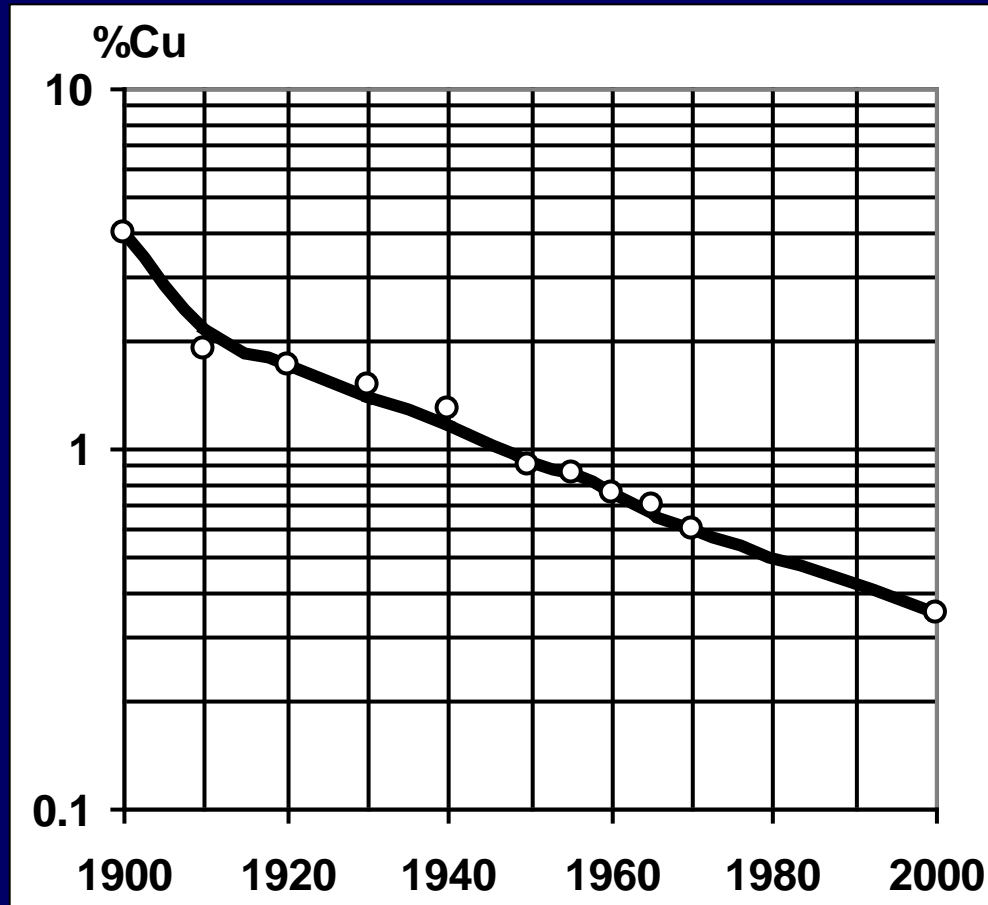
ENCI Maastricht

November 2012

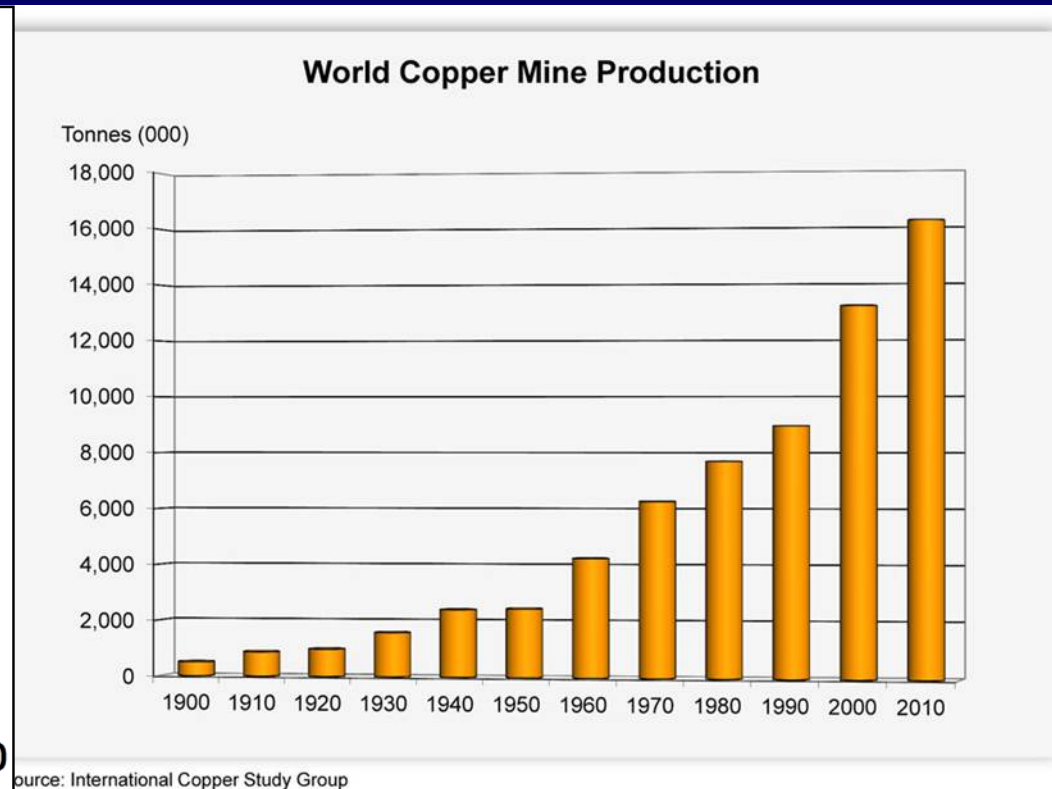
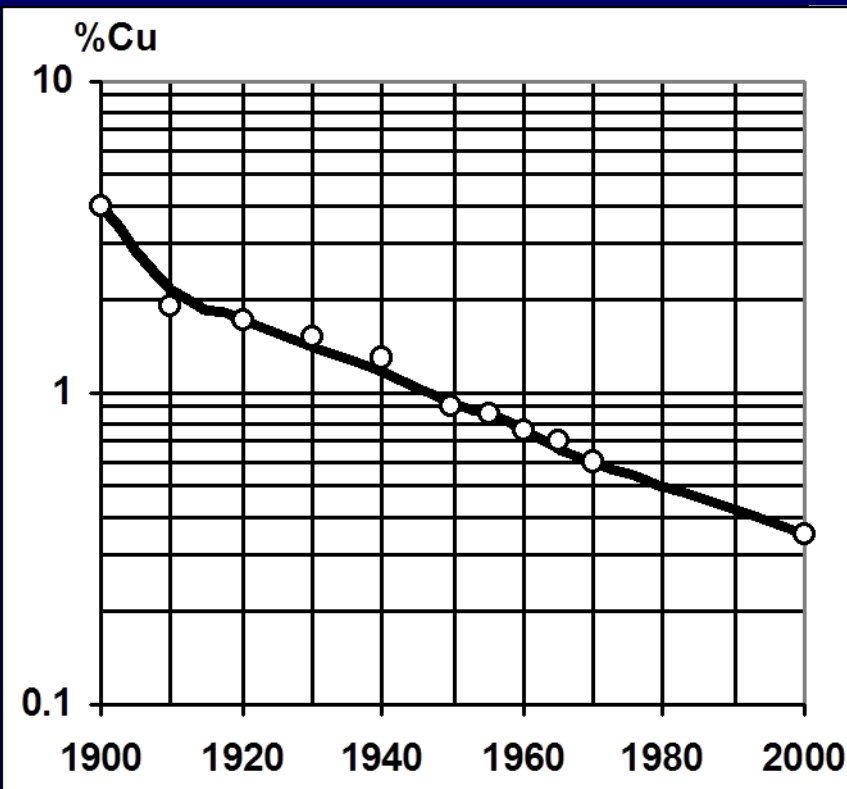
Why important ?

- $3 \cdot 10^9$ tons milled per year worldwide
 - This increases strongly every year
- It takes 5% of worlds electricity
- 5% to 15% of the total mine-to-metal costs (metal ore).

Decreasing grades → more milling

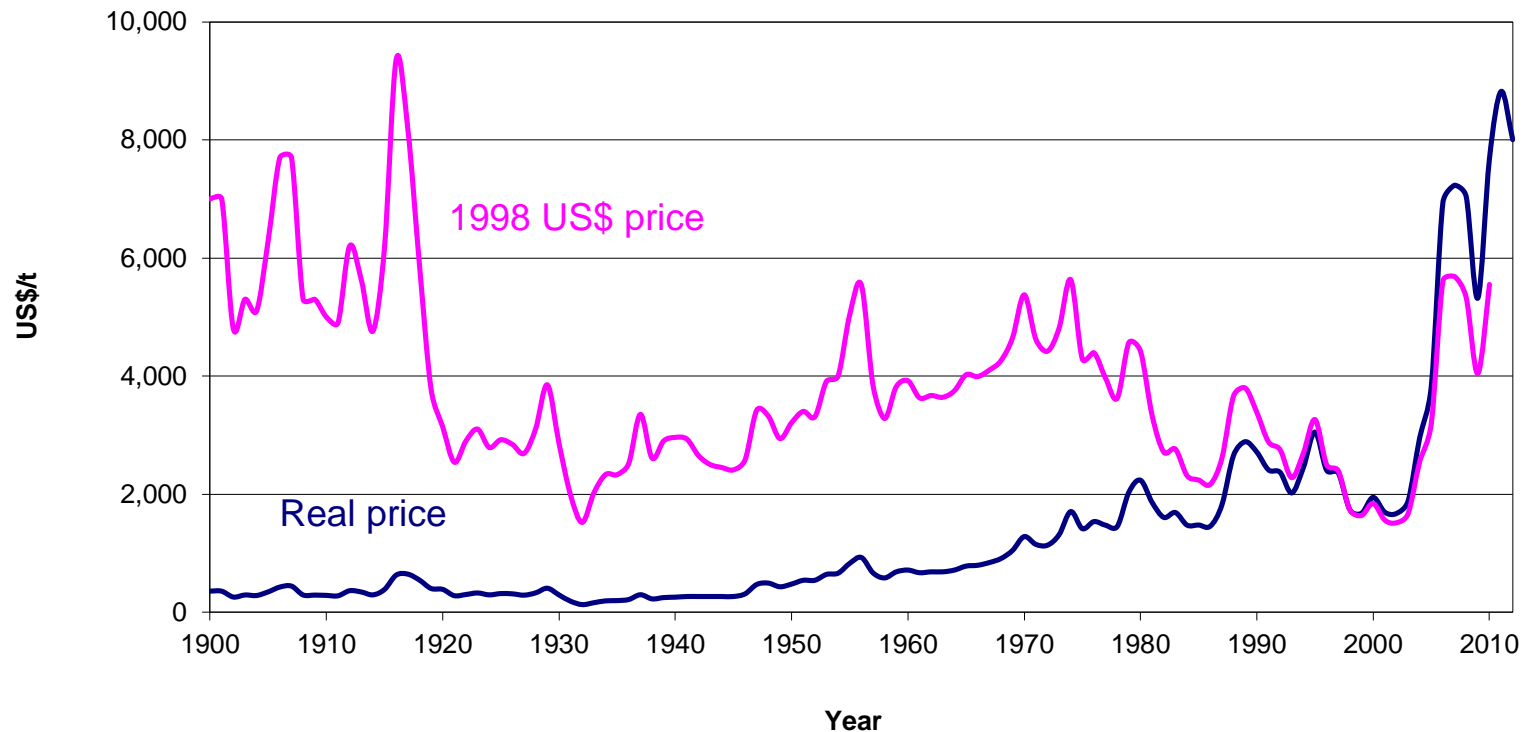


Decreasing grades, increasing demands.....



But what happened with the price?

Copper price evolution



Objective

- Liberation of valuable mineral(s)
- Increase reaction surface
- Create desired treatment-, use-, & storage properties

- Metal ore: 50 – 200 μm
- Gravel for road construction: max. 15-50 mm
- Coal: max. 50 mm

→ Uniform particle size is desired

Grade/recovery: Iron oxide example

1 ton ROM

Headgrade: 40% Fe

Concentrate: 65% Fe

Tailings: 20% Fe

Grade:

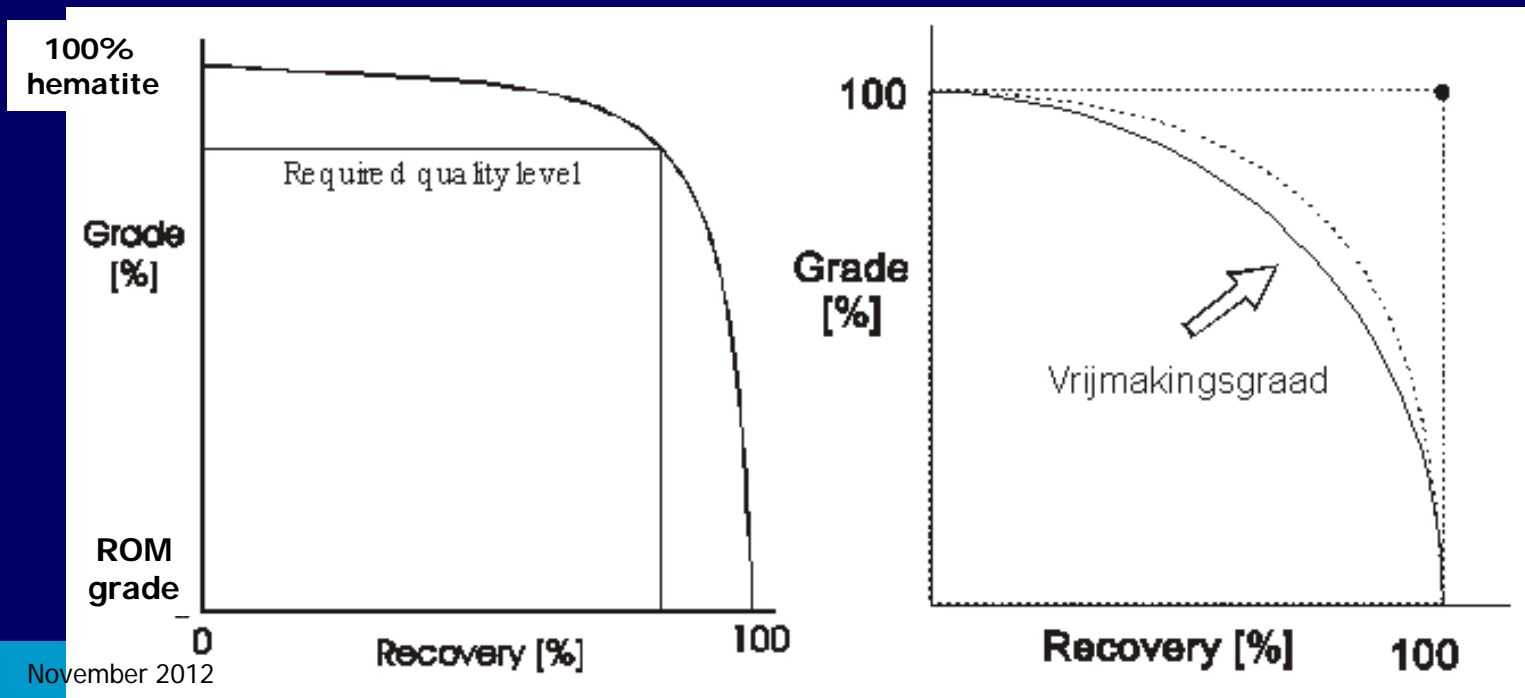
$$\frac{290}{444} * 100 = 65\%$$

Recovery:

$$\frac{290}{400} * 100 = 72.5\%$$

$$\frac{\text{kg Fe in concentrate}}{\text{total kg concentrate}}$$

$$\frac{\text{kg Fe in concentrate}}{\text{kg Fe in feed}}$$



First step

- Drilling / blasting
- Cutting

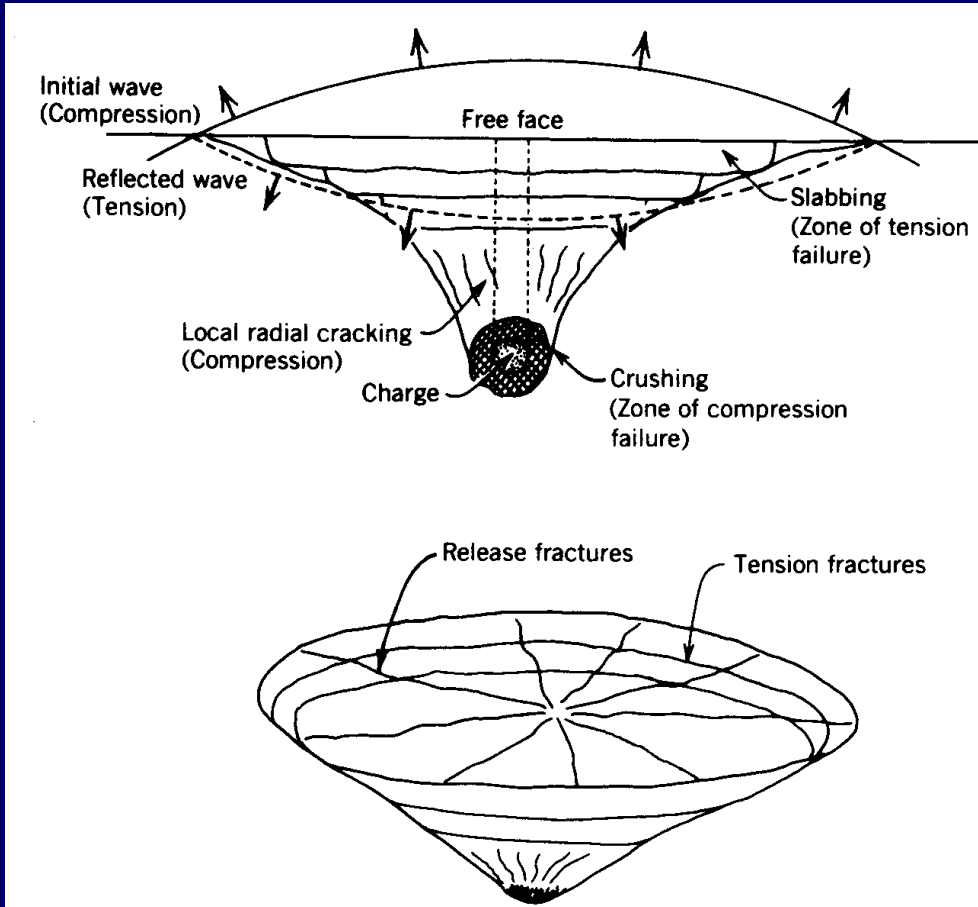
→ Reduction from infinite down to 100 – 1000 mm

Blasting



No uniform size
can be achieved:

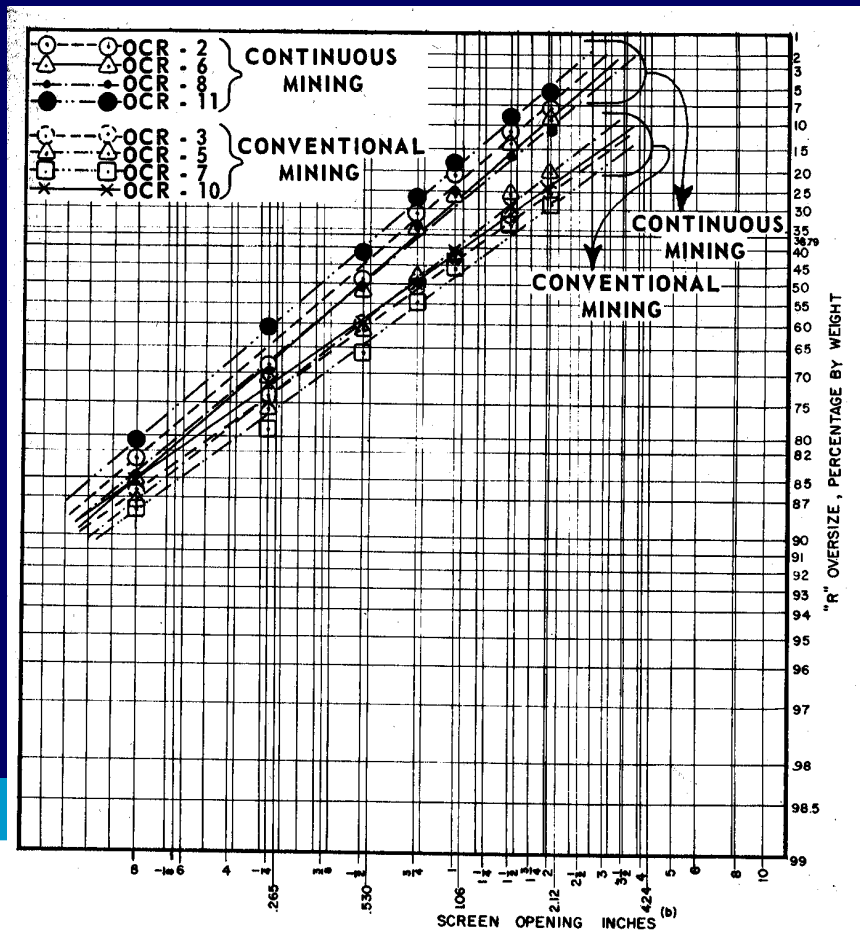
1 m³ blocks & ultra fines
generated at the same time





Cutting

- More uniform
- But still a wide range



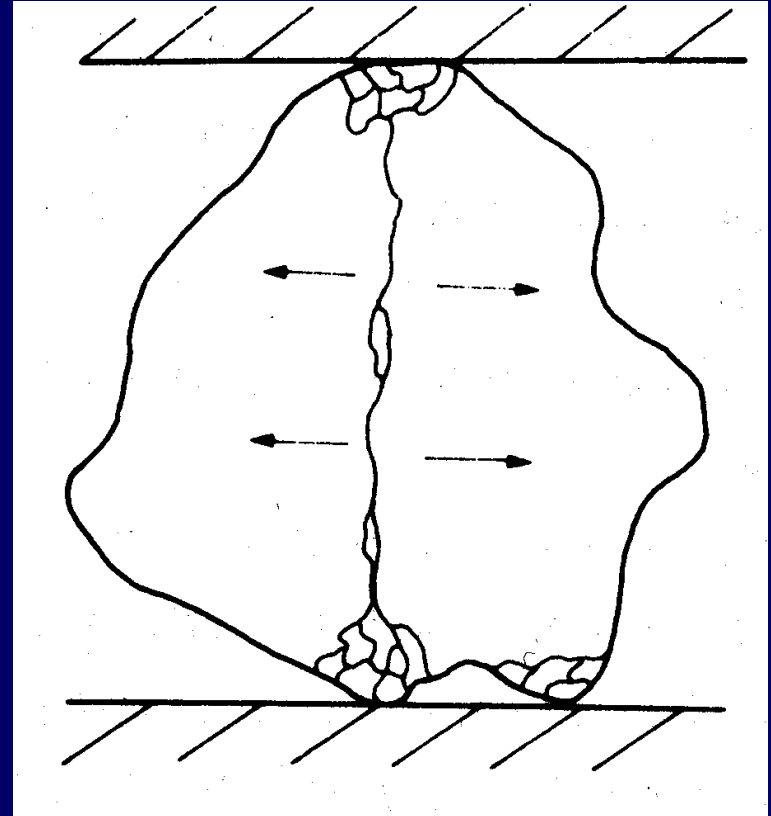
(Except dimension stone cutting)





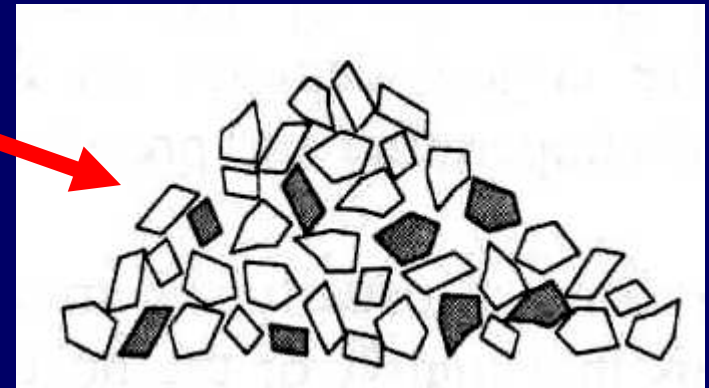
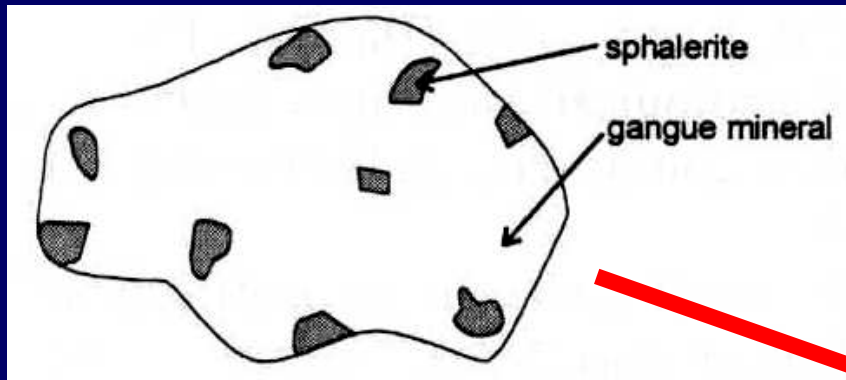
Size reduction

- Controlled conditions, but ...
- **Still no uniform product !**



Ideal liberation

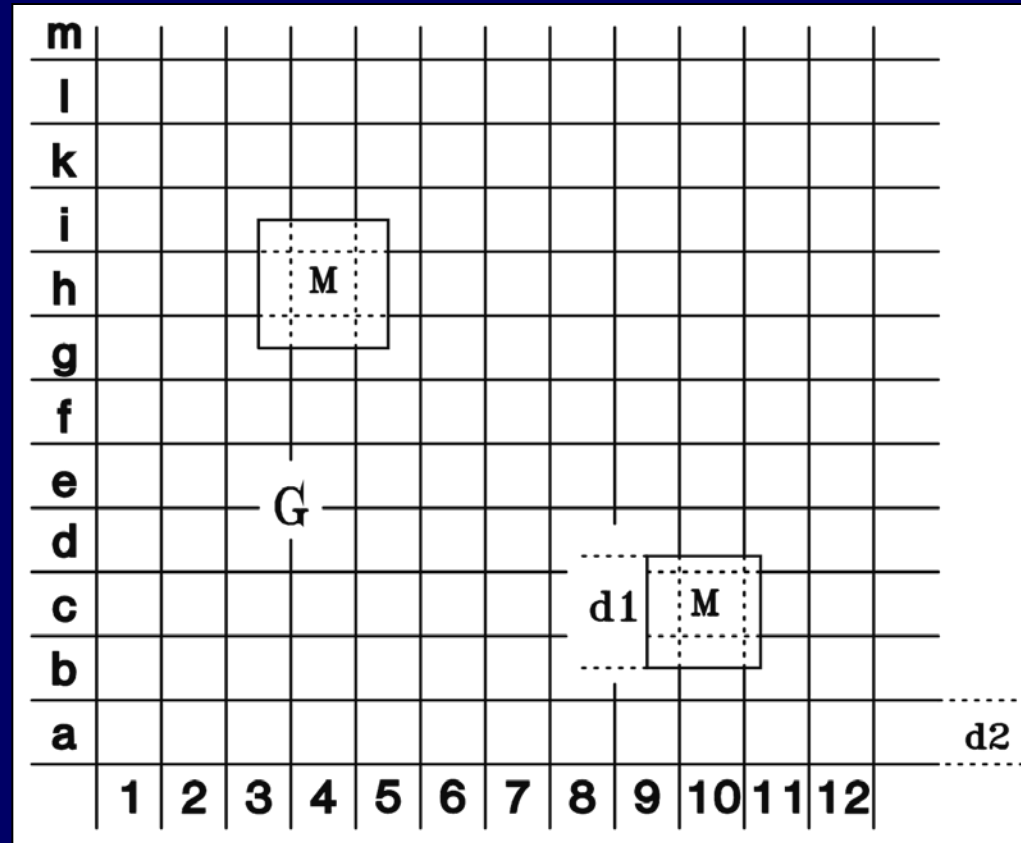
- Size reduction along grain boundaries
- Usually not possible



Liberation practice

Dominant component
(usually gangue) better
liberated

→ **Concentrator must
recover partially
liberated minerals
as well**

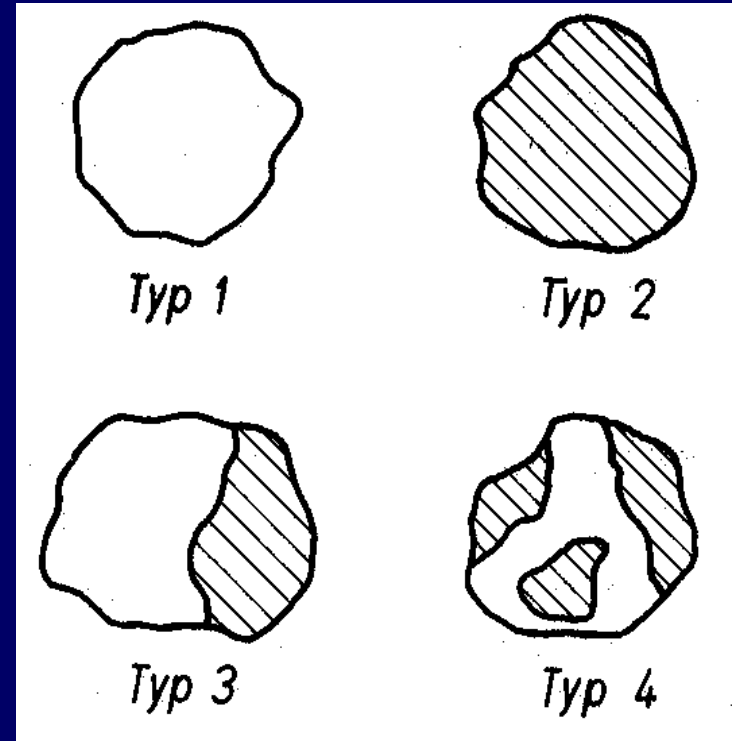


Liberation rate

- Objective: **Minimize** necessary size reduction
- Liberation as function of progressing size reduction depends on
 - Mineral
 - Type, properties, texture, shape
 - Volume concentration
- Determine optimum size
 - Not too large → insufficient liberation
 - Not too small → concentration too expensive or ineffective
- How?
 - Liberation analysis (or washability)
 - Prediction of mill output (simulation)

Liberation model

- Meloy (2 phase model)
 - Easily extendable to 3 and more phases)
- Particles of type 1,2,3 only
- Geometrically similar
- No selective (easier) breakage of inter mineral contacts
 - Magmatic & igneous, be careful with sedimentary deposits



Results

- U = volume unliberated
- L = volume liberated

$$d_1/d_2 = U_2/U_1$$

$$L+U=1$$

$d=1 \rightarrow$ Assume $L=60\%$

$d=0.5 \rightarrow L=80\%$

$D=0.25 \rightarrow L=90\%$

Note that in leaching, electrostatic separation, flotation etc. only minerals that are at surface are determining.

Results

- U = volume unliberated
- L = volume liberated

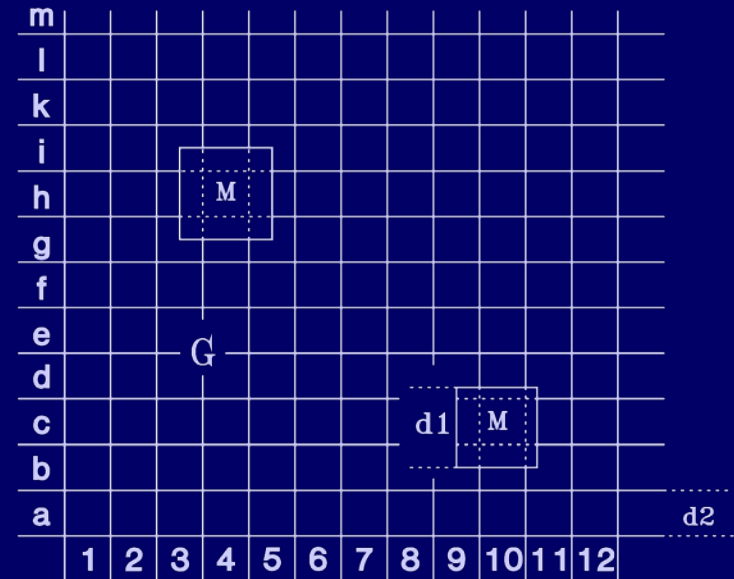
$$L+U=1$$

$$D = d_1/d_2 = U_2/U_1$$

$$D=1 \rightarrow \text{Assume } L=60\%$$

$$D=2 \rightarrow L=80\%$$

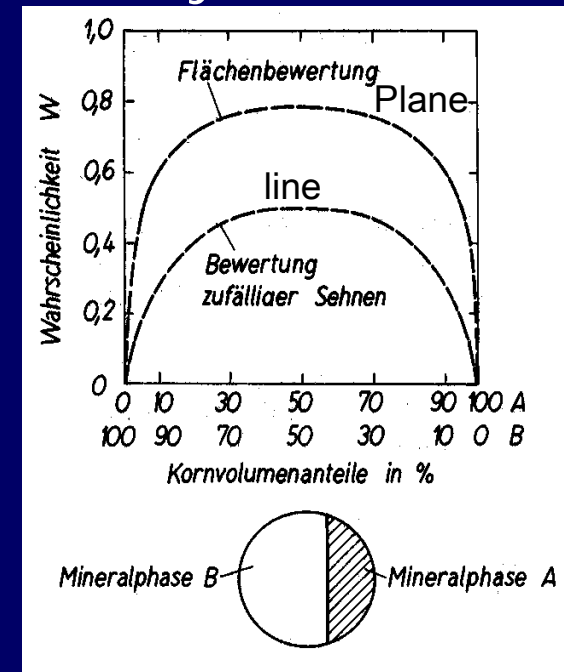
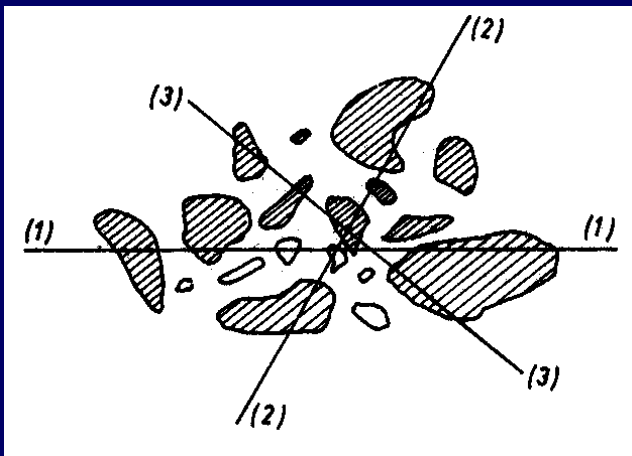
$$D=4 \rightarrow L=90\%$$



Note that in leaching, electrostatic separation, flotation etc. only minerals that are at surface are determining.

Liberation analysis

- By (microscopic) imaging: too optimistic
 - An intergrown particle has a high probability to be seen as one type of mineral
 - By density analysis
- See course notes pages 56 - 58

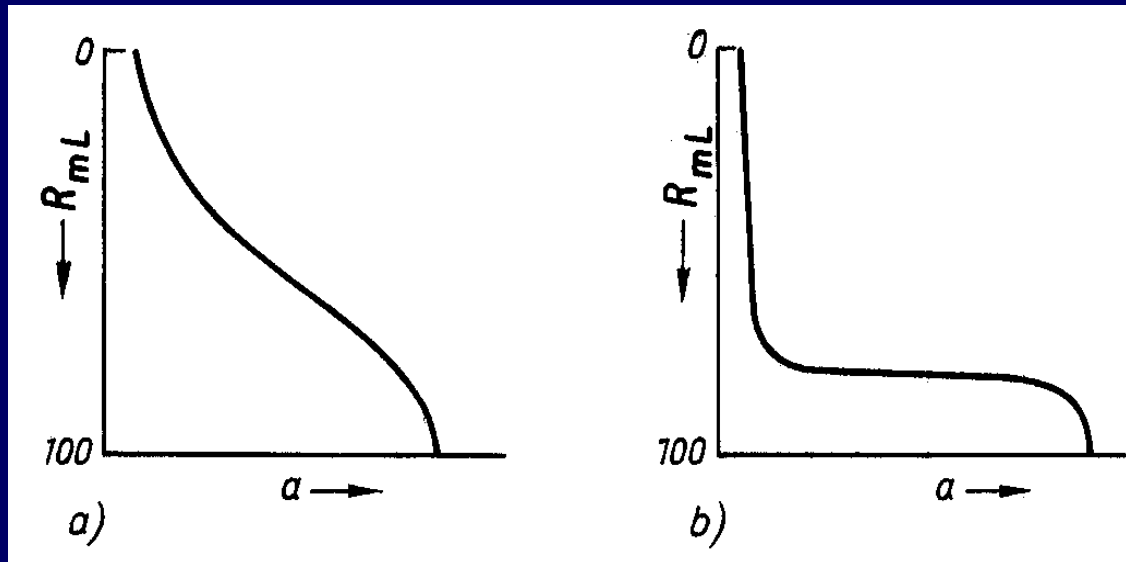


W = probability to observe two phases

Liberation (washability)

Ash distribution of coal/shale system

Coal ash content "a" (x-axis), plotted against cumulative total mass recovery of floats (y-axis).



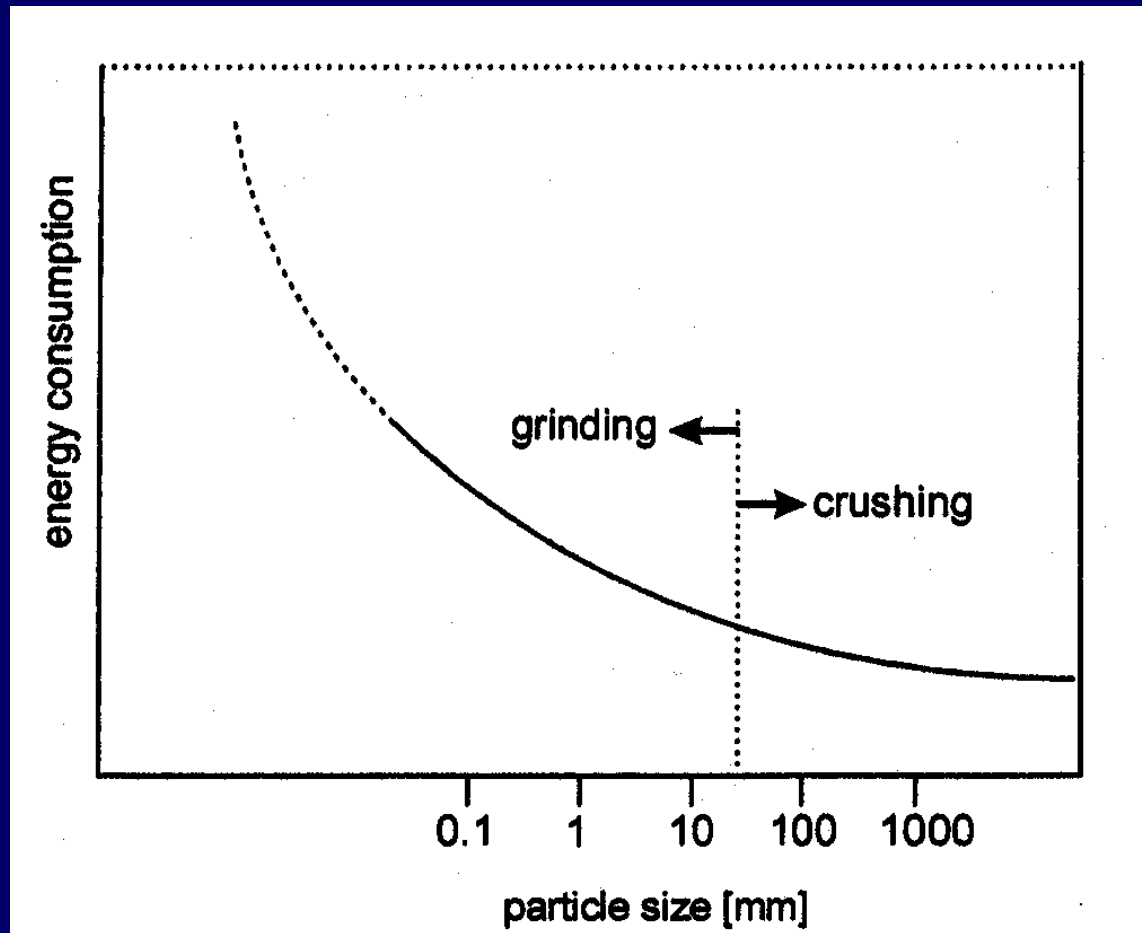
Poor liberated

Well liberated

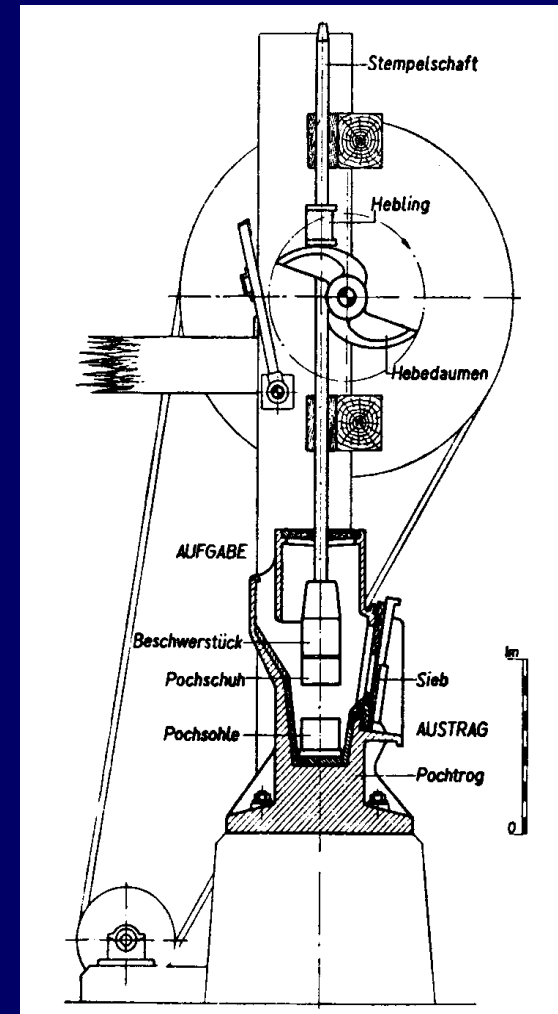
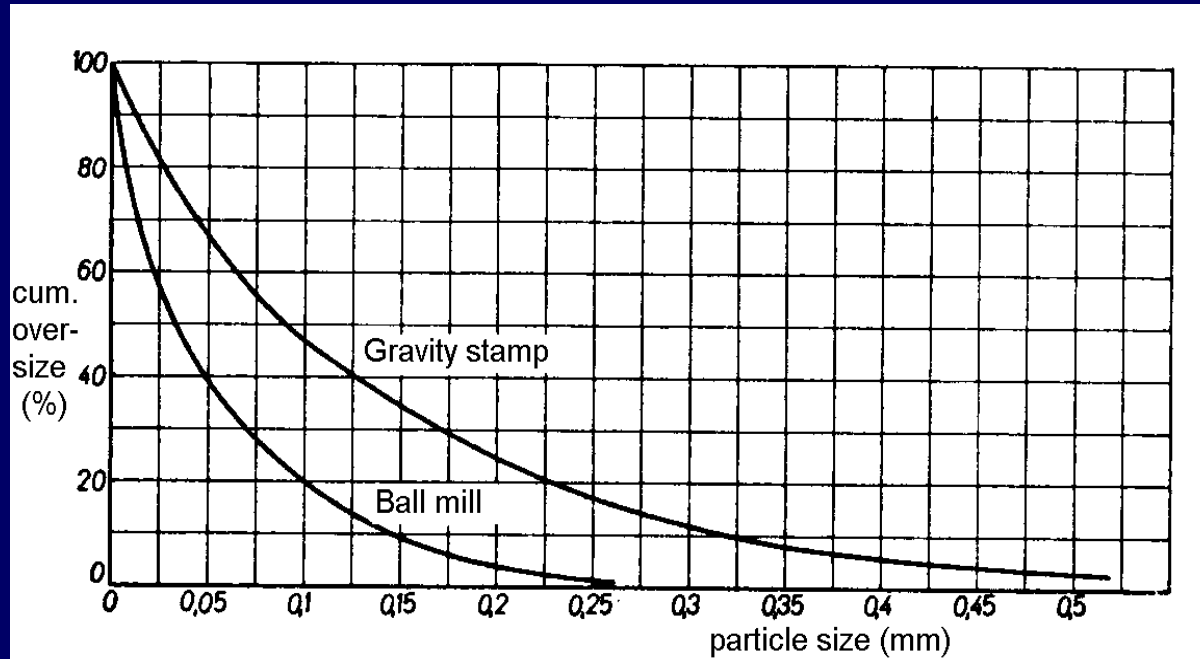
Crushing methods

- Explosives
- Ball breaking
- Pneumatic hammer (hand-held / crane mounted)
- Cutting
- Hammer/impact crushers
- Jaw crushers
- Roll crushers
- Grinding

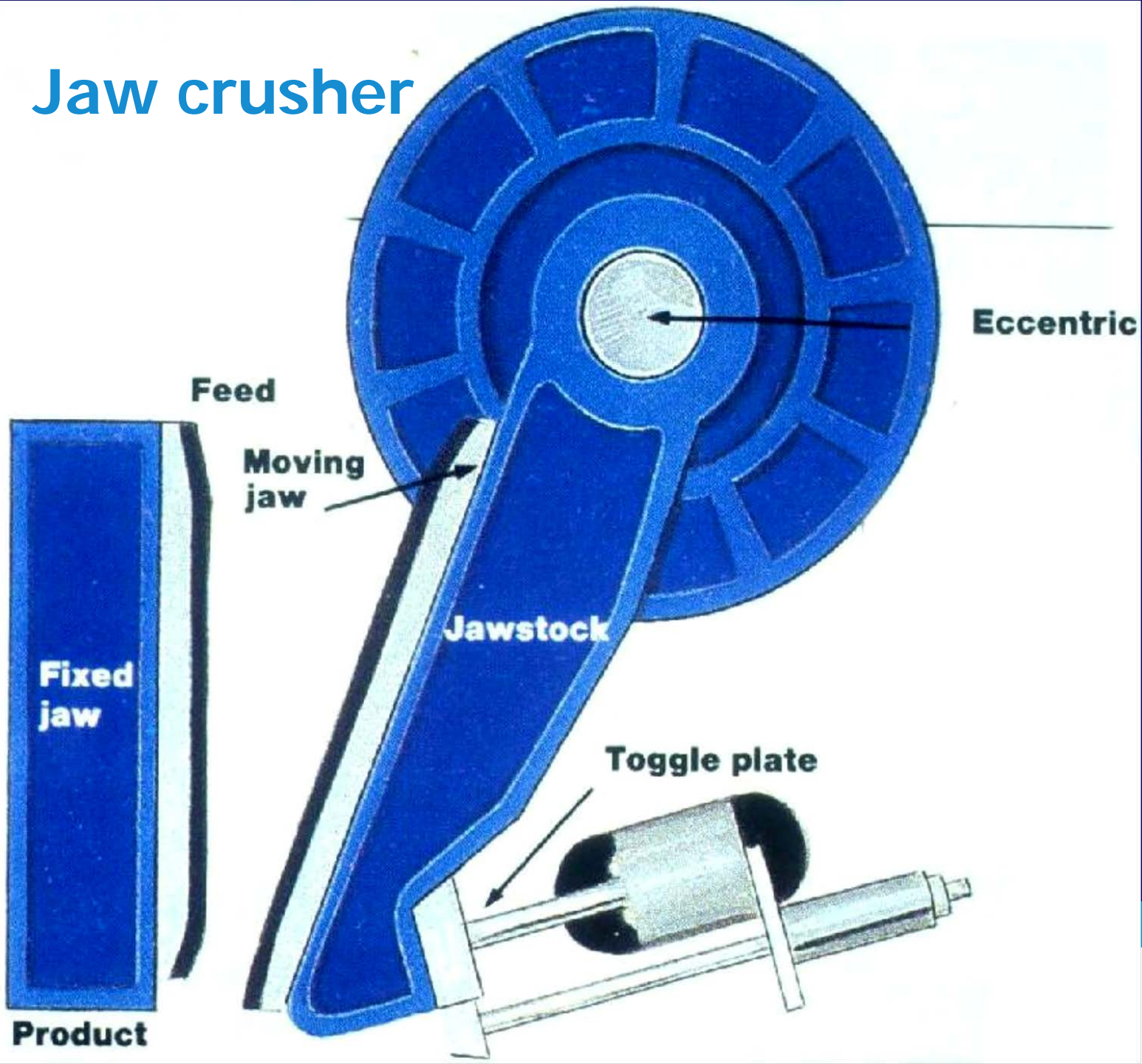
Crushing & grinding



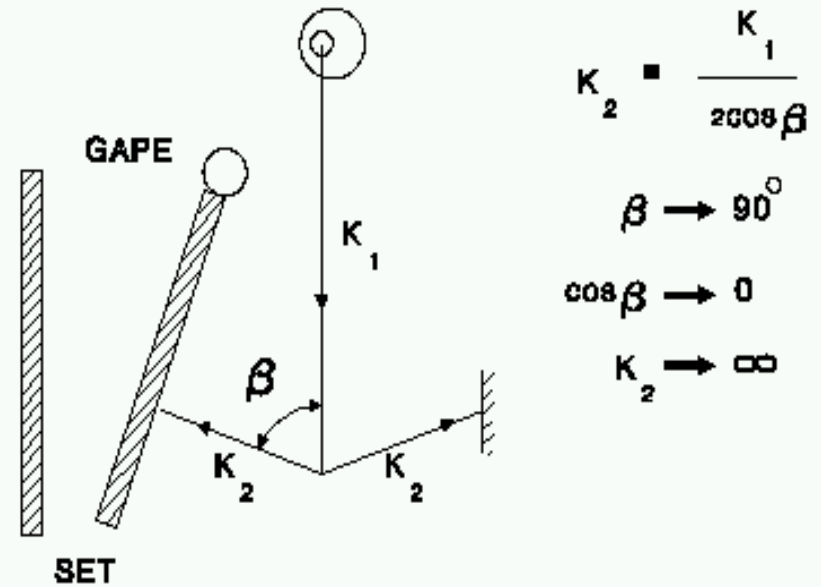
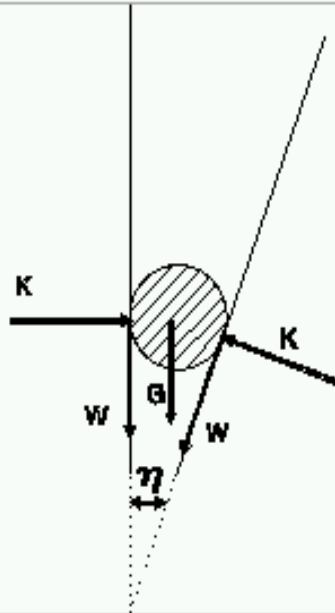
Gravity stamp



Jaw crusher



Jaw crusher principle



$$K_2 = \frac{K_1}{2 \cos \beta}$$

$\beta \rightarrow 90^\circ$
 $\cos \beta \rightarrow 0$
 $K_2 \rightarrow \infty$

1. Negligible weight G relative to other forces
2. K = crushing force
3. W = friction force

$$K \cdot (1 - \cos(\eta)) - W \cdot \sin(\eta) = 0$$

$$\frac{W}{K} = \tan(\varphi) \quad (\text{Friction angle})$$

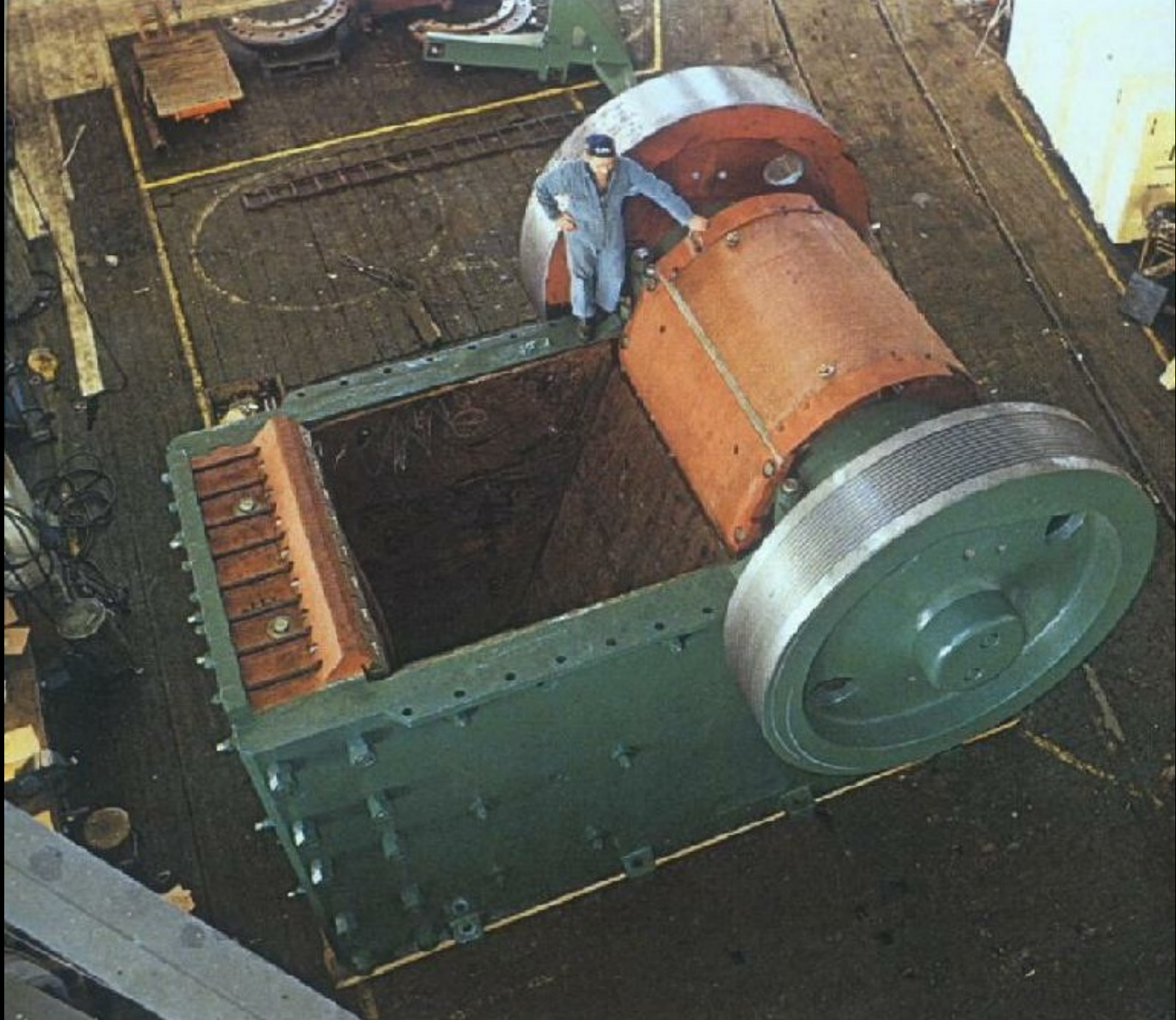
$$K - K \cdot \cos(\eta) - W \cdot \sin(\eta) = 0 \quad (1)$$

$$K \cdot \sin(\eta) - W - W \cdot \cos(\eta) = 0 \quad (2)$$

$$W \leq K \quad \text{boundary case } W = K \quad (3)$$

$$\frac{W}{K} = \frac{1 - \cos(\eta)}{\sin(\eta)} = \frac{2 \sin^2\left(\frac{\eta}{2}\right)}{2 \sin\left(\frac{\eta}{2}\right) \cos\left(\frac{\eta}{2}\right)} = \tan\left(\frac{\eta}{2}\right)$$

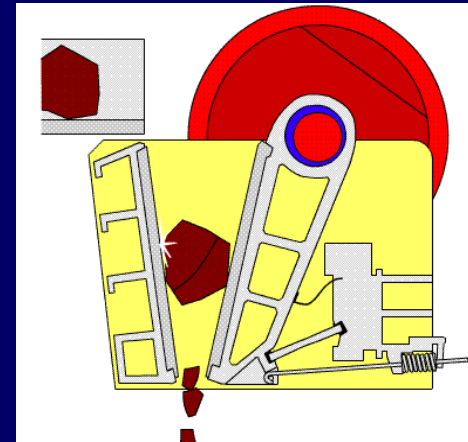
hence: $\eta < 2 \cdot \varphi$





Jaw crusher

- Intermittent breaking action
- Based on pressure
- Heavy fly wheels
- Nip angle 16° to 22° (larger angles for softer rock)
- Optimized stroke frequency n
 - Too high: material does not fall down
 - Too low: material densifies
 - n higher for smaller crushers
- Stroke length 10 – 50 mm.
- Constant feeding required
- Capacities 1 – 1000 m³/h
- Power 1 ~ 2 kWh/t



Jaw crusher advantages

- Little head room required, favourable for underground crushing
- Easy replacement of worn parts
- Easy adjustment of set opening

Jaw crusher disadvantages

- Expensive, heavy foundations necessary due to intermittent crushing action
- Emergency stopping impossible due to fly wheels
- Re-start with choked crushing chamber impossible
- Flat objects may pass uncrushed
- A special feeder for constant feed rate is needed to prevent choking

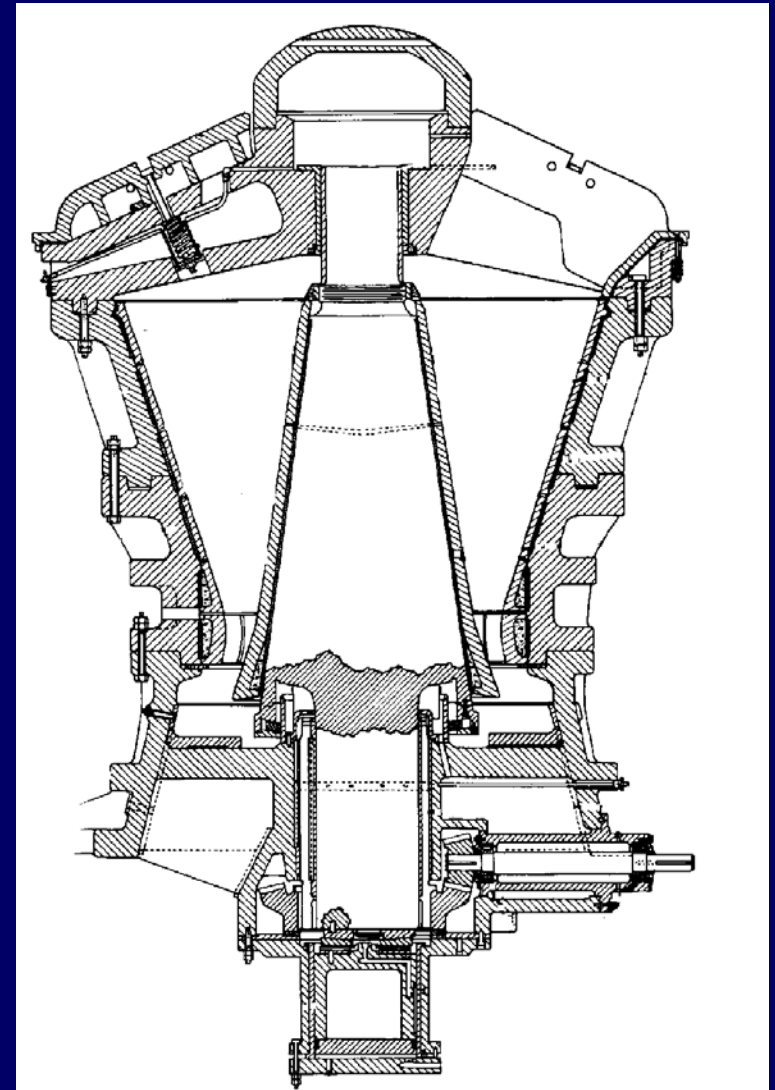
Gyratory crusher

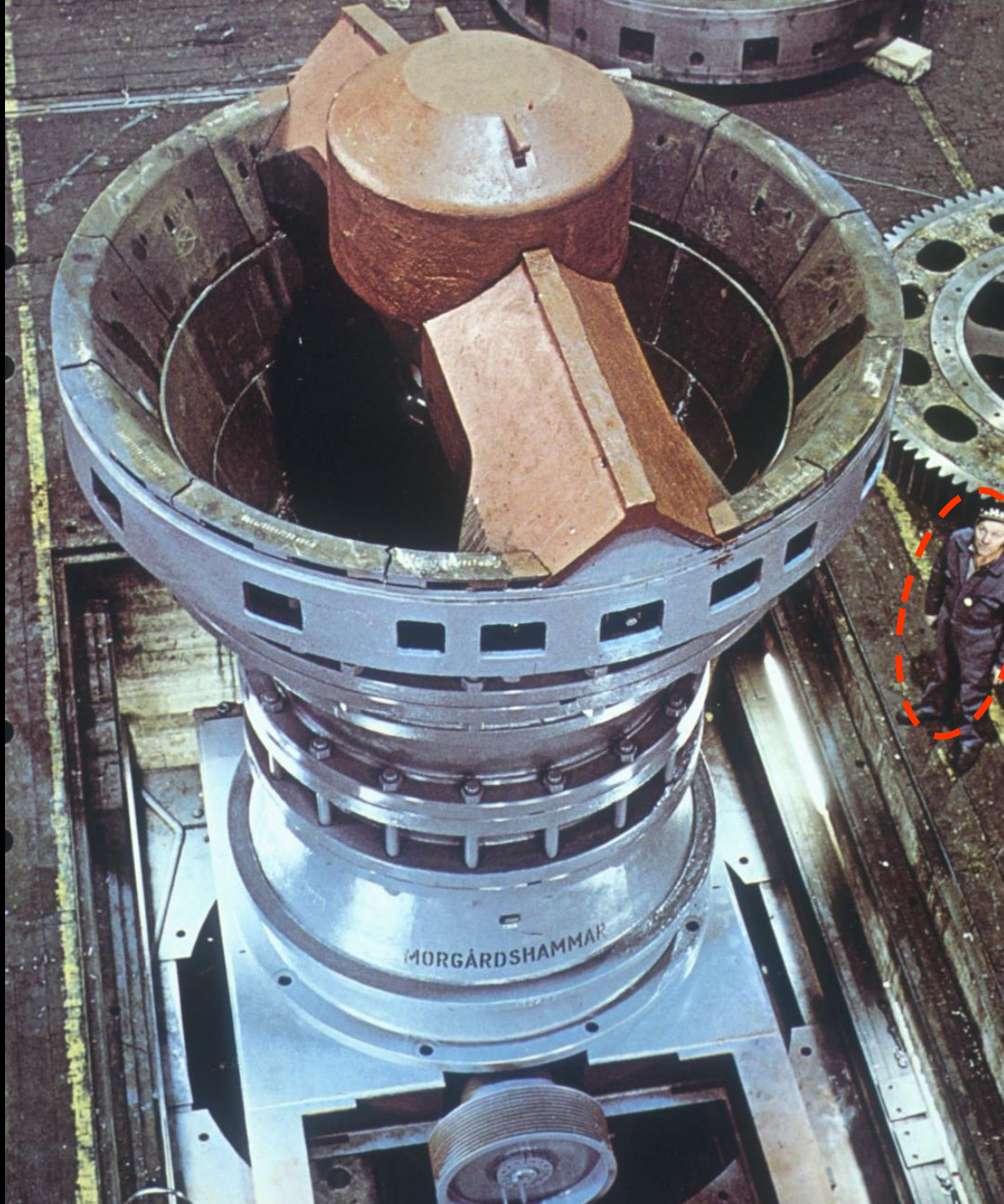
- Principle similar as jaw crusher
- Continuous breaking
- Based on pressure
- No feeder required
- Capacity in m³/h:

$$V = 0.8D^{2.5}S$$

D = lower cone diameter [m]

S = set_{min,max} [mm]





MORGÅRD SHAMMAR

Gyratory crusher advantages

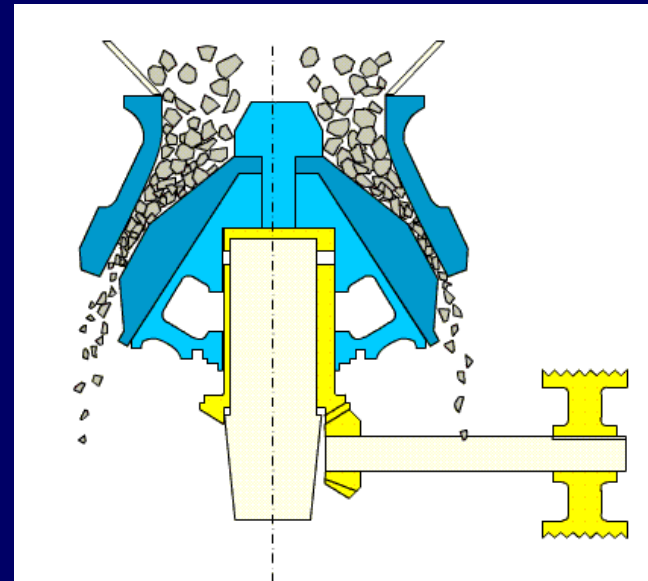
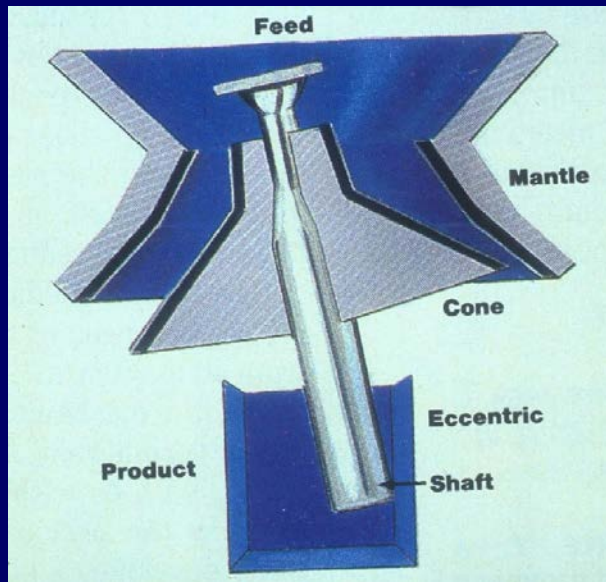
- Very high capacities (up to 8000 t/h)
- High energy efficiency
- Costs of foundation lower (continuous crushing action)
- Less choking problems
- Less sensitive for unbreakable material (no fly wheels)
- Direct feeding from different sides with 200 – 300 tonne mine dumpers is possible
- Irregular feed and choke feed no problem
- Emergency stop possible, no fly wheels
- Re-start with filled crushing chamber is possible

Gyratory crusher disadvantages

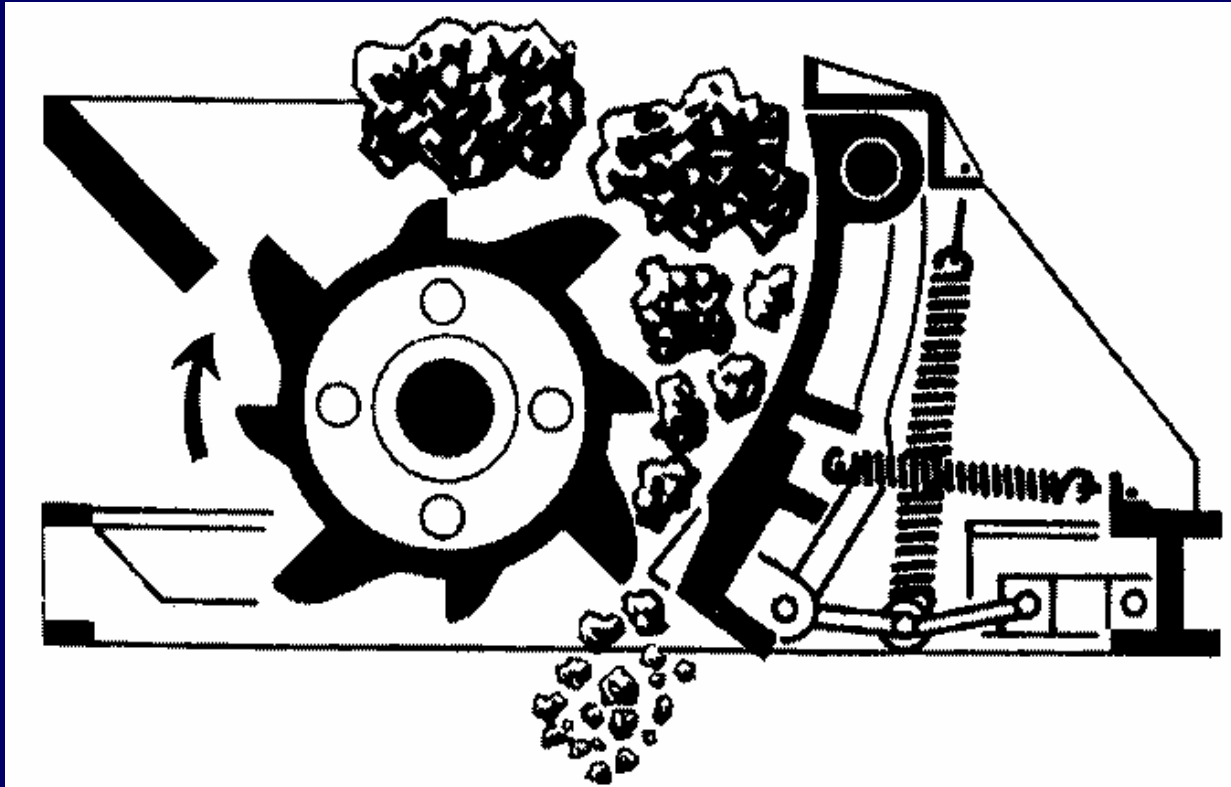
- Complex construction
- Feeding with soft material is impossible (rule of thumb: “if it can’t be screened, it can’t be gyratory crushed”)
- High wear of bearing
- Specific shape of the crushing plates: turning of liner plates is impossible, higher replacement costs.

Cone crusher

- Secondary crusher, down to 3 mm product
- Runs faster as gyratory crusher 300 – 600 min⁻¹
- Crushing by impact between the cone and mantle

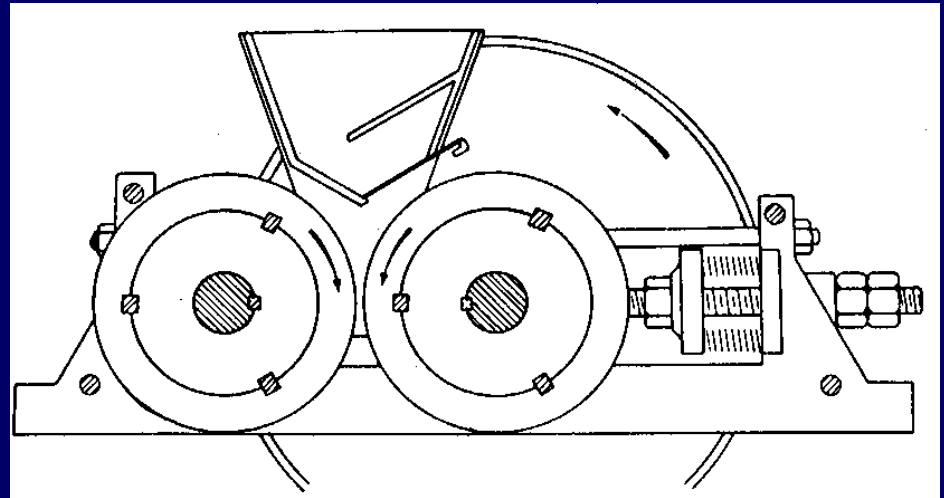


Primary coal crusher

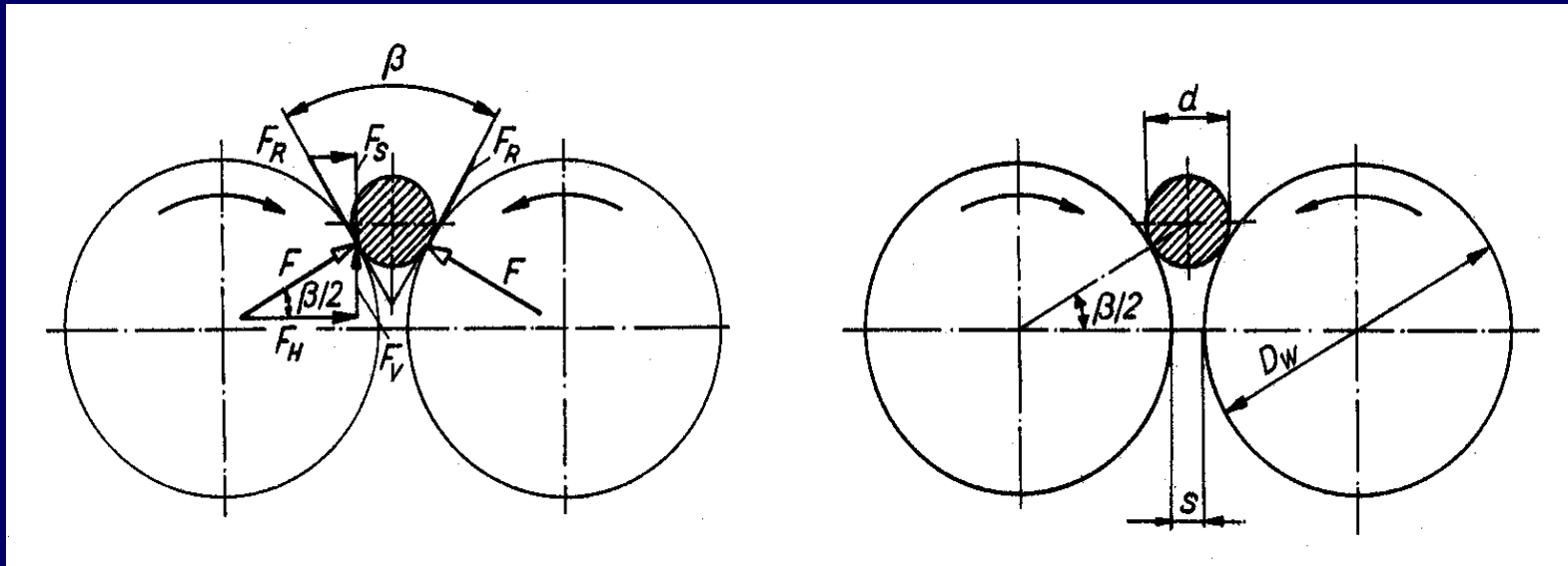


Grinding rolls

- Typical secondary crusher
- High pressure grinding rolls (cement industry)
- 1 to 2 kWh/t energy use (= low)
- For friable, sticky, froze, less abrasive feeds
 - Limestone
 - Coal
 - Chalk
 - Gypsum
 - Salt
 - Phosphate
 - Soft iron ores

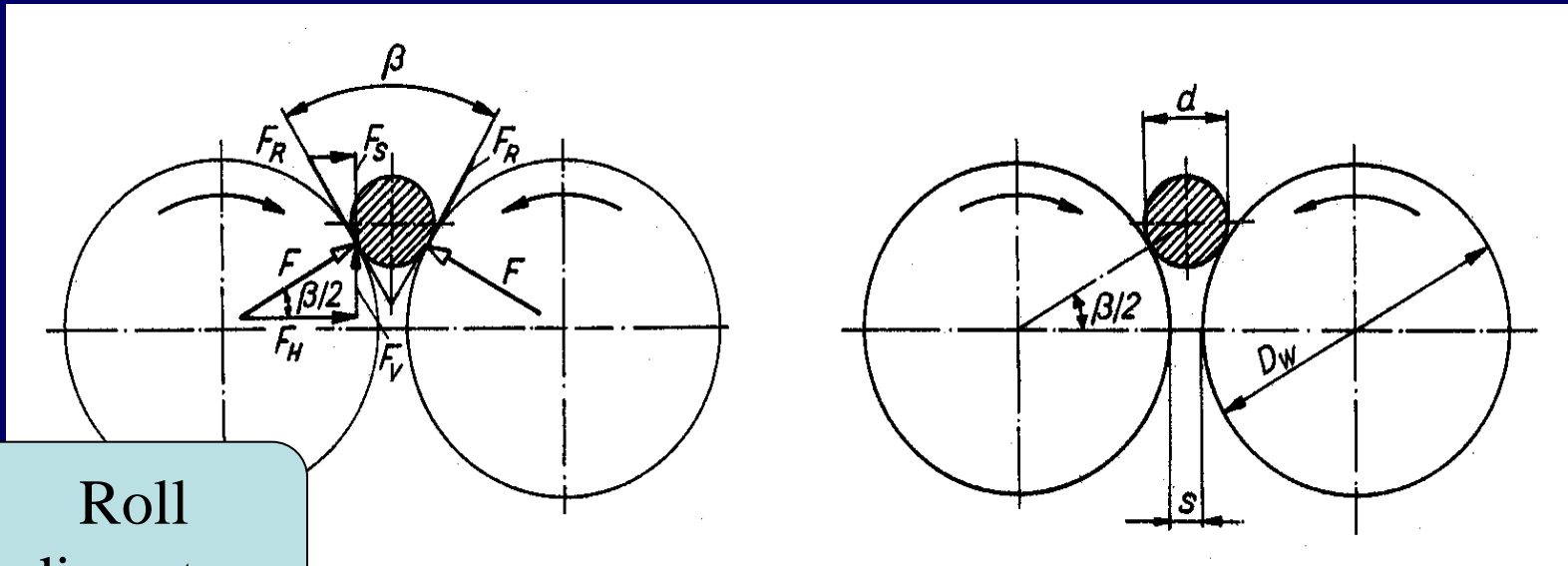


Roll crusher



$$D_w \geq \frac{d_0 - S\sqrt{1 + \mu^2}}{\sqrt{1 + \mu^2} - 1}$$

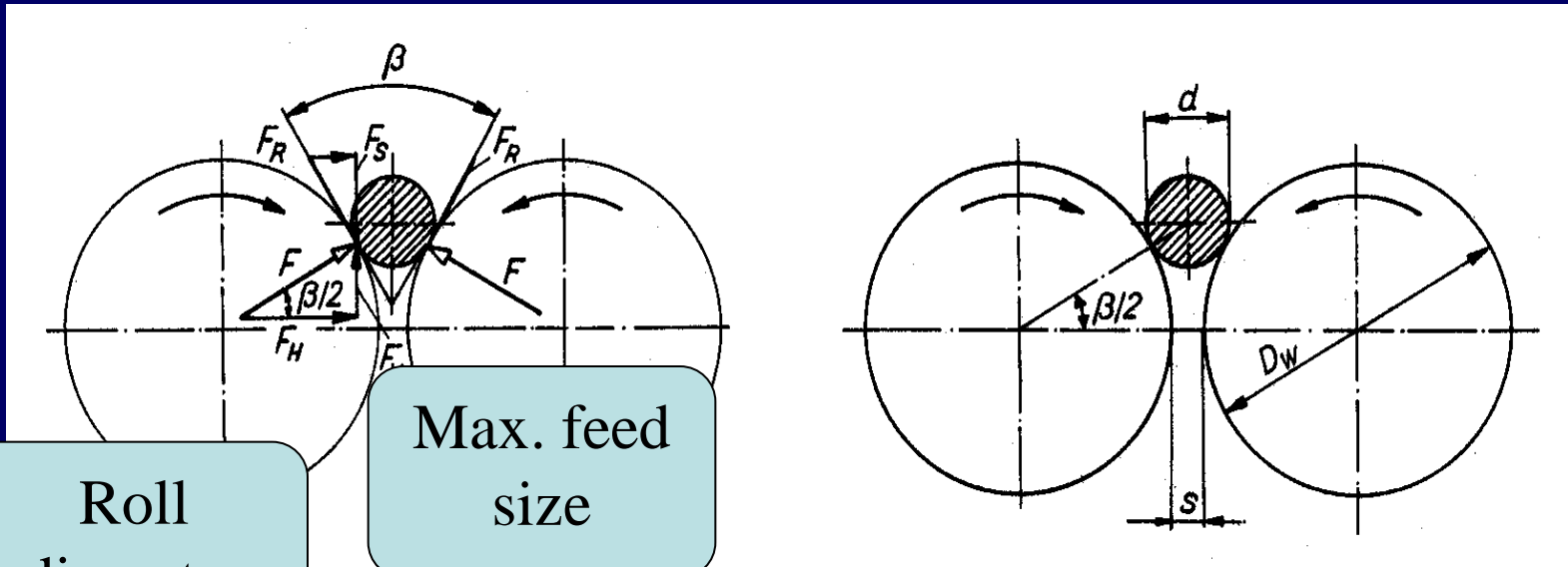
Roll crusher



Roll diameter

$$D_w \geq \frac{d_0 - S\sqrt{1 + \mu^2}}{\sqrt{1 + \mu^2} - 1}$$

Roll crusher

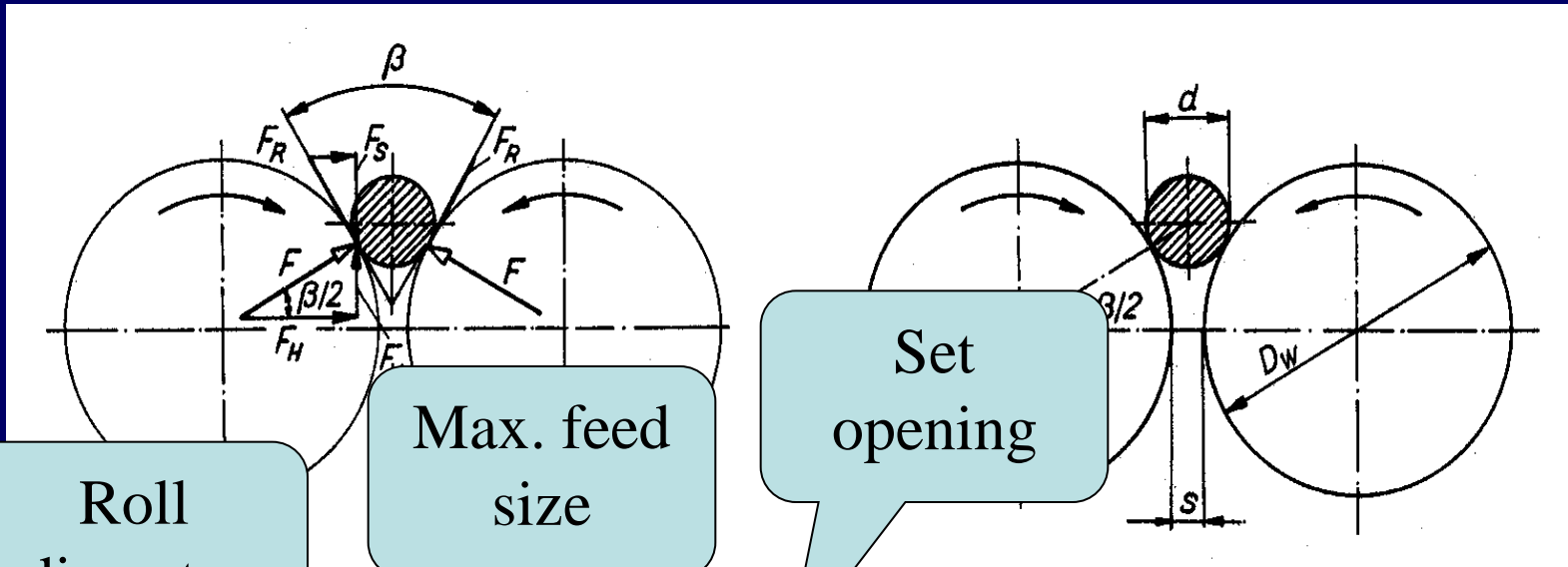


Roll diameter

Max. feed size

$$D_w \geq \frac{a_0 - S\sqrt{1+\mu^2}}{\sqrt{1+\mu^2} - 1}$$

Roll crusher



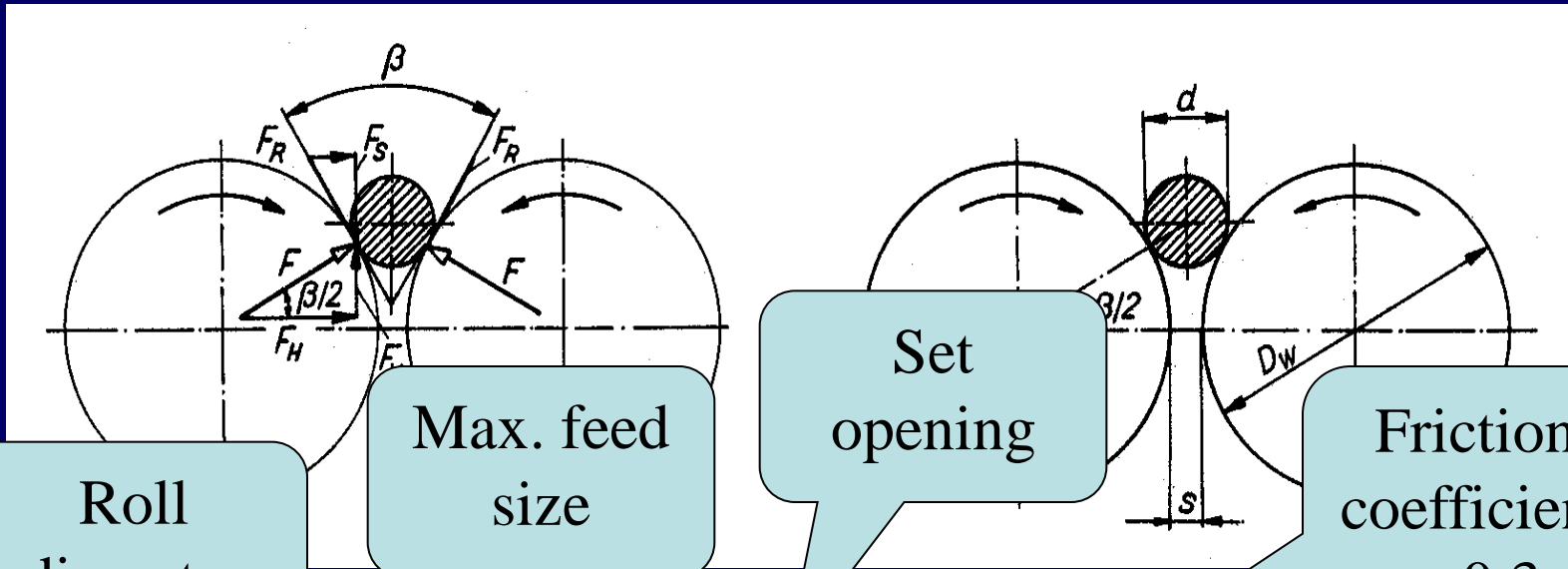
Roll diameter

Max. feed size

Set opening

$$D_w \geq \frac{a_0 - S\sqrt{1+\mu^2}}{\sqrt{1+\mu^2} - 1}$$

Roll crusher



Roll diameter

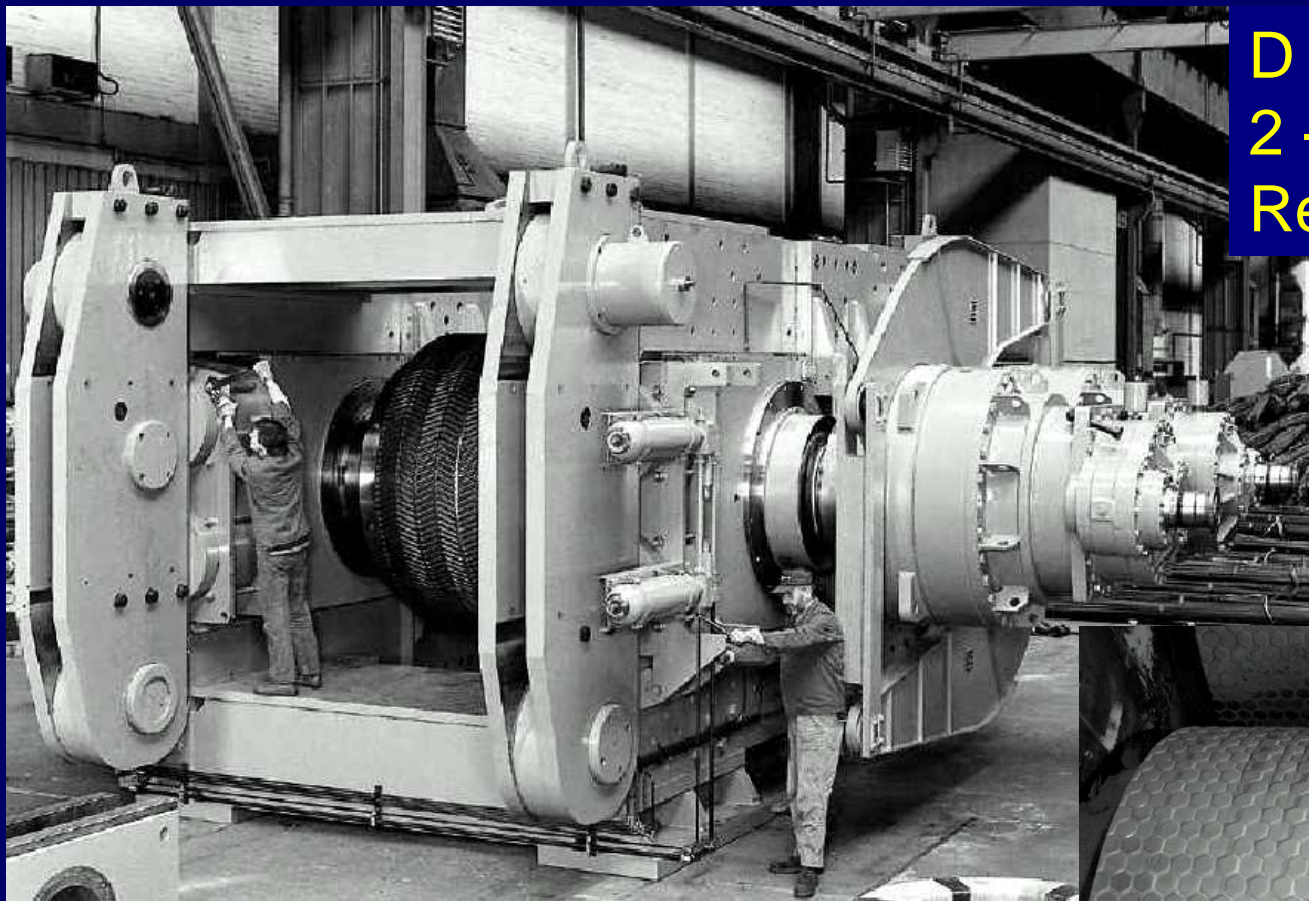
Max. feed size

Set opening

Friction coefficient = 0.3

$$D_w \geq \frac{a_0 - S\sqrt{1+\mu^2}}{\sqrt{1+\mu^2} - 1}$$

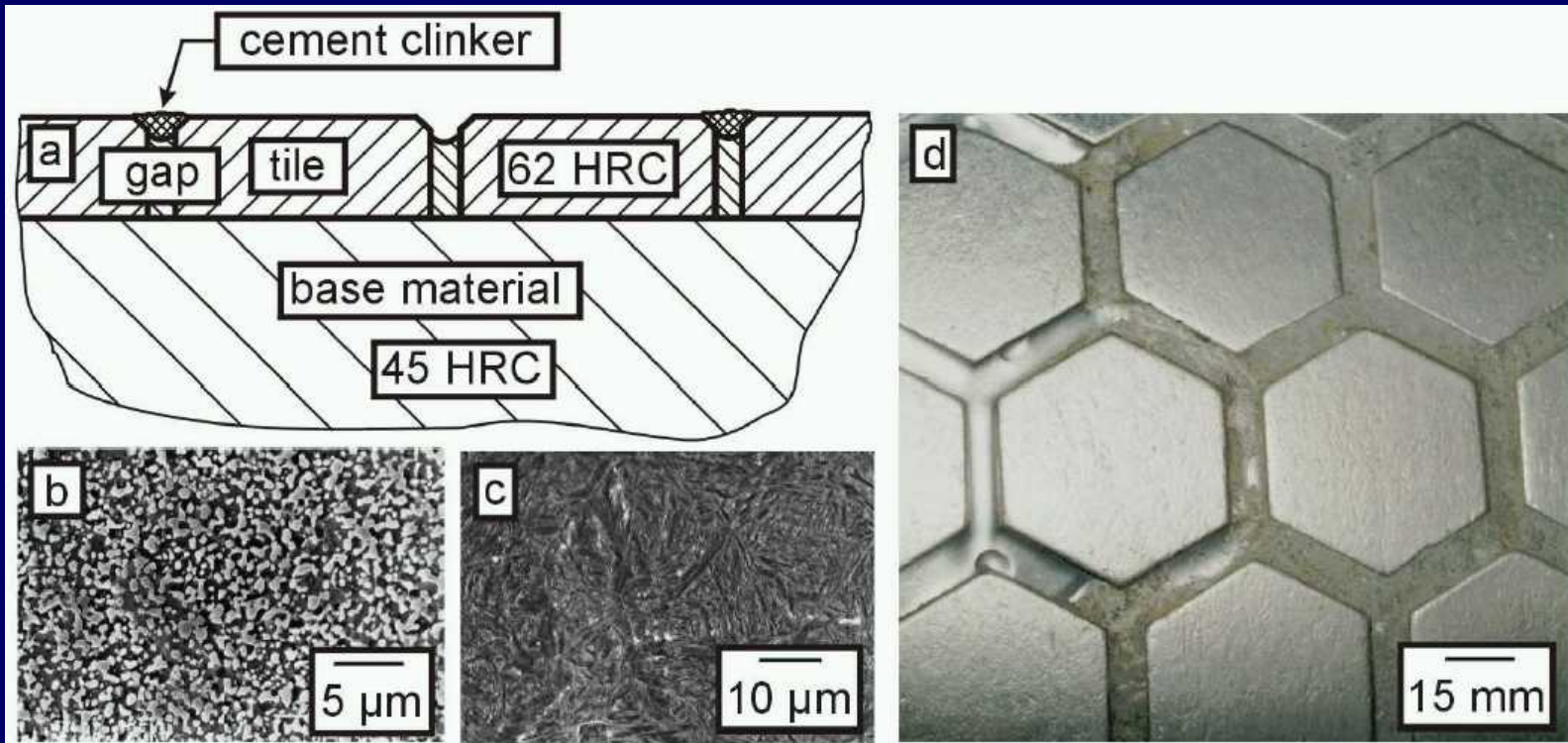
HPGR



$D / W \sim 1$
 $2 - 6 \text{ N/mm}^2$
Reduction ratio 5 : 1



Liner design



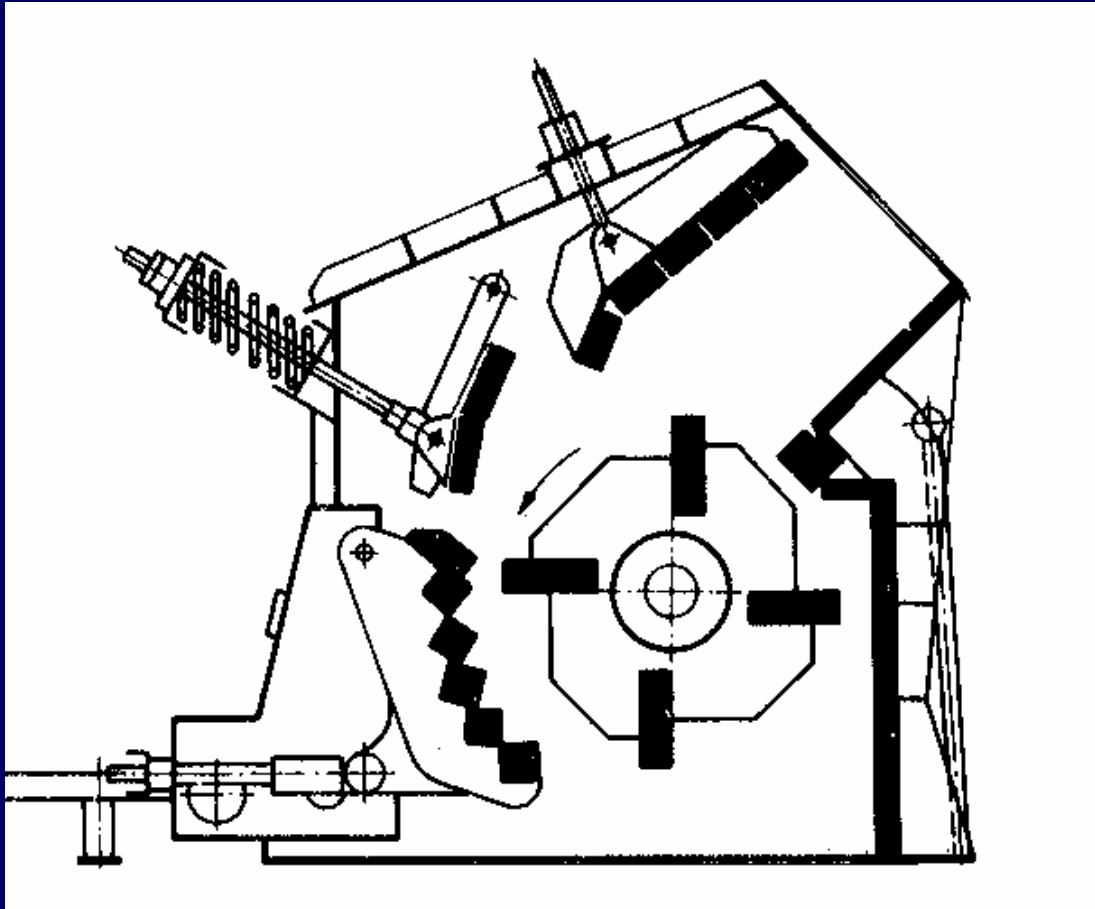
Advantages of roll crushers

- Simple construction and trouble free operation
- Easy maintenance and repair, especially for fines crushing
- Handles frozen, sticky or agglomerated feed
- Uniformity of product
- Low energy consumption
- Often the most economic solution in the 3...10 mm range
- Simultaneous heat transfer via the rolls is possible

Disadvantages of roll crushers

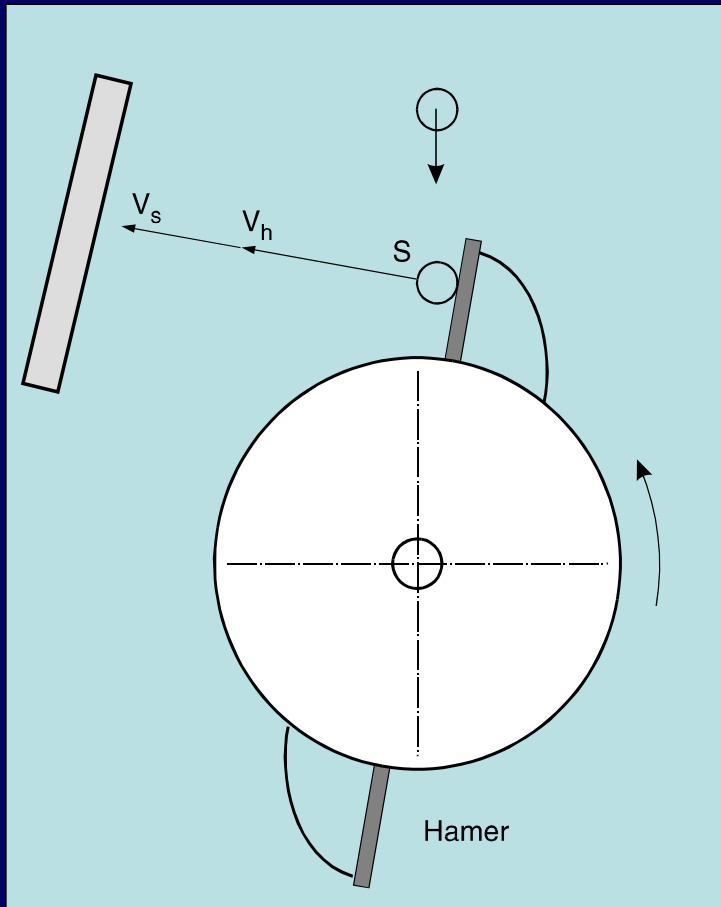
- Low reduction ratio
- Low capacity in relation to its unit dimensions (not compact)
- Continuous feed rate is necessary, no choke feed

Impact crusher



High wear if silica content exceeds 15%

Impact crusher



20-60 m/s
Reduction ratio 40:1

Initial particle speed

Particle diameter

Particle density

$$s_0 = \frac{v_r d_p^2 \rho_p}{18\eta}$$

Braking distance

Air viscosity

Advantages of impact crushers

- High reduction ratio
- Easy adjustable to variable feed material or different applications
- Lower capital costs in comparison to jaw, gyratory or roll crushers
- Small head room requirements
- Selective crushing possible in some cases

Disadvantages of impact crushers

- Constant feed rate required
- Only crushing of soft or middle hard rock
- No material that tends to agglomerate should be fed
- High wear (especially on rotor edges), and the need to use advanced wear resistant materials

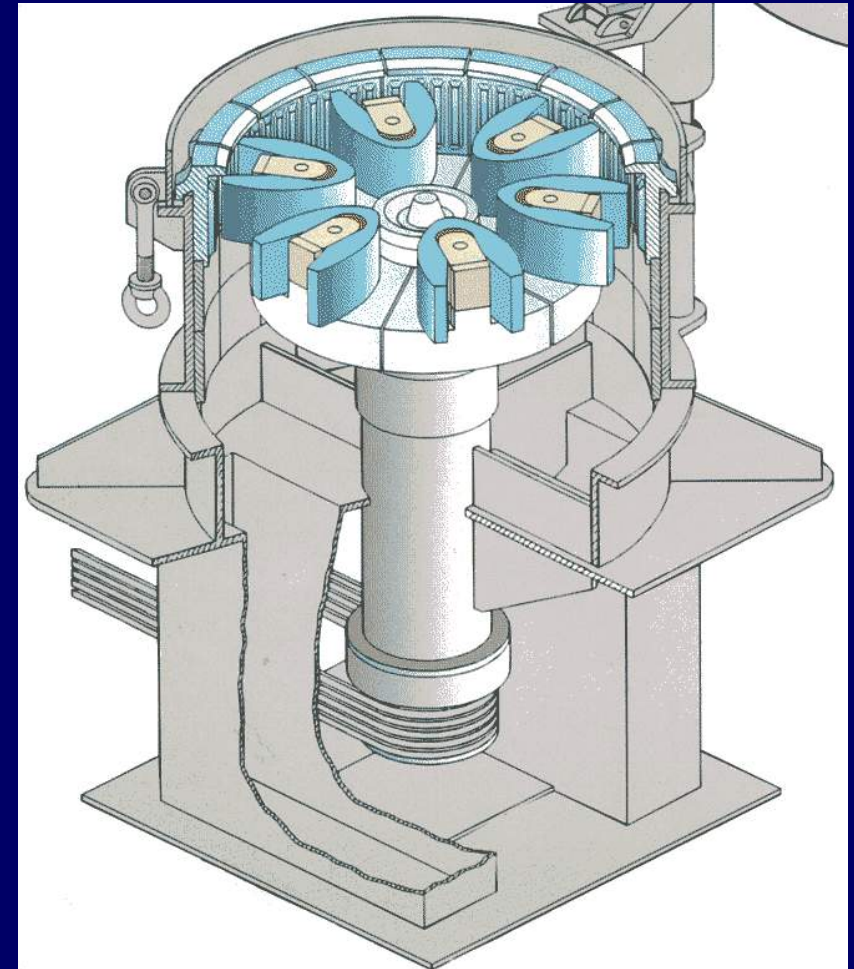
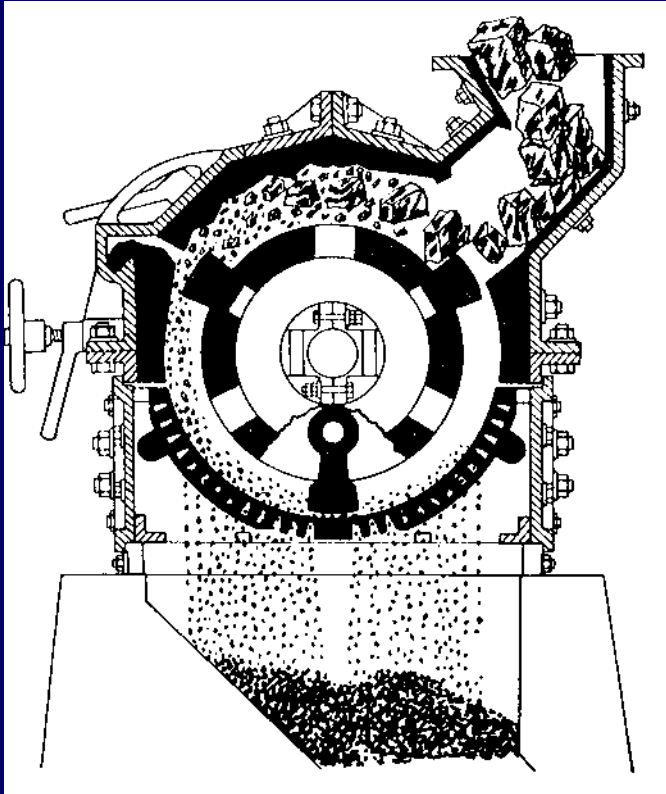




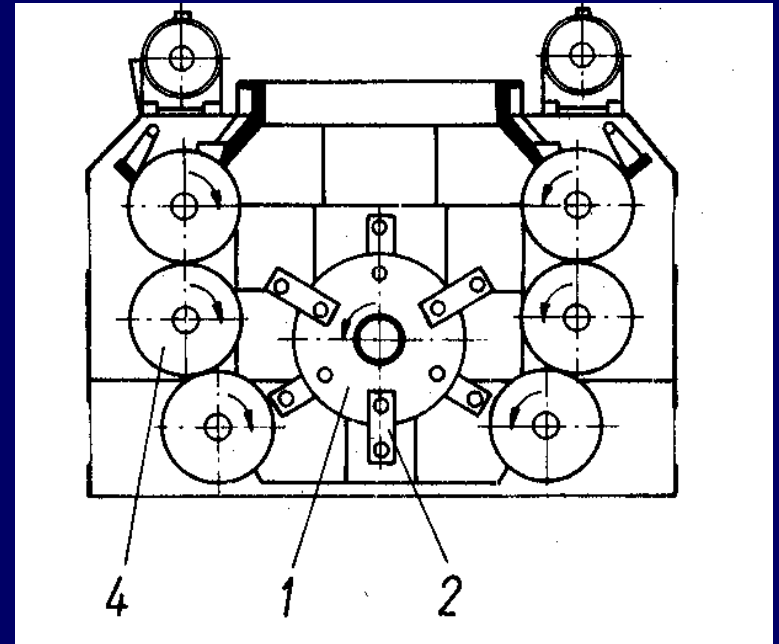
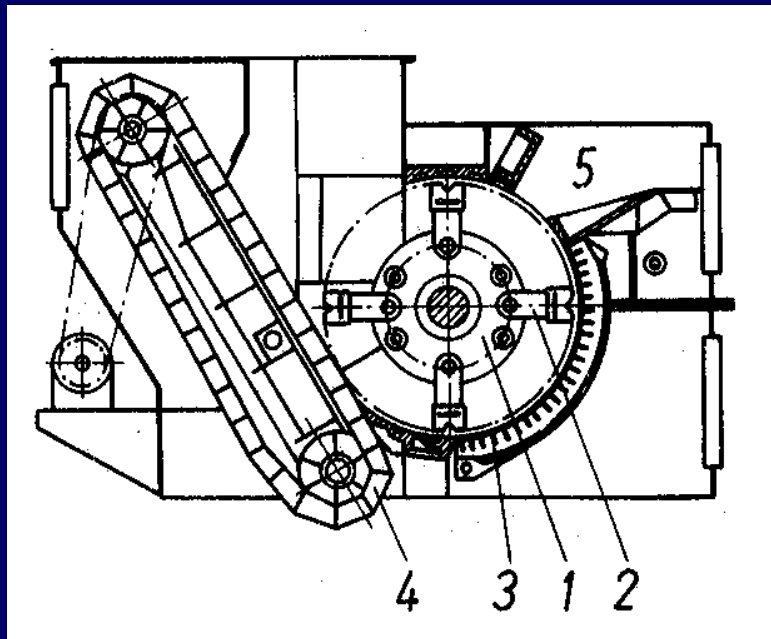


Hammer crusher / cage mill

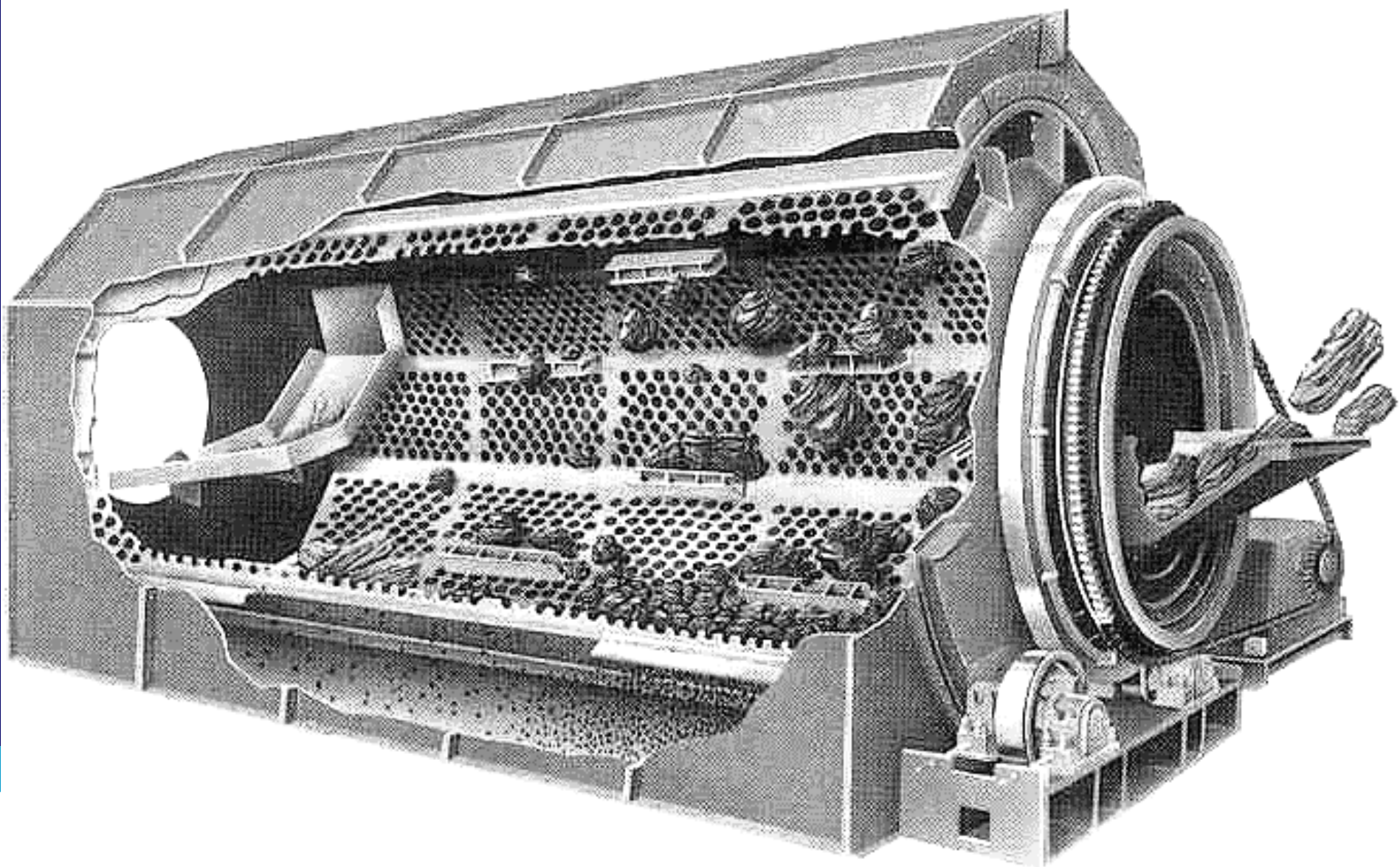
- Usually bottom screen



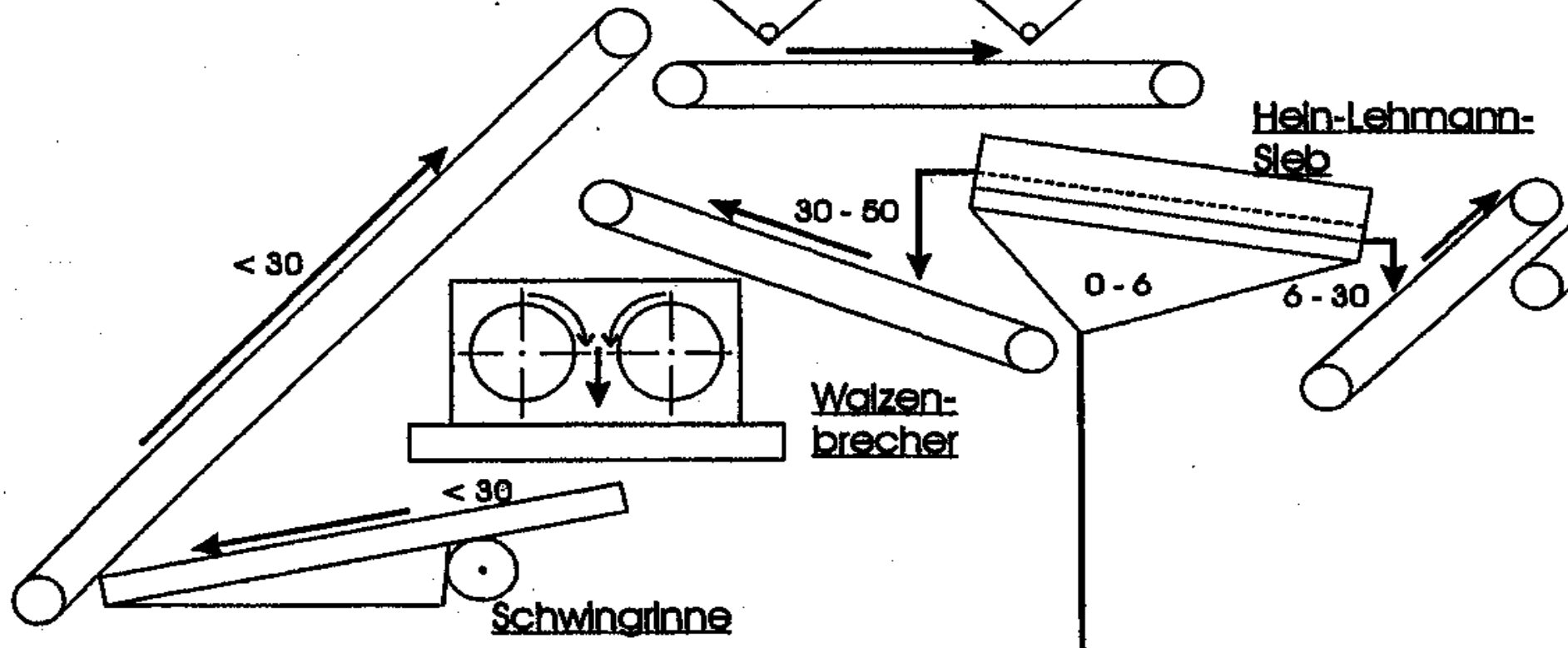
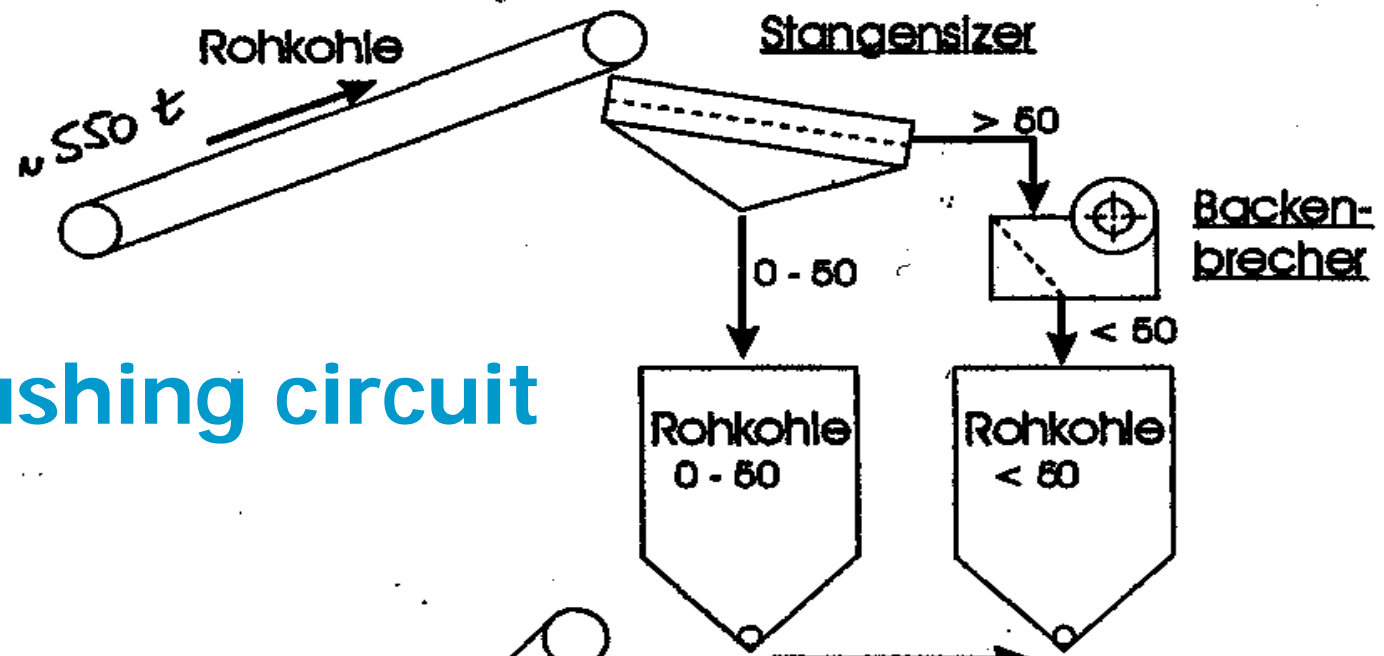
Hammer crusher for sticky feed



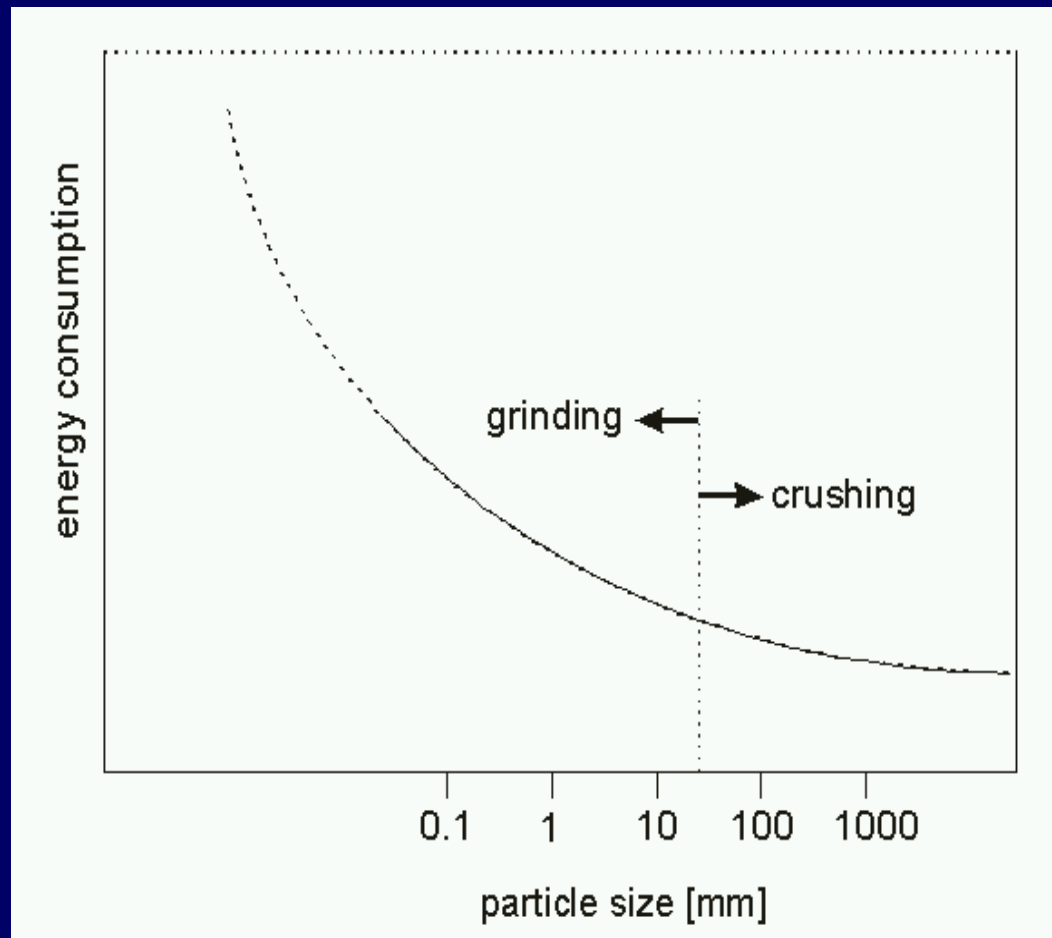
Rotary breaker



Crushing circuit



Grinding vs crushing



Energy consumption

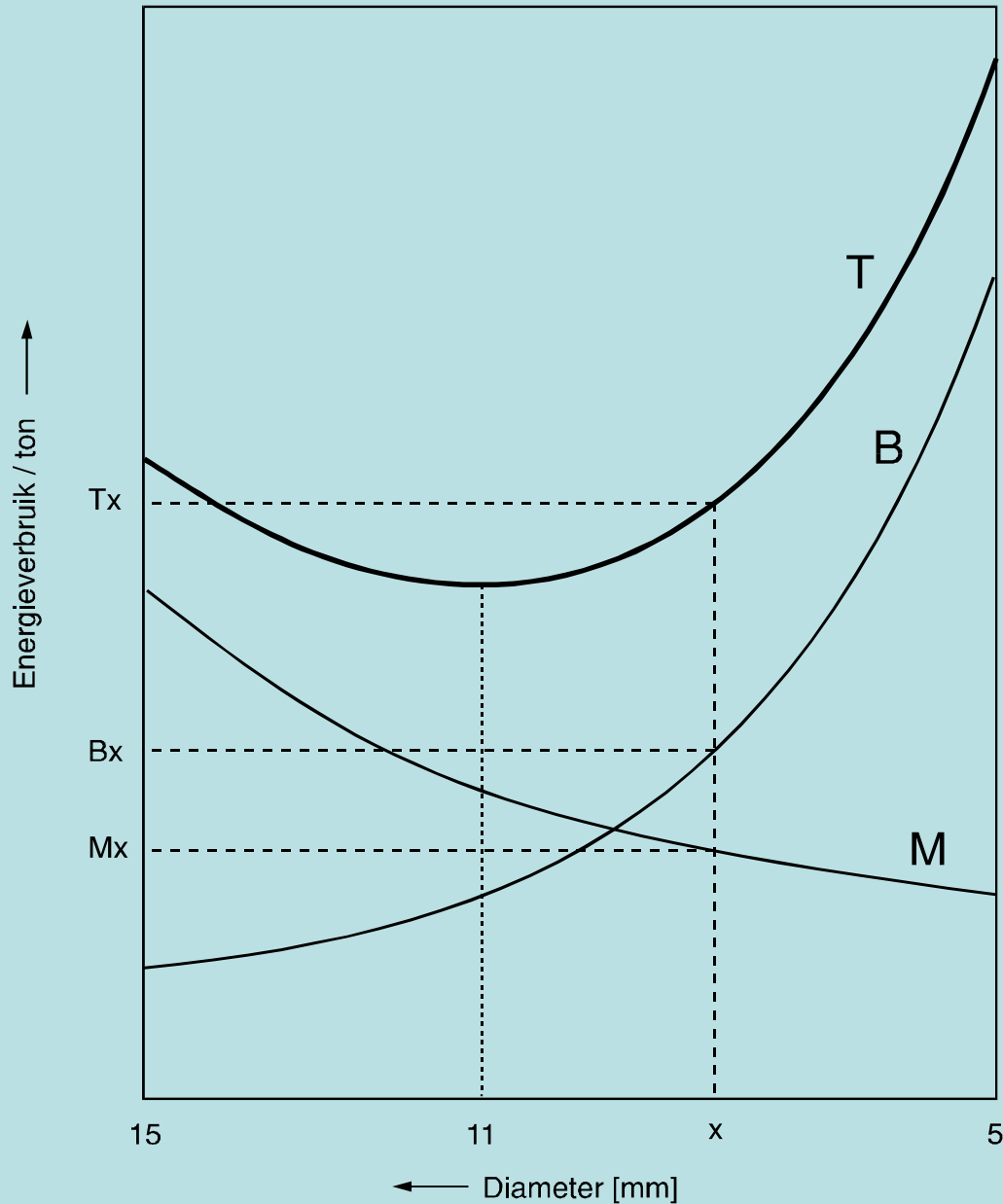


Fig 9-1 : Aansluiting/Energieverbruik breken en malen.

$$T = B + M$$

De totaal-curve heeft in dit voorbeeld een minimum bij $x = 11$ mm. Energetisch is het dus het meest gunstig om tot 11 mm te breken en verder te malen.

Grinding

Tumbling mills:

- Ball mill (steel balls)
- Rod mill (steel rods)
- Tube mill (rods&balls, or balls only)
- Pebble mill (hard, rounded rocks, e.g. flint stone or porcelain)
- Autogeneous mill (large pieces of ore)
- Semi-Autogeneous (SAG) mill (Large pieces of ore and steel balls)

Other mill types:

- Roller mill, pan mill
- Vibratory mill

Dry or wet ?

Advantages of wet grinding:

- Less energy consumption per tonne of product
- No dust generation
- Moist feed does not need to be dried prior to grinding (contrary to dry grinding)

Disadvantages of wet grinding:

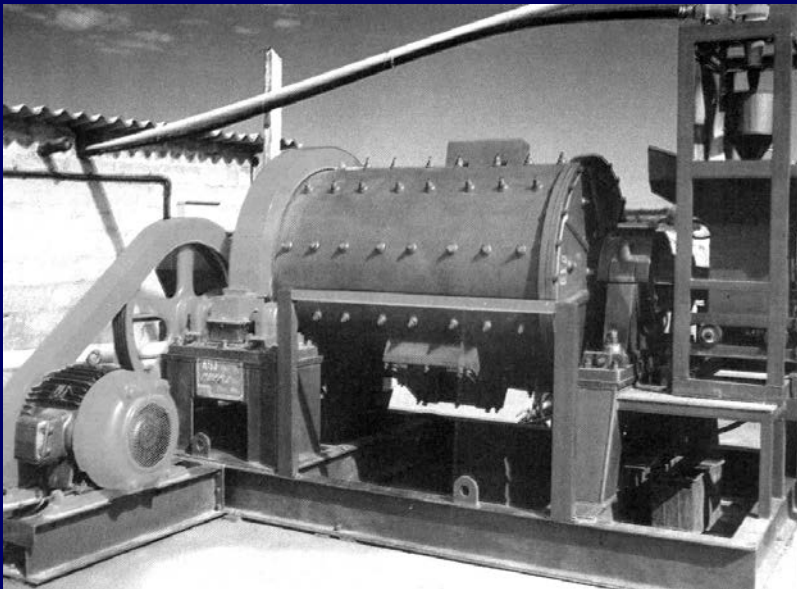
- Higher wear of grinding media and liner
- Corrosion
- Product is wet and must be dewatered
- Some products are not allowed to contact water (cement!)

Tumbling mill

Power consumption

$$P = C_1 D^{2.5} L = C_2 V_m D^{0.5}$$

$$(V_m = \frac{1}{4}\pi D^2)$$



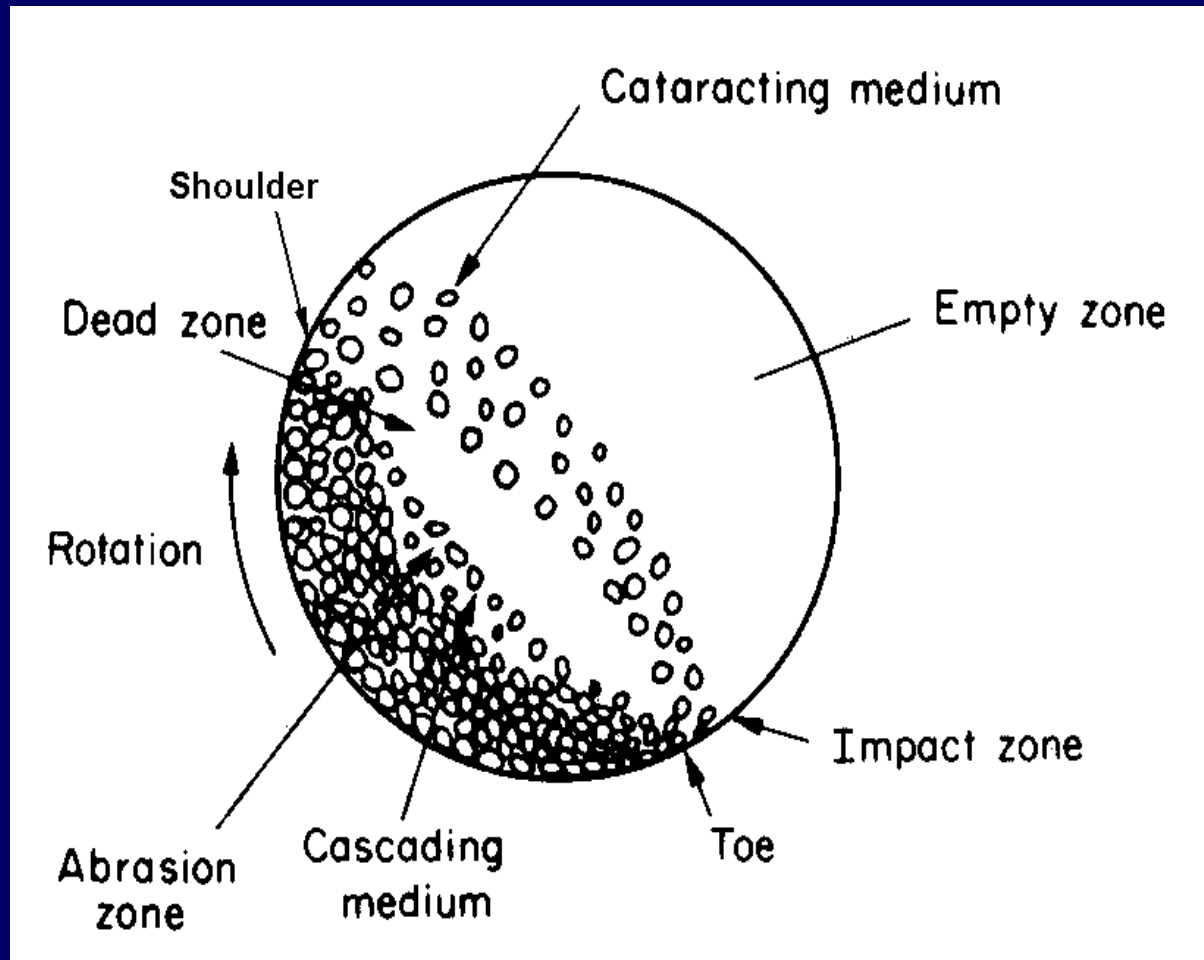
1.2 m internal, 27 kW



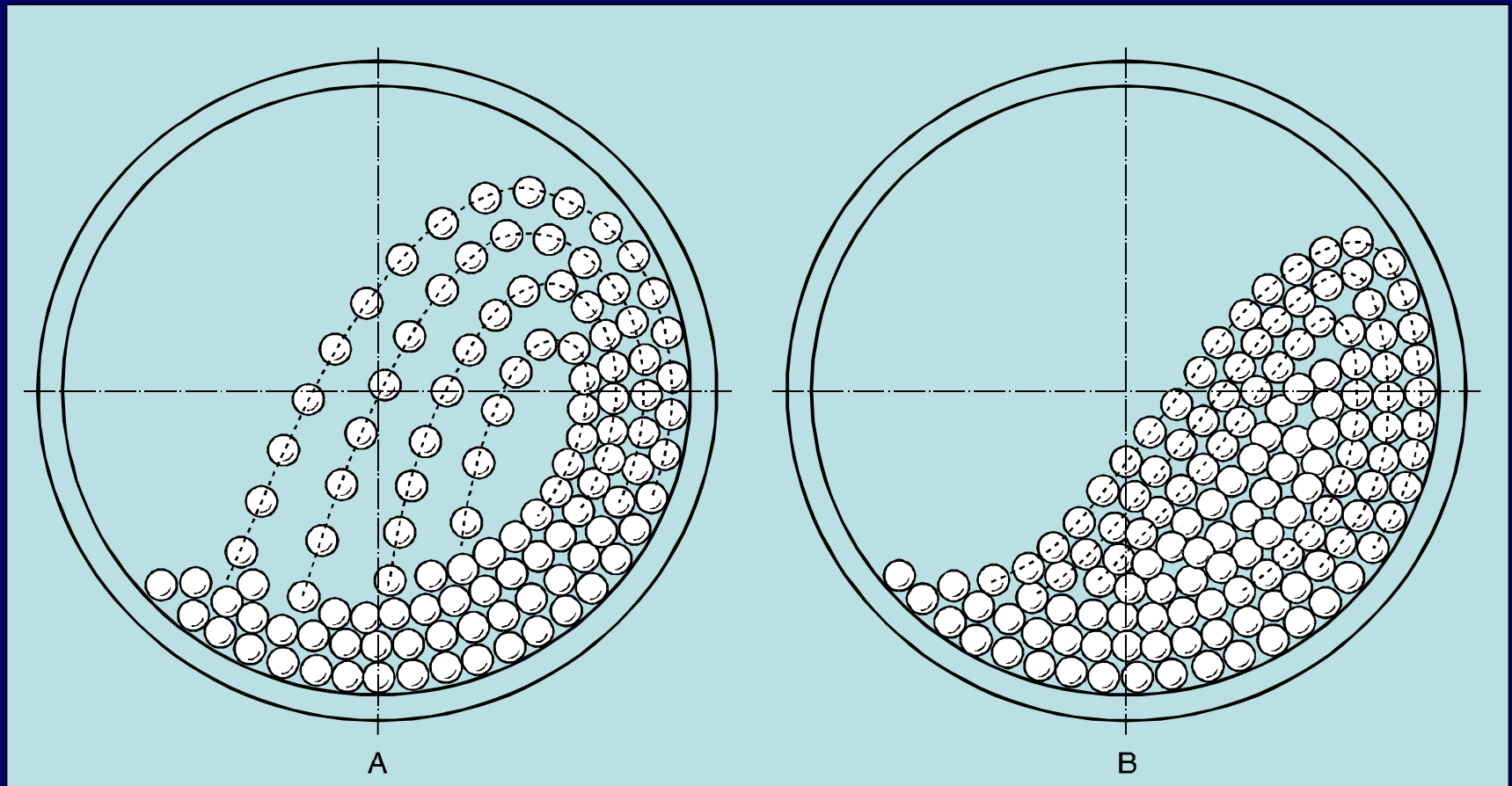
12.2 m internal, 20 MW

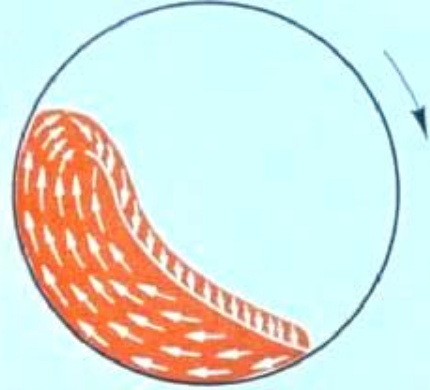
- Power independent of feed!
- Capacity proportional to power consumption

Mill terminology

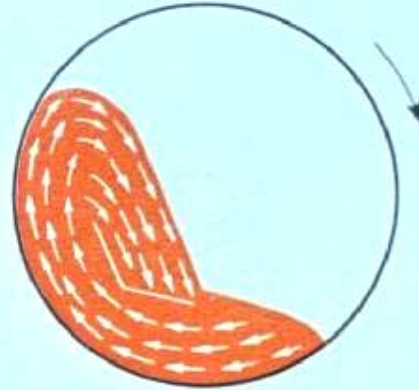


Cascade / cataract

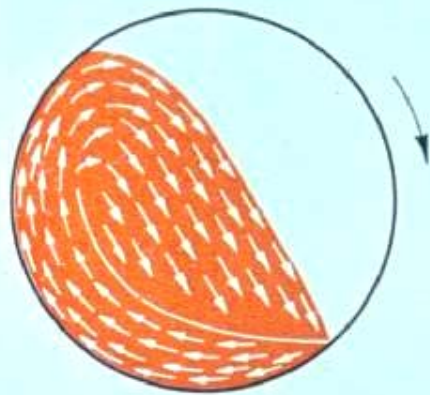




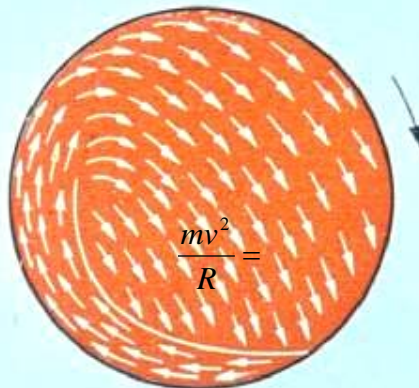
CASCADE



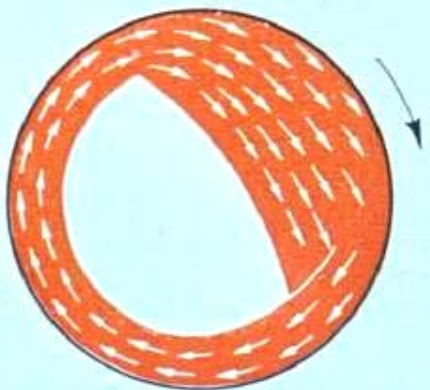
CATARACT



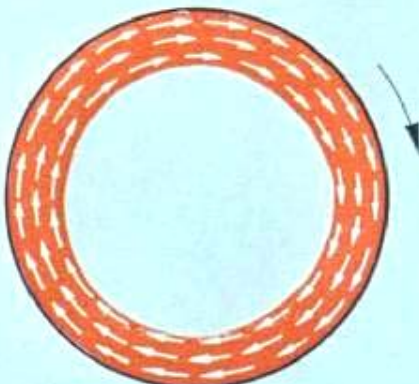
EQUILIBRIUM



SATURATION



PRECENTRIFUGAL STATE



CENTRIFUGAL STATE

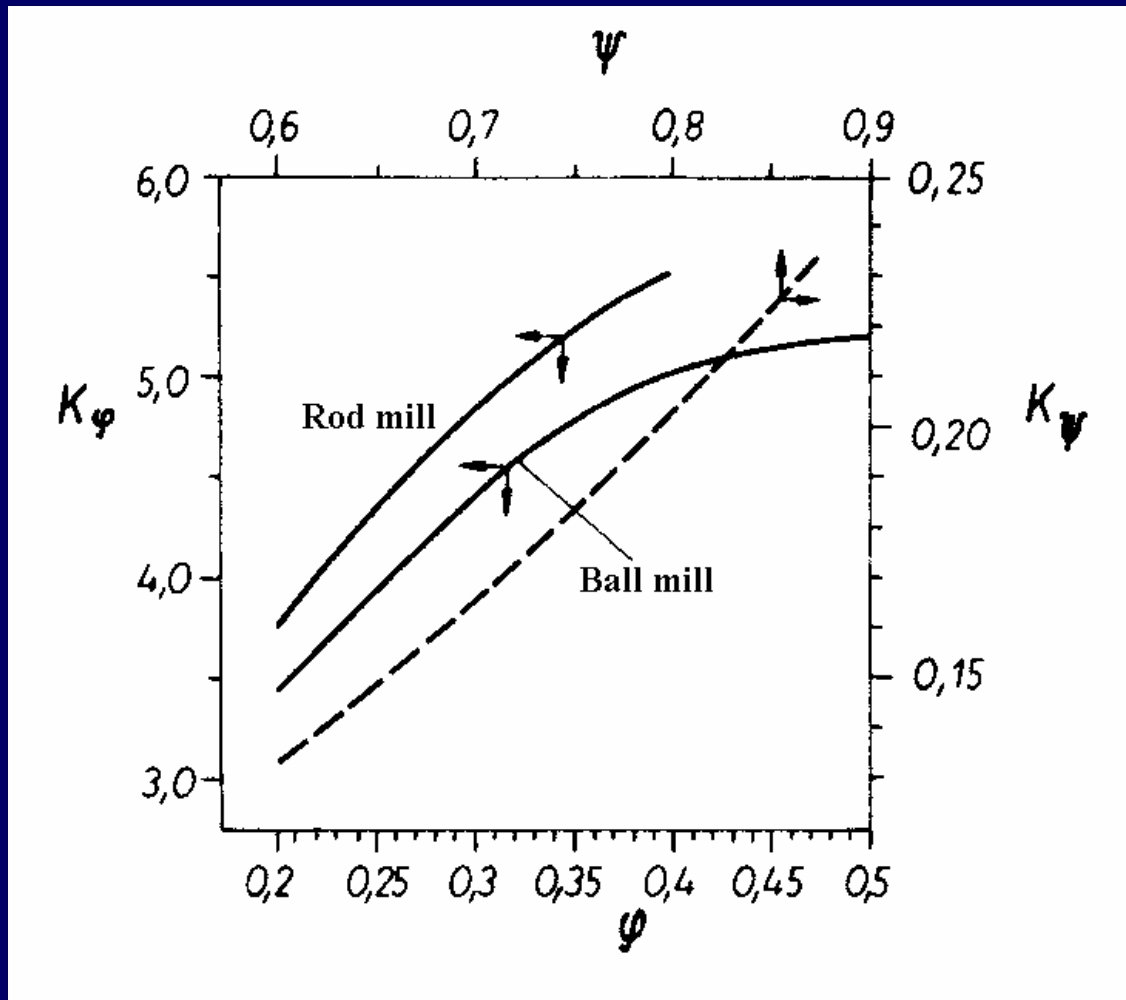
$$\frac{mv^2}{R} = mg \cos \alpha$$

$$n_{crit} = \frac{42.3}{\sqrt{D-d}}$$

Fig. 1. State of motion in a mill (according to RONCO)

$$P = 8.44 K_T K_\phi K_\psi L D^{2.5}$$

Mill Power (empirical)



$$\psi = n/n_{crit}$$

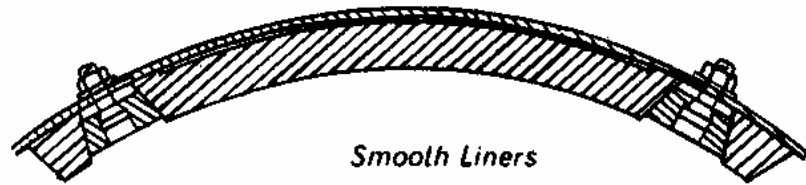
$$\phi_m = \frac{V_m}{V_M} = \frac{m_m}{\rho_m (1 - \epsilon_m) V_M} \frac{1}{V_M}$$

$K_\psi = f(\psi)$ and mill dependent

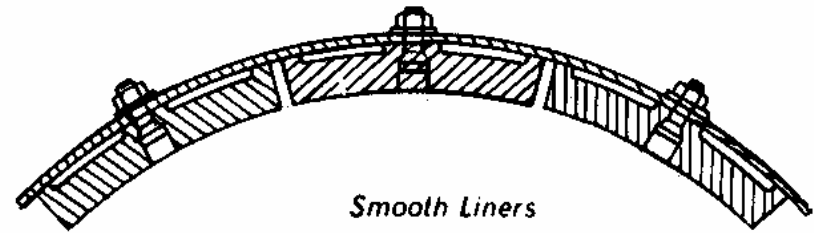
$K_\phi = f(\phi)$

K_T = mill specific factor

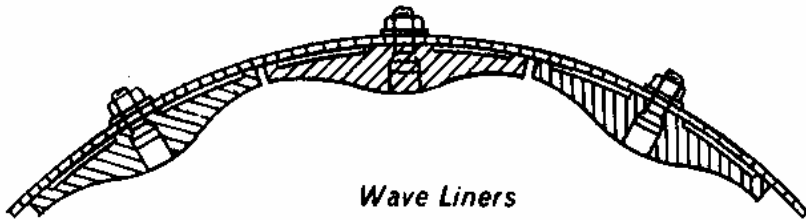
Liner types



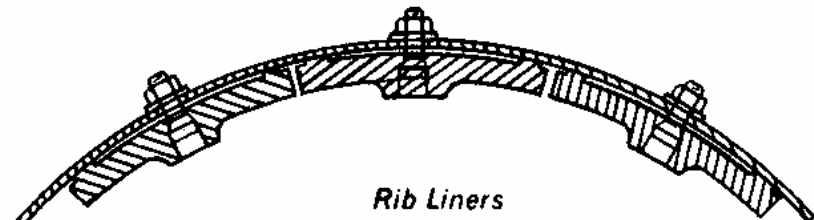
Smooth Liners



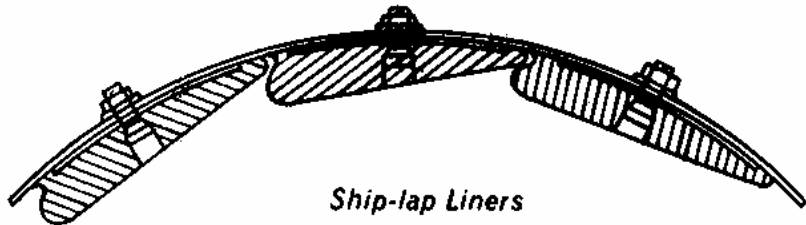
Smooth Liners



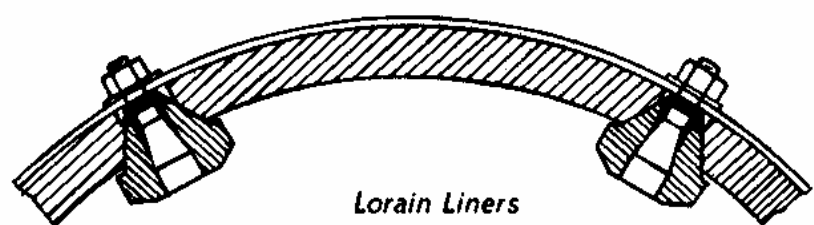
Wave Liners



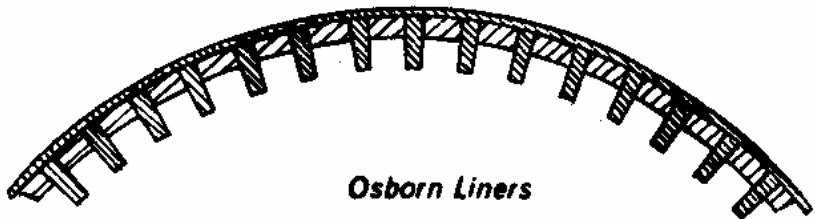
Rib Liners



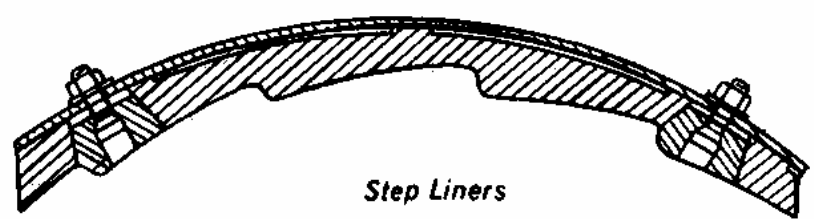
Ship-lap Liners



Lorain Liners

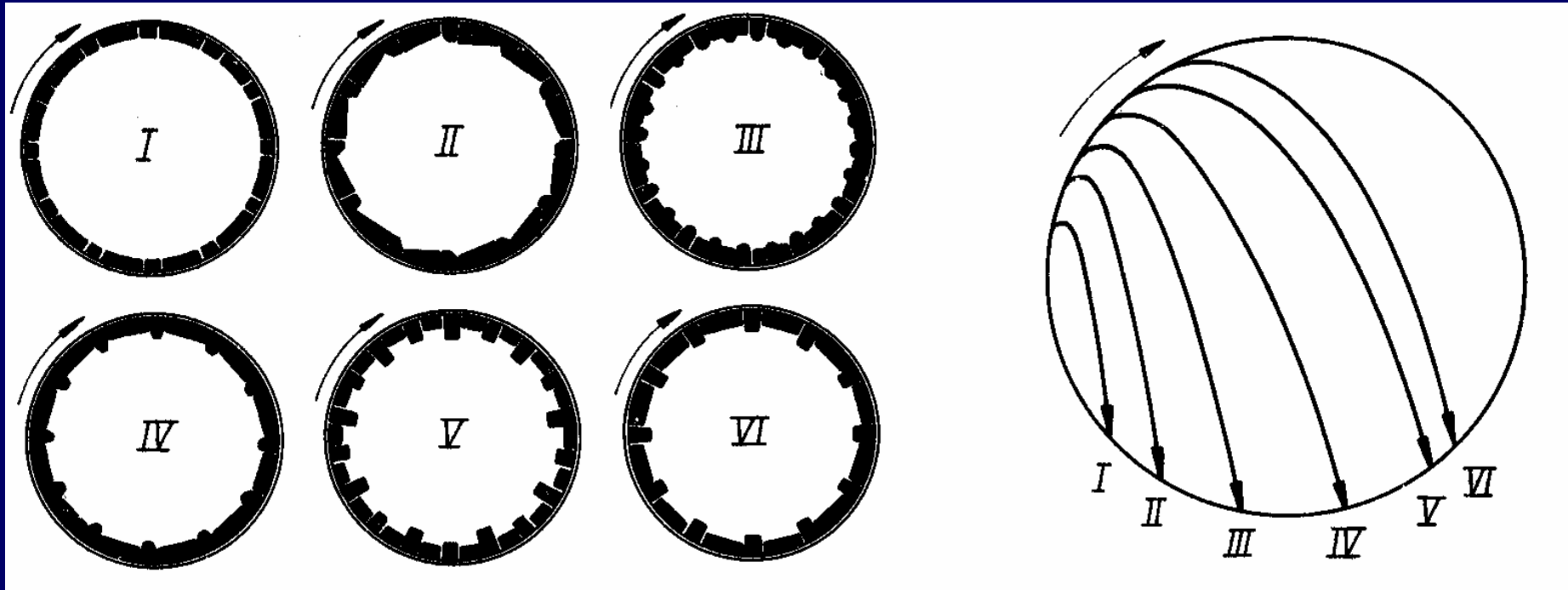


Osborn Liners



Step Liners

Effect of liner and liner wear



Lisheen mine boasts continuous milling

MILL linings at the Lisheen zinc-lead mine near Moyne in County Tipperary, Ireland, can now be replaced without any loss of production, thanks to optimised design of the Skega components from Metso Minerals.

The company's UK subsidiary has worked closely with the mine's operator, Anglo American plc, to develop grinding-mill linings that can be replaced during scheduled shutdowns.

The grinding-mill's feed, central and discharge sections are each fitted in separate months during the mine's normal maintenance periods. Splitting the relining into three shorter parts avoids the need for any special closures that would halt the output of lead and zinc concentrates. Lisheen surface engineer Jack Smith said: "We have got to the stage where the lining



downtime is not impacting on production."

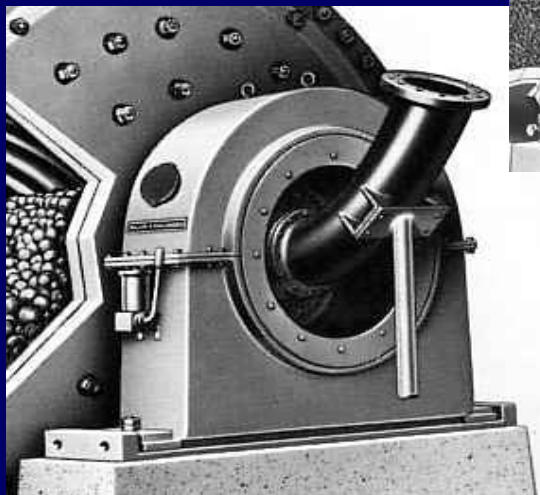
Lisheen opened in 1999 and produces some 300,000 t/y of zinc concentrates and 40,000 t/y of lead concentrates. Material is crushed underground then stockpiled in a teepee-shaped building (below left) before being conveyed to the semi-autogenous grinding (SAG) mill (below left, inset).

The SAG mill, supplied by Australia's ANI, reduces the feed from a nominal size of 150 mm to 0.4 mm. Further processing takes place in a ball mill, also fitted with Skega linings, before the pulverised slurry is fed into the floatation process that extracts the zinc and lead concentrates.

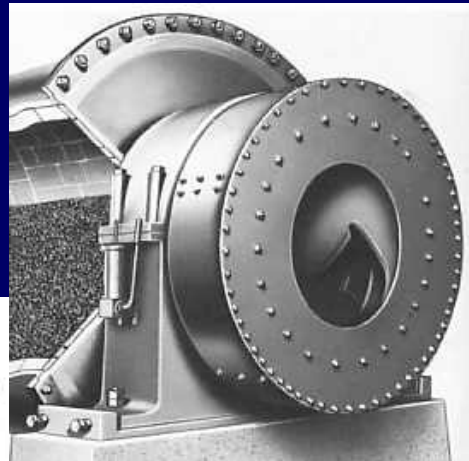
The SAG mill has a diameter of 6.1 m and a length of 4.1 m, and contains four rows of 34 lifters. It is equipped with a powerful 2.3 MW motor and runs with 8-12% of the mill's volume filled with 125 mm-diameter steel grinding balls. Incoming ore varies in zinc and lead levels and is sometimes hard, sometimes soft. The mill is therefore designed to operate at variable speeds, with feed rates of up to 250 t/hr.

There are typically just 40 hours of downtime a month (the bulk of which are allocated for the plant's scheduled maintenance period) and so the lining system has to be robust enough to withstand at least nine months without replacement.

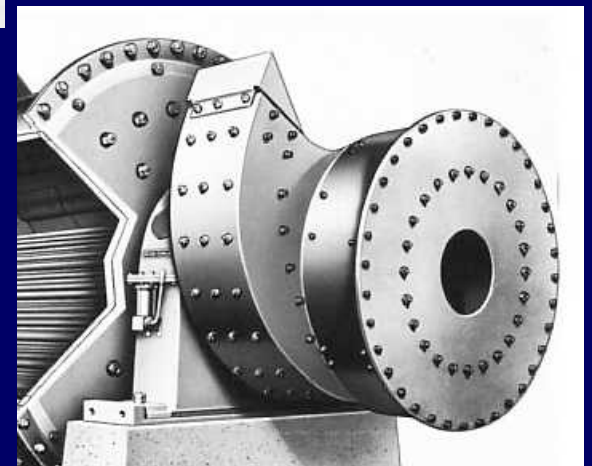
Feeders



Spout

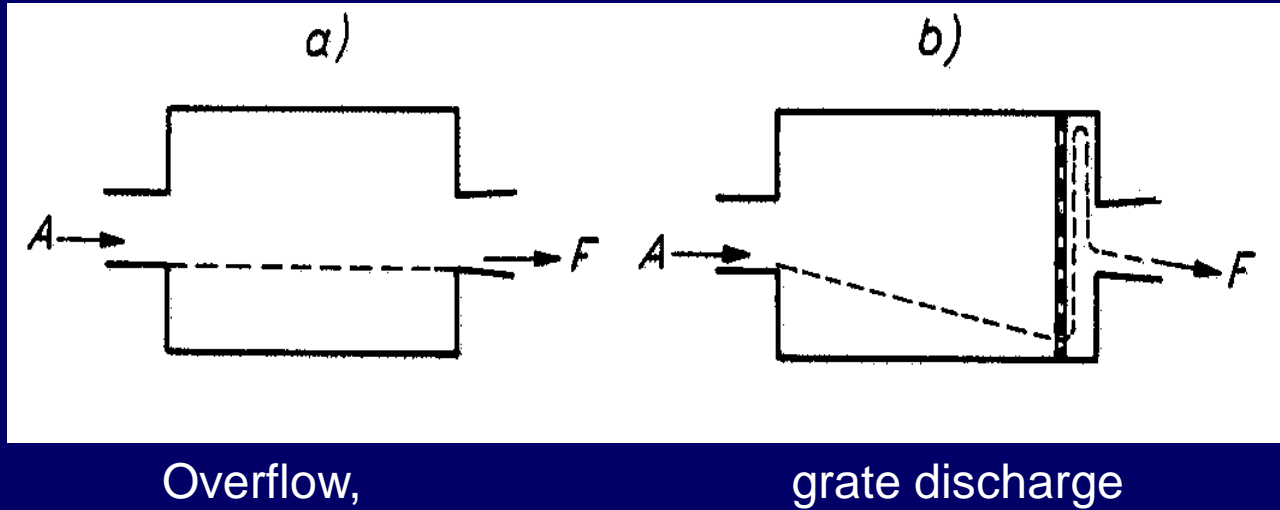


Drum

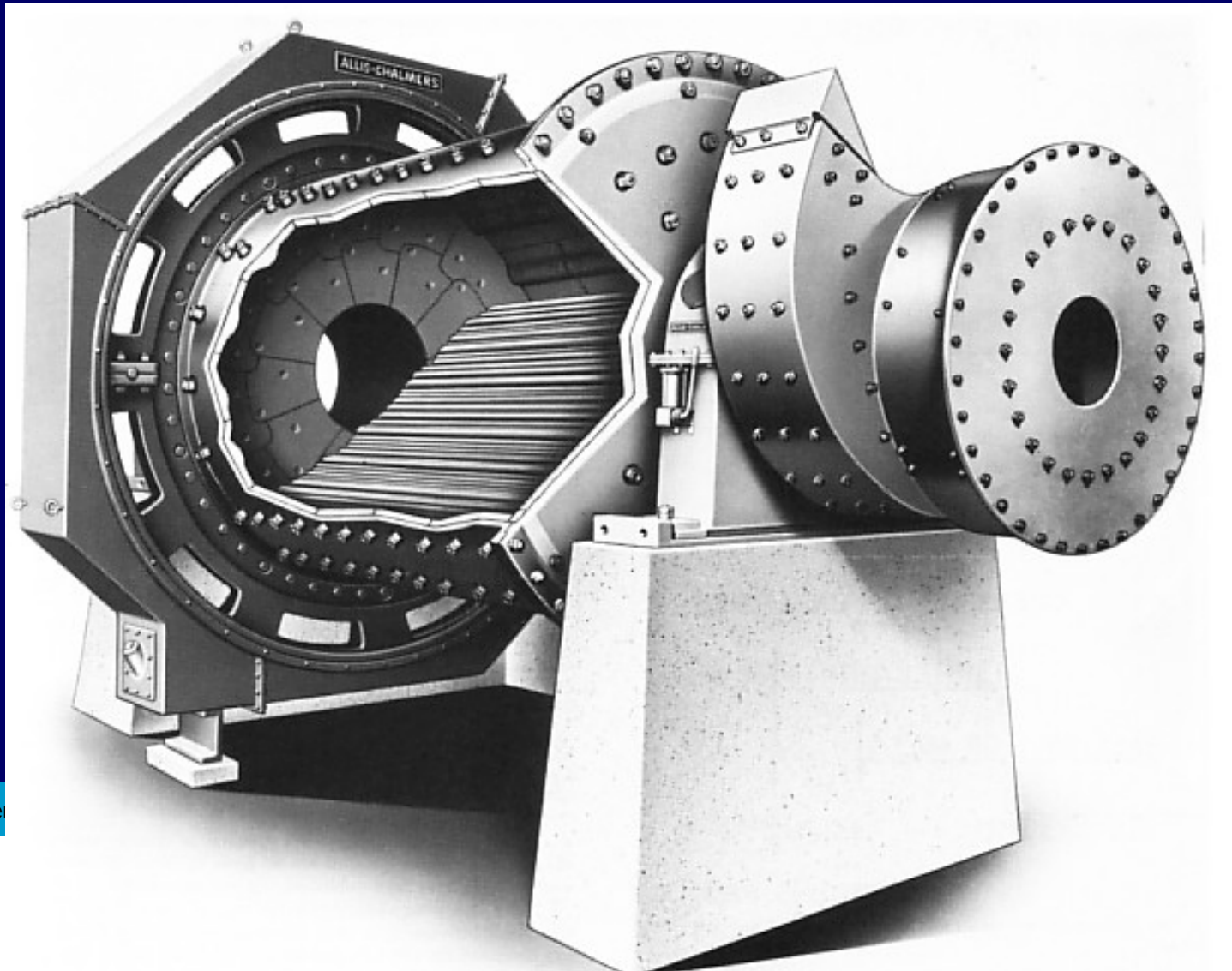
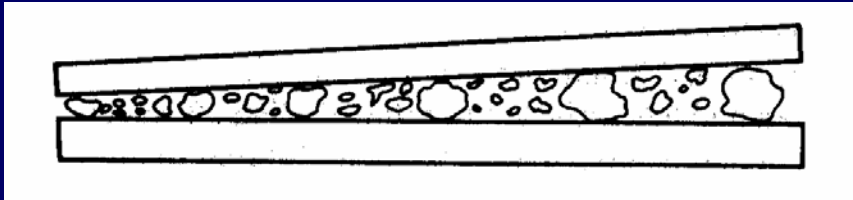


Drum / scoop

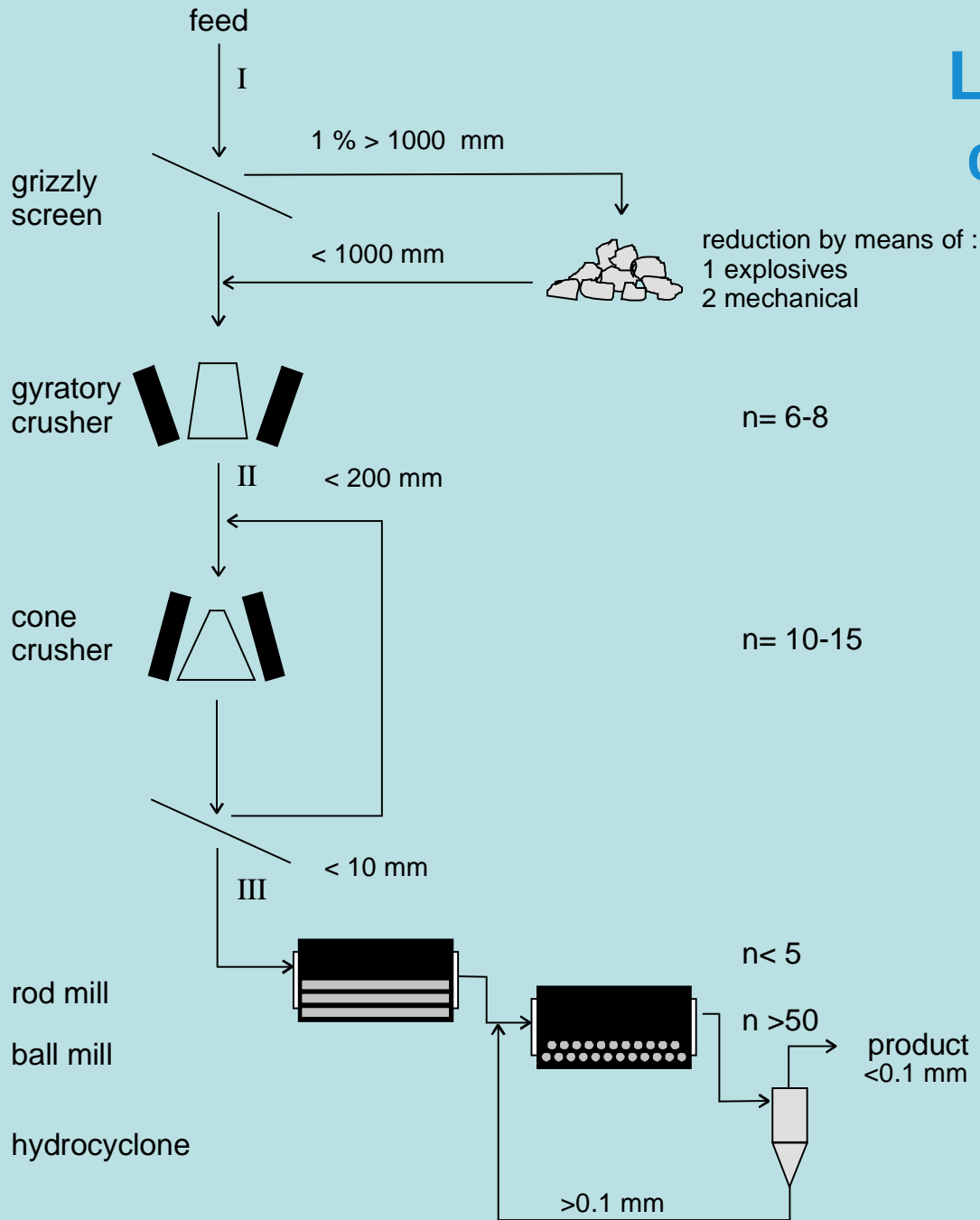
Discharge



Rod mill



Liberation circuit



Energy & cost of comminution

Bond has published a W_i for taconite of 14.87 (1961). This value is used in these base case calculations.

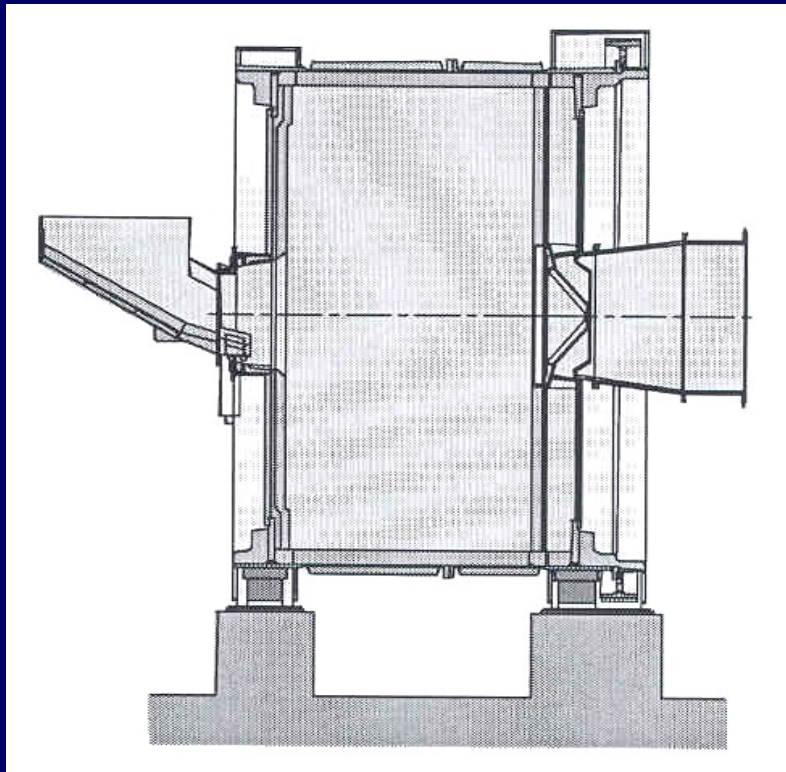
Table 1 shows the feed and product size, the calculated total energy input, and the energy cost for each unit operation. The explosive cost is based on the powder factor of 0.33 kg/tonne (0.65 lbs/ton) and an explosive cost of \$0.264/kg (\$0.12/lb). Electric energy cost is assumed to be \$0.07 per kwh.

Table 1: Energy and cost calculations by unit operation

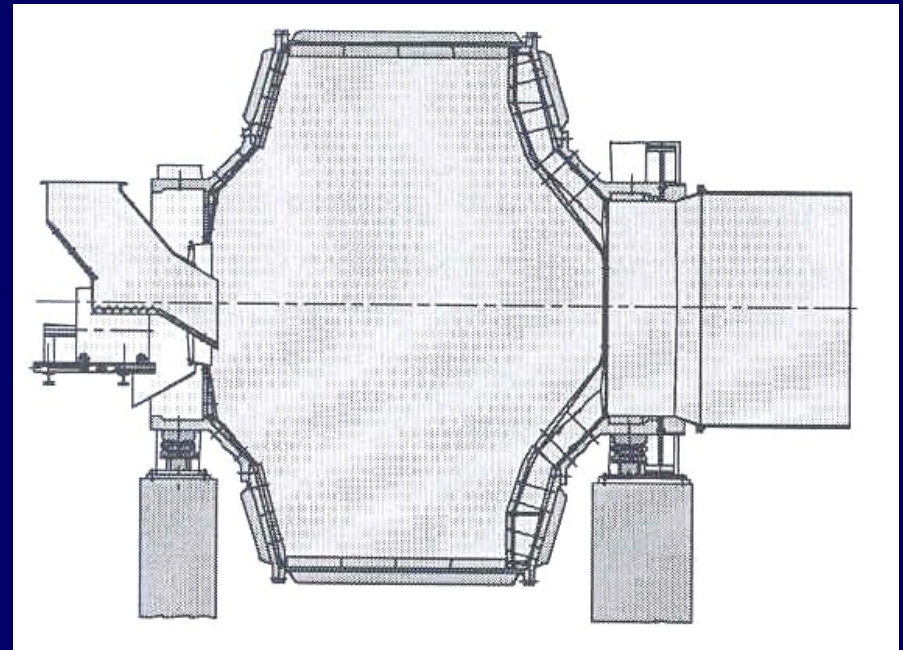
Operation	Feed size	Product size	Work input	Energy cost
	cm	cm	kwh/ton	\$/ton
Explosives	∞	40	.24	.087
Primary crushing	40	10.2	.23	.016
Secondary crushing	10.2	1.91	.61	.043
Grinding	1.91	.0053	19.35	1.35
Totals			20.43	1.50

By far the greatest work input is in grinding. Size is reduced by a factor of 360. In primary crushing, it is reduced by a factor of four and in secondary crushing by about five times. Clearly, changes in blasting that reduce grinding requirements will have the biggest impact for energy savings.

SAG mills

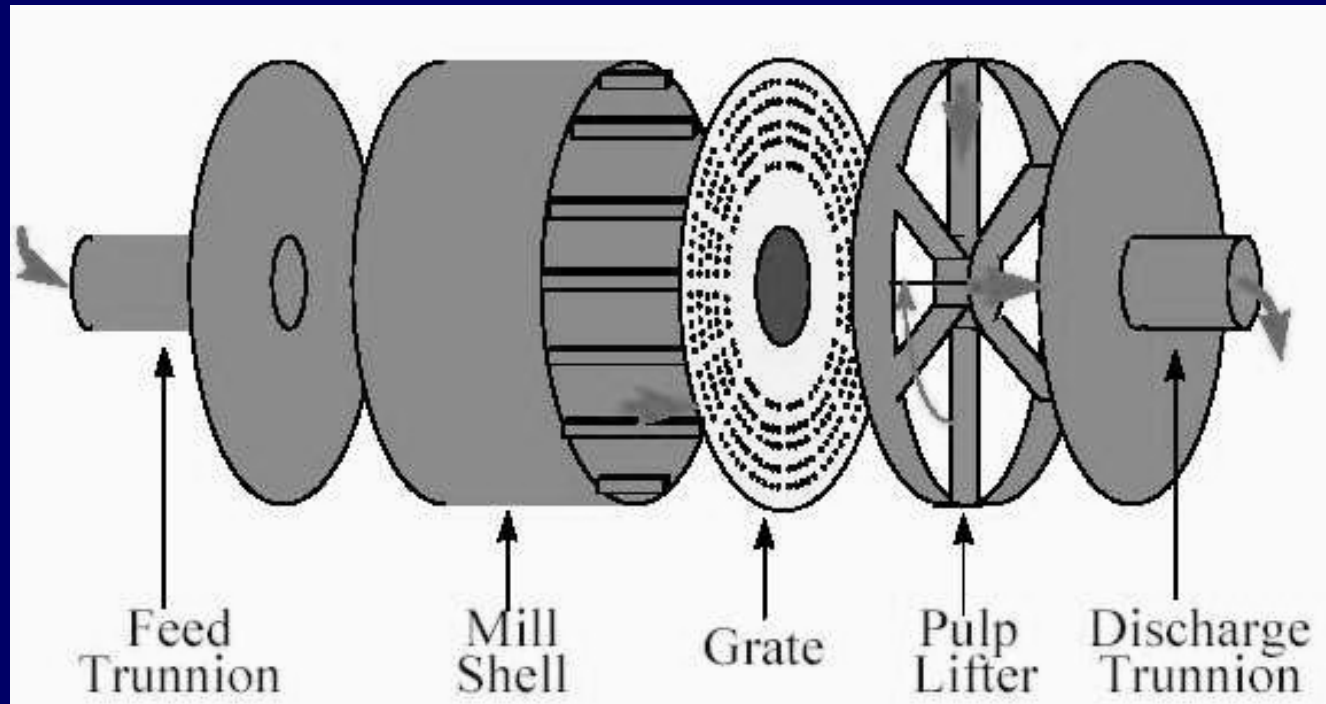


Shell supported

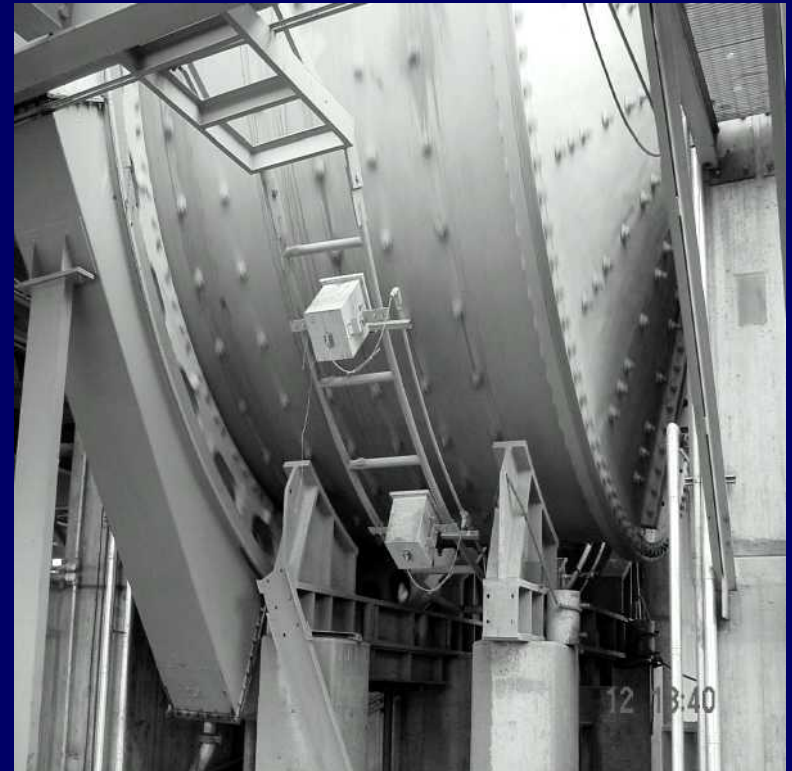
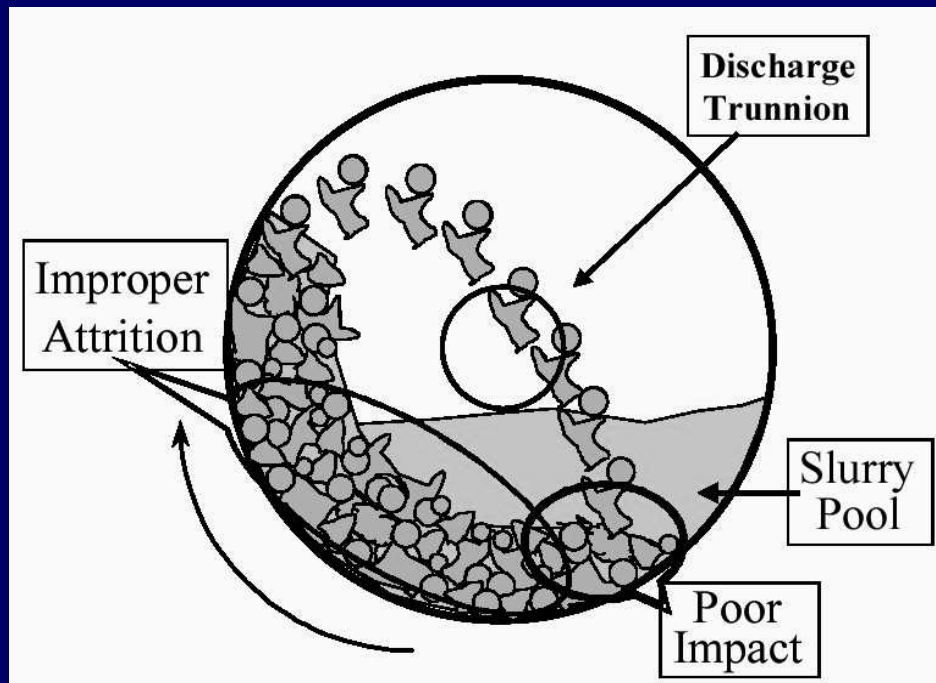


Conventional

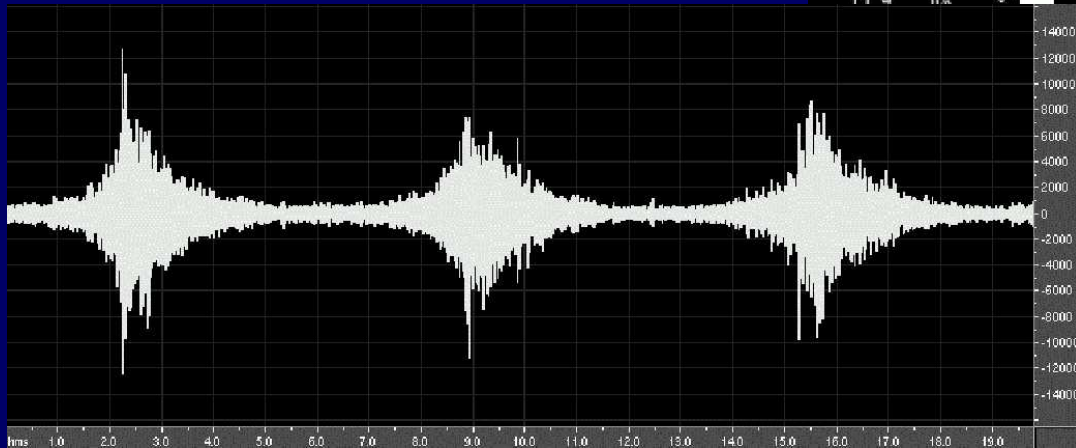
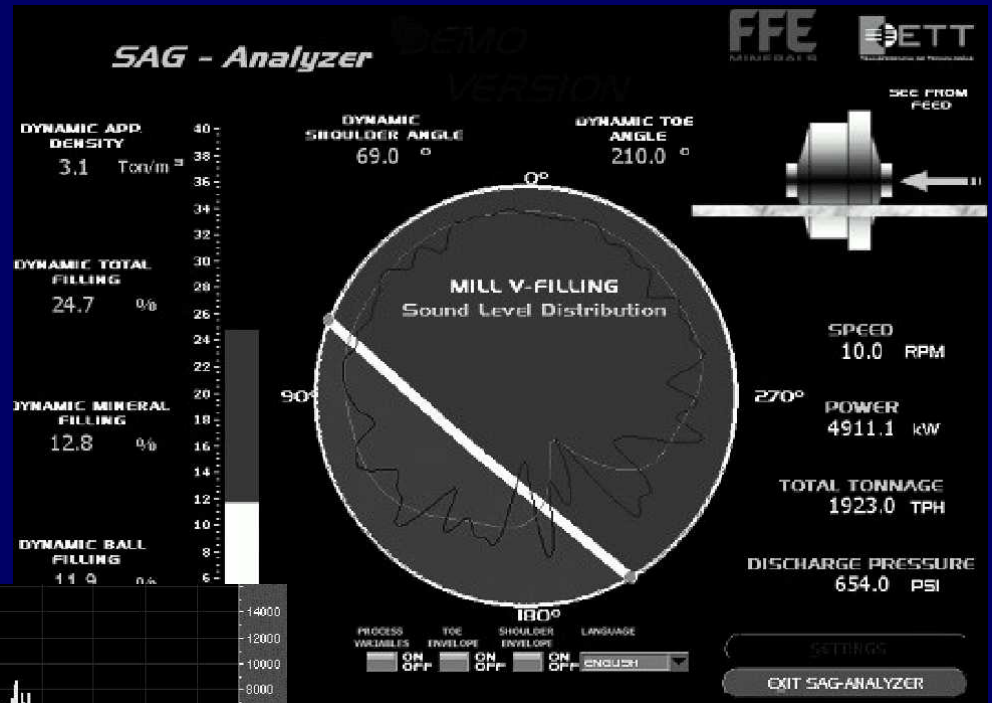
SAG mill discharge



SAG mill control



SAG mill control



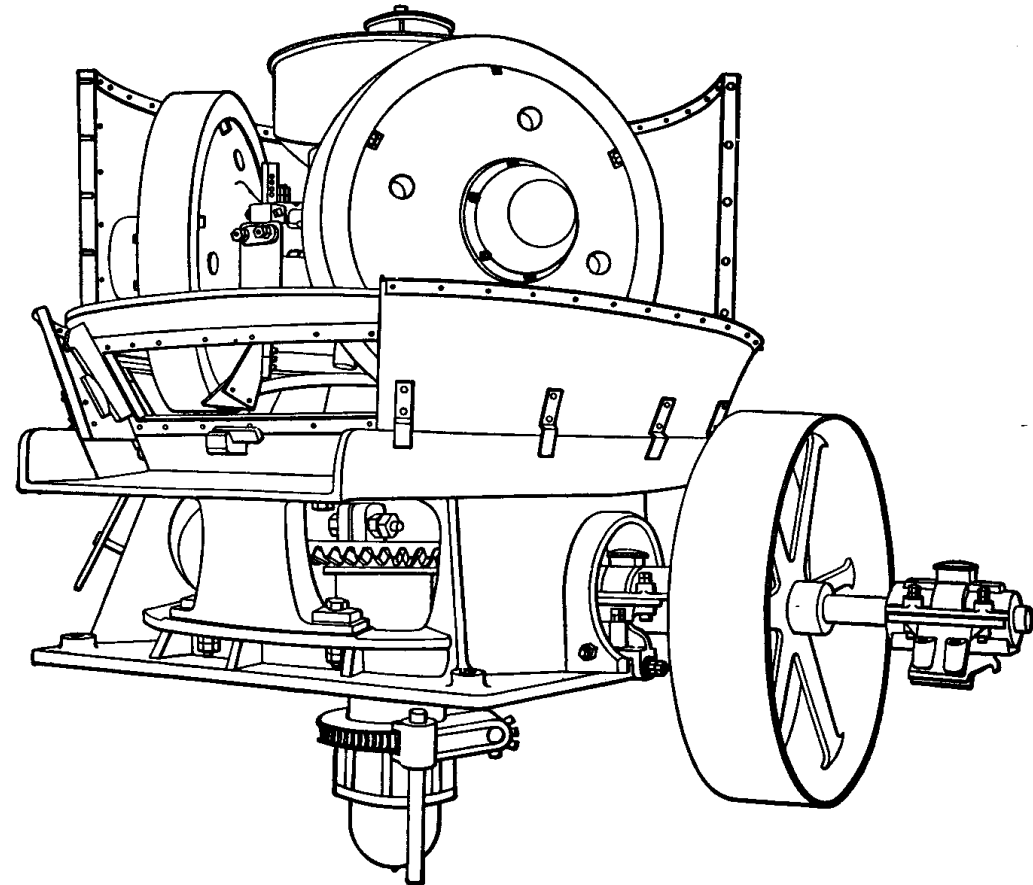
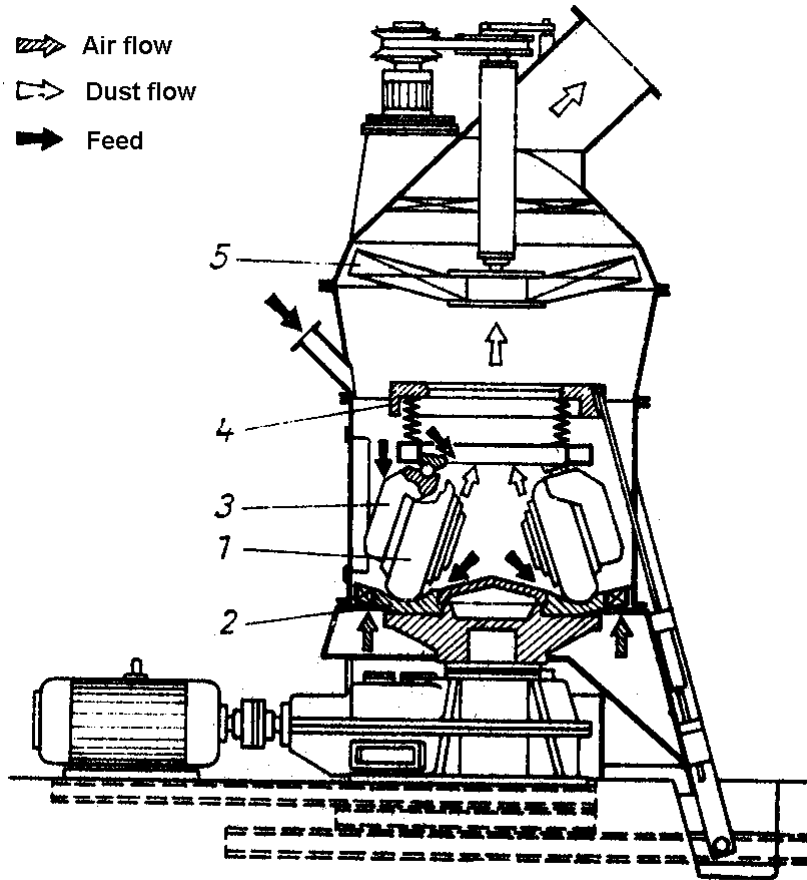
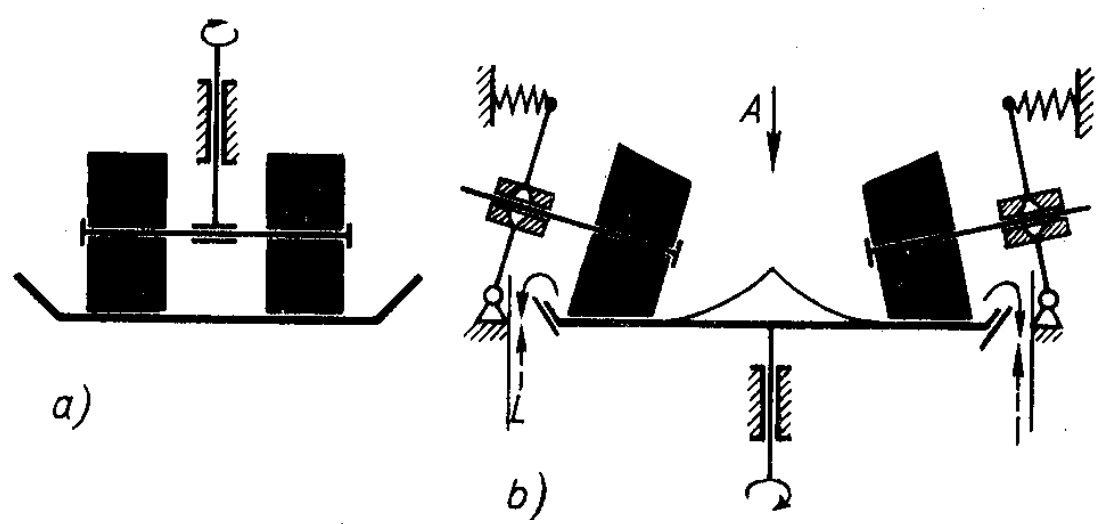
Advantages of autogeneous mills

- Simple flow sheet
- Lower operational costs as ball mills
- Less contamination of product with Fe³⁺ (less steel consumption), favourable for flotation efficiency
- Increased breaking along grain boundaries when ore minerals are stronger than the matrix due to lower impact load compared to ball mills. This results in optimised liberation and more efficient flotation (better adherence of air bubbles).

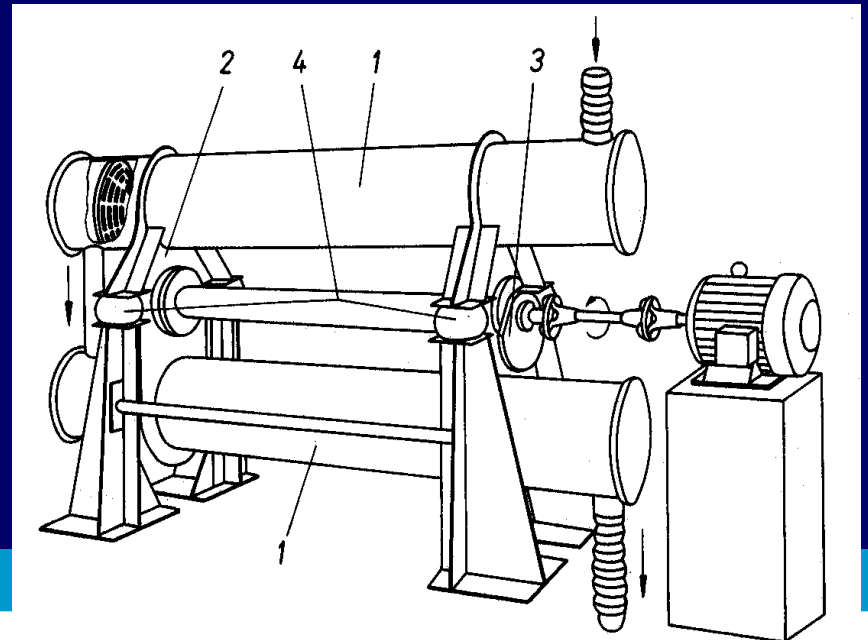
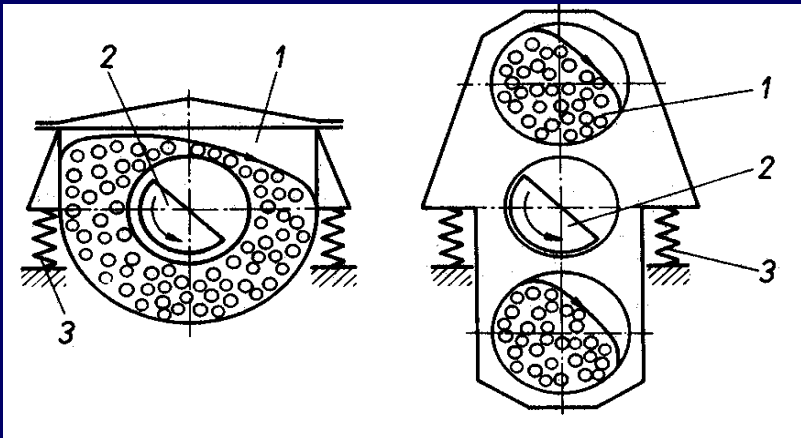
Disadvantages of autogeneous mills

- Not suitable for all ore types
- Autogeneous mills cannot be designed using lab-scale tests results alone: expensive pilot scale testwork is necessary (scaling up).
- Higher energy consumption for fine grinding
- Higher slimes generation may occur due to attrition, which may cause high reagent use in downstream flotation stages
- Capacity per unit of mill volume is lower, due to lower density of grinding media and lower ϕ
- At a variable ore body supply of ore that grinds autogeneously may be problematic

Roller mill



Vibratory mill



November 2012

Power draw

Autogenous mills, rod mills, ball mills

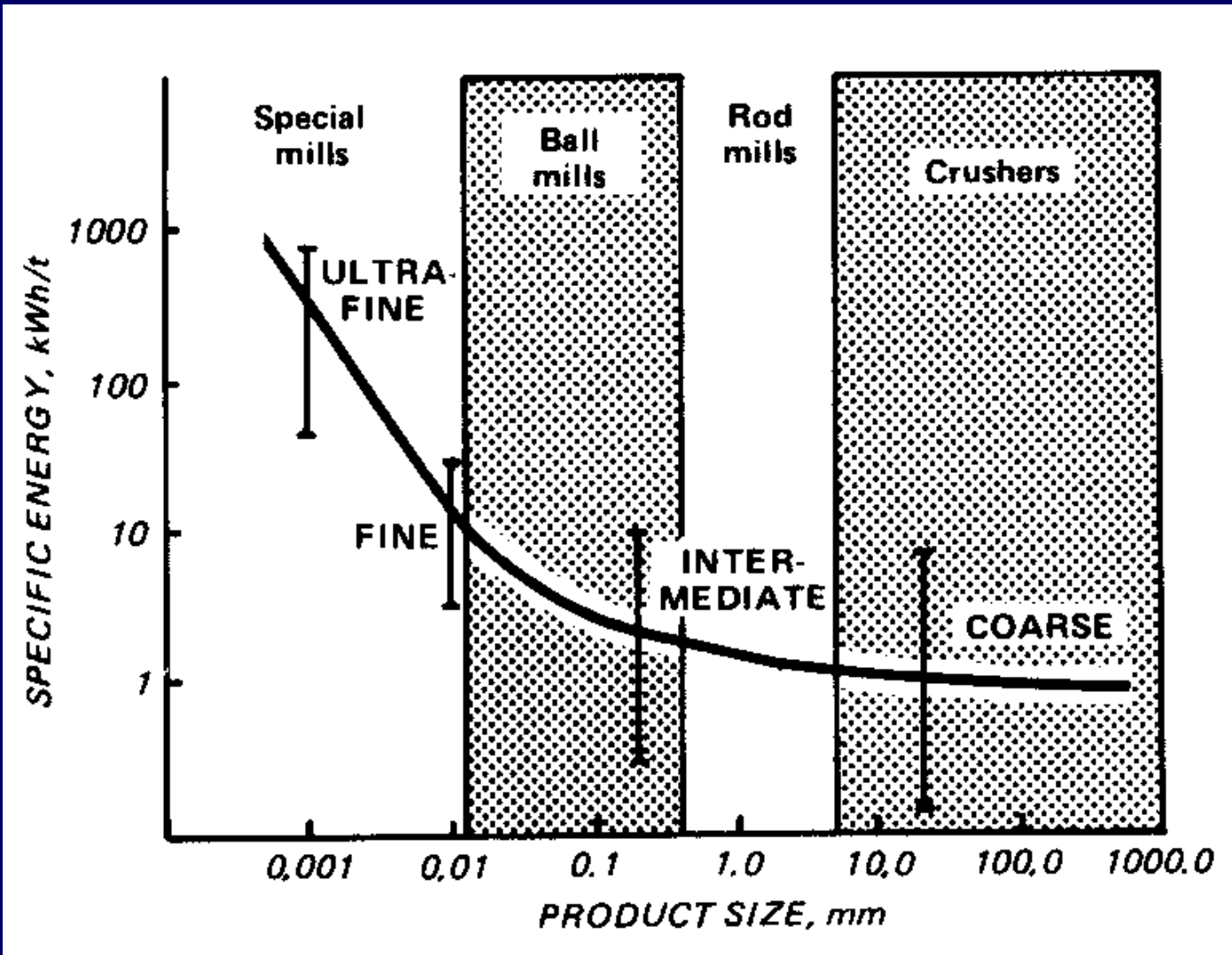


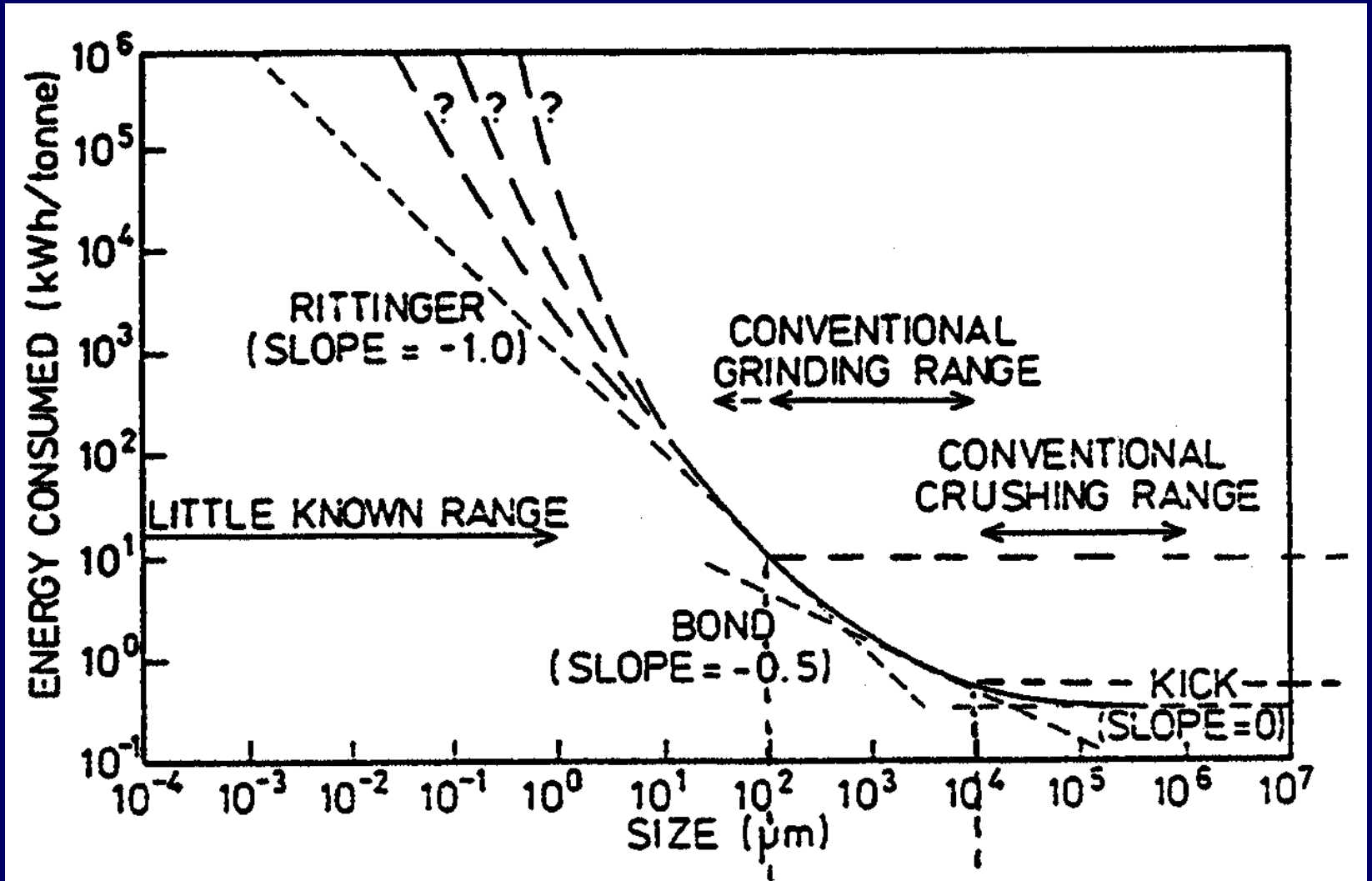
Materiaalkarakteristiek:

W_{im} = Work Index (Bond):

W_{im} = theoretische arbeid die verricht moet worden om een oneindig stuk materiaal ($D_{80(\text{voeding})} \rightarrow \infty$) met een gewicht van één short ton (907 kg) te vergruizen tot $d_{80(\text{product})} = 100 \mu\text{m}$.

Theory: Bond, Rittinger, Kick
→ See lecture notes





$$W_{Bm} = W_{im} \sqrt{\frac{100}{d'_{80}} \frac{\sqrt{d_{80}} - \sqrt{d'_{80}}}{\sqrt{d_{80}}}}$$

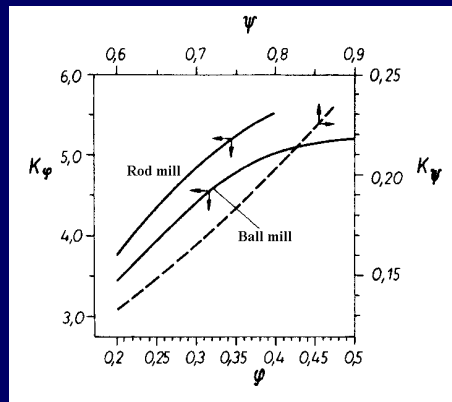
Indicative values of Bond's work index W_{im}

<i>Material</i>	W_{im} [kWh/st] <i>average</i>	W_{im} [kWh/st] <i>range</i>
Quartz	12.8	6.8...22.1
Cement clinker	4.2	1.4...8.8
Limestone	11.1	3.3...27.6
Bauxite	5.3	2.5...12.2
Iron ore	10.0	2.3...33.6
Copper ore	12.4	1.8...40.2
Molybdenum ore	12.5	5.8...18.6
Lead ore	15.5	11.0...21.8
Shale	10.6	5.8...19.0
Gypsum	6.9	4.3...11.7

1. Determine W_{im} (see Section 1.5.2)
2. Determine size distribution of the feed d_{80} and product d'_{80} of the grinding stage to be designed
3. Calculate W_{Bm} in [kWh/t] by using Eq. 1.5.7. and correcting for metric tonnes: $W_{Bm}[\text{kWh/t}] = 0.907 W_{Bm}[\text{kWh/st}]$.

$$W_{Bm} = W_{im} \sqrt{\frac{100}{d'_{80}}} \frac{\sqrt{d_{80}} - \sqrt{d'_{80}}}{\sqrt{d_{80}}}$$

4. Determine power of the motor using $P=W_{Bm}Q$ (Q in t/h)
5. Determine L and D, e.g. by using Eq. 1.3.7 and Fig. 1.3.7.



$$P = 8.44 K_T K_\phi K_\psi L D^{2.5}$$

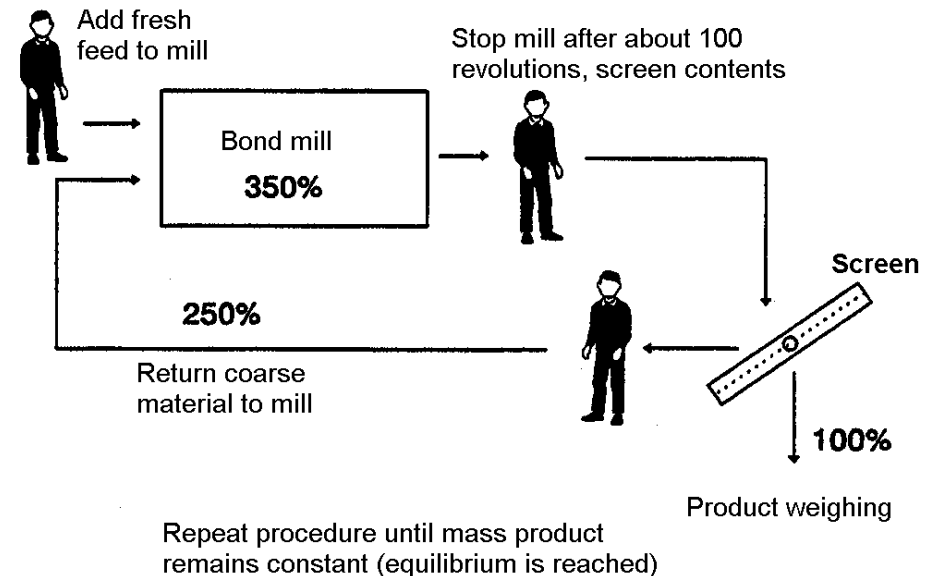
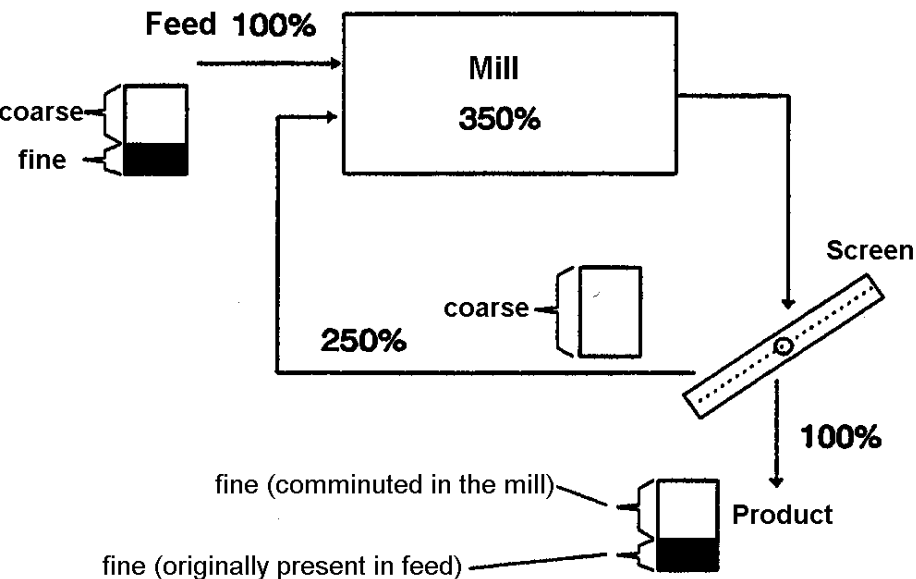
Grindability test (Bond)

“Old” method:

- Size reduction below a specific d80
- Determine size distribution (set of screens)
- Grind in a defined laboratory mill for a fixed time interval
- Determine new size distribution
- (When needed) repeat 2, 3 and 4 until required d'80 is obtained

→ What is wrong with this method???

Bond's method, determine W_{im} (see course notes):



Mass specific grinding efficiency W_m

Consider the **increase in surface energy** ($2\gamma\Delta A$) and relate to the consumed grinding energy W_m :

$$\eta_1 = \frac{2\gamma\Delta A}{W_m} \approx 0.1...1\%$$

If in addition to the increase in **surface energy**, **structural changes of the flaws** are included: $\eta_2 \approx 1...2\%$

If **losses due to plastic deformation** are also included: $\eta_3 \approx 1...12\%$

When **all energy, excluding friction**, that is needed for size reduction (W_v) is related to grinding energy W_m : $\eta_4 = W_v/W_m$

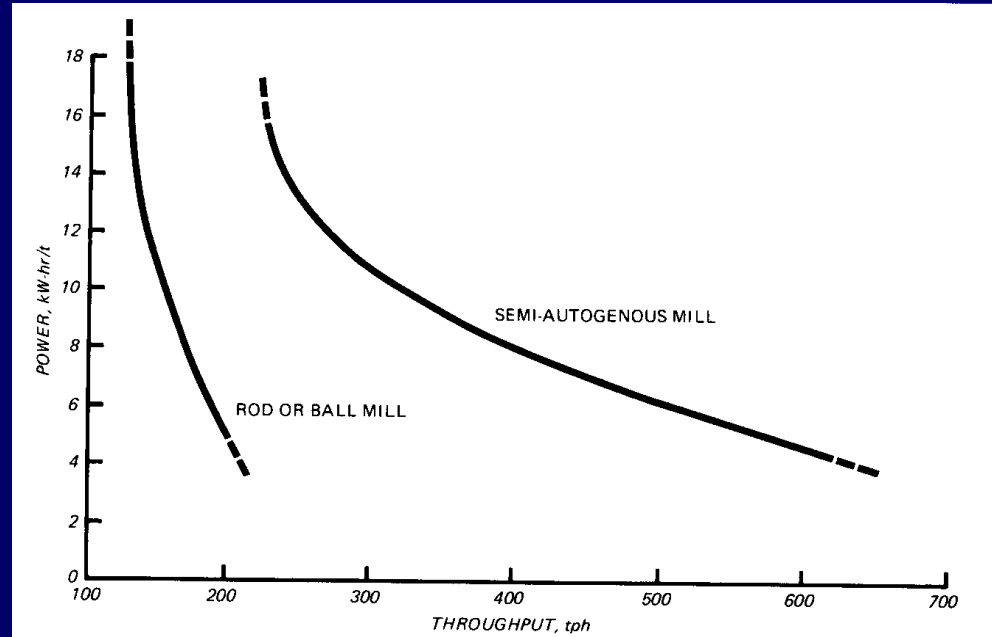
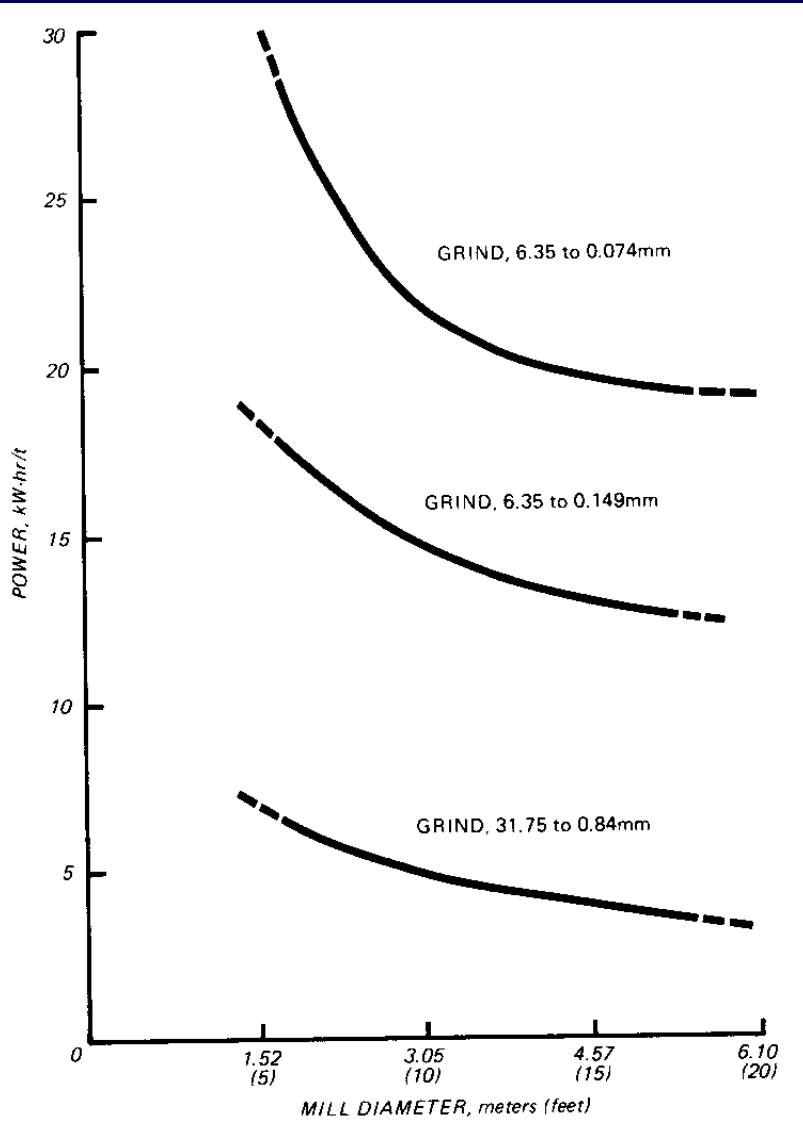
Grinding efficiency

	η_4 (%)	Losses are due to:
roll crusher	70...100	<ul style="list-style-type: none">• Plastic deformation of particles• Plastic deformation of crusher/mill and media surface• Friction• Elastic deformation not leading to breakage• Kinetic energy of material• Machine wear• Generation of noise and vibration
impact crusher and mills	25...40	
roller mill	7...15	
ball mill	6...9	
pneumatic stream mills	1...2	

$$W_{\text{wet}} \approx \frac{1}{3}W_{\text{dry}}$$

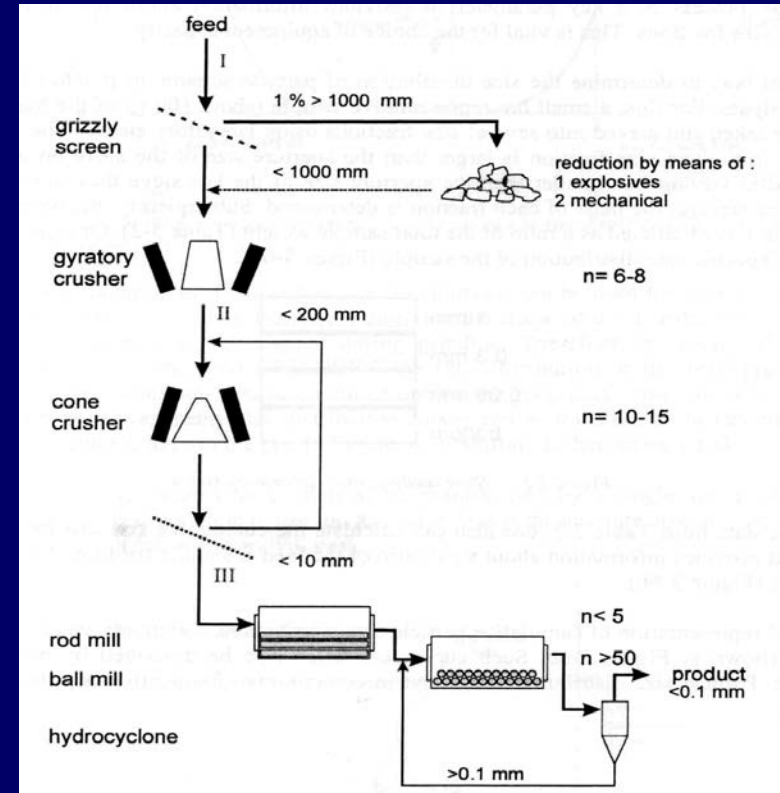
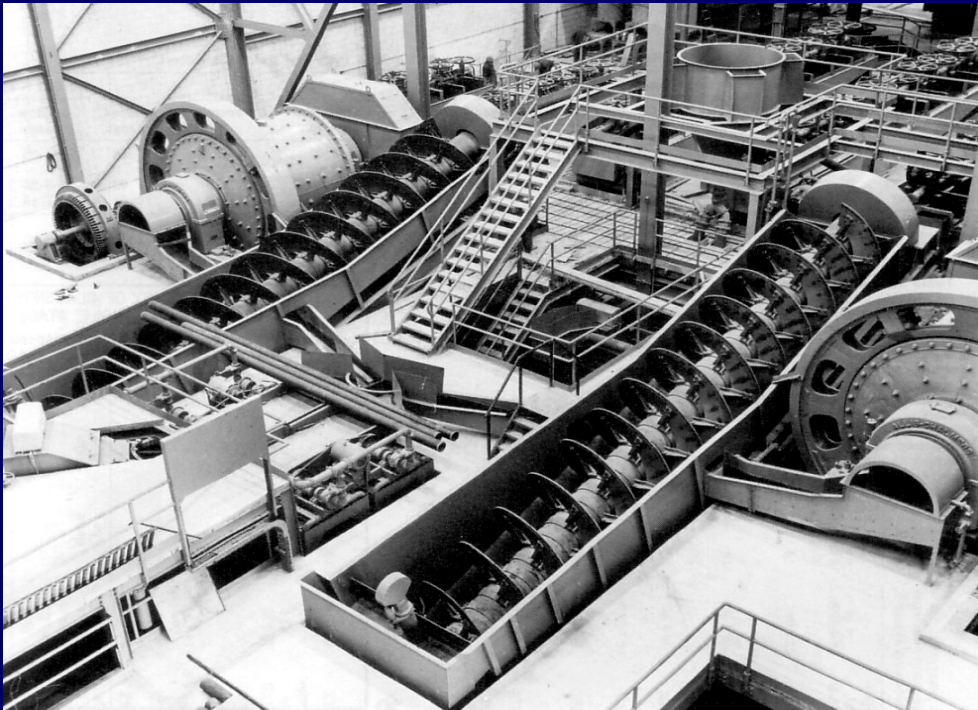
$$W_{\text{wet}} > W_{\text{wet}} + \text{Grinding aids}$$

$$W_{\text{dry}} > W_{\text{dry}} + \text{Grinding aids}$$

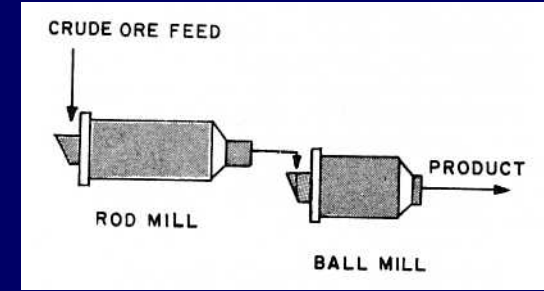
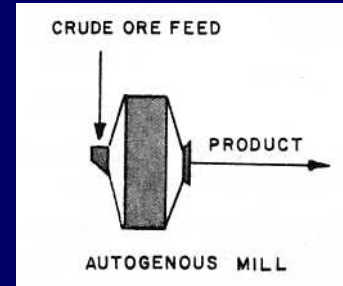
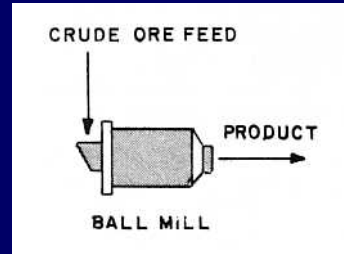
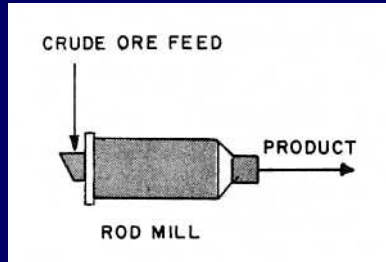


Wet closed loop

Sulphidic ore

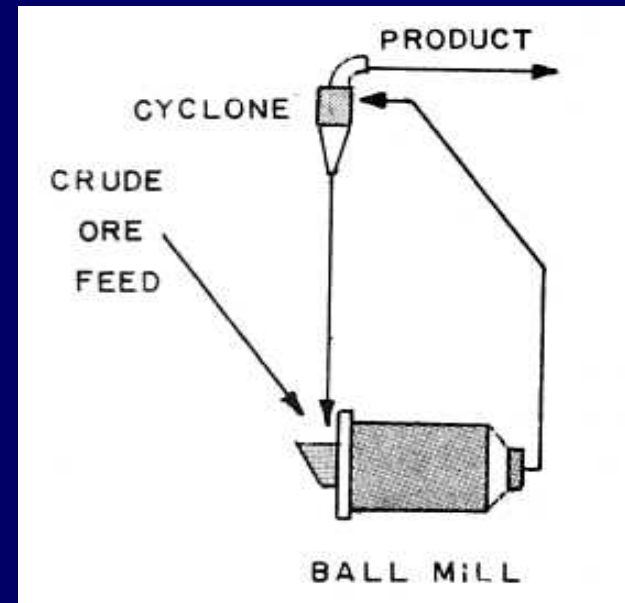
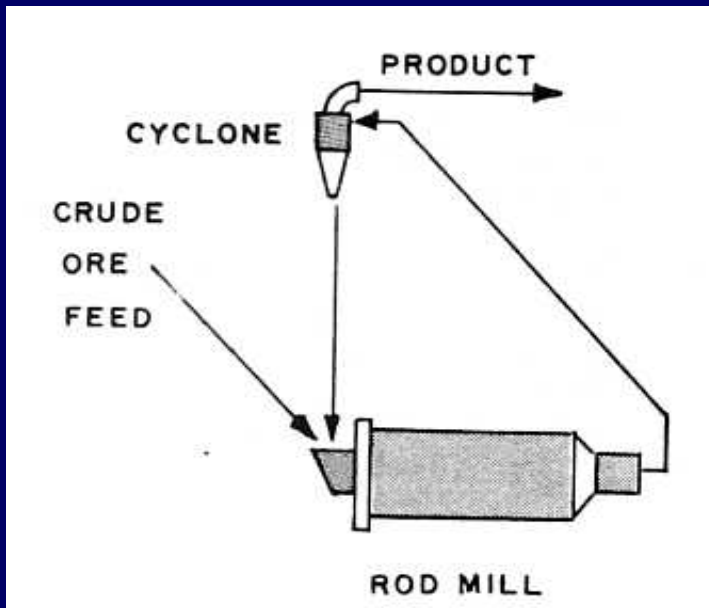


Wet open circuit

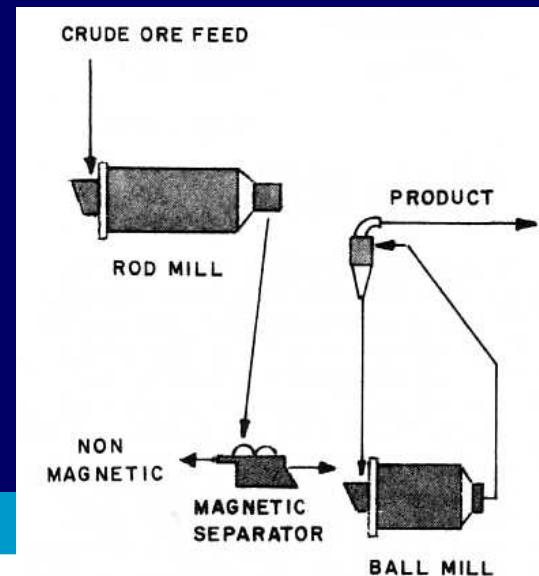
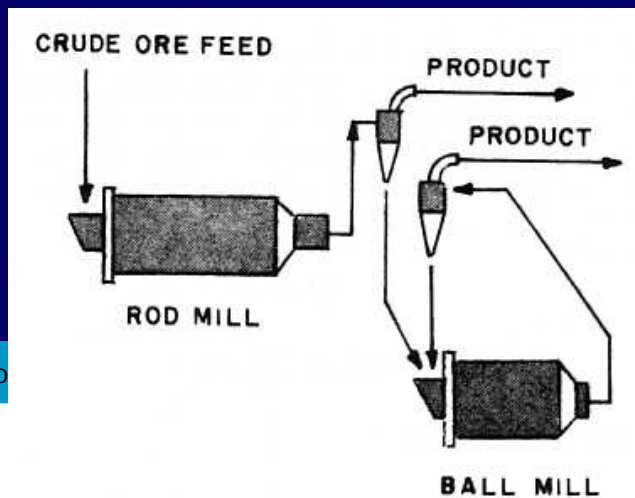
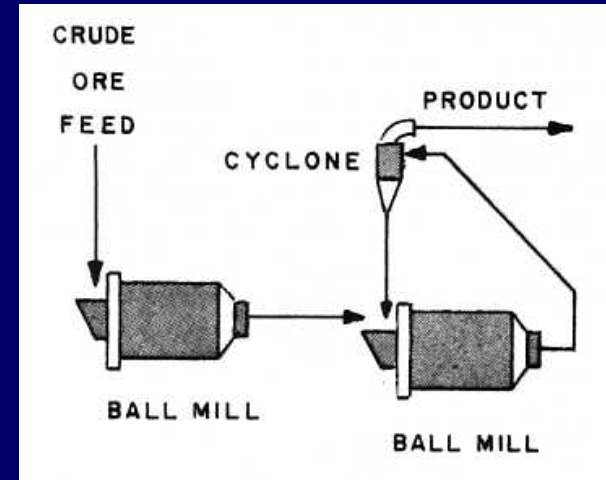
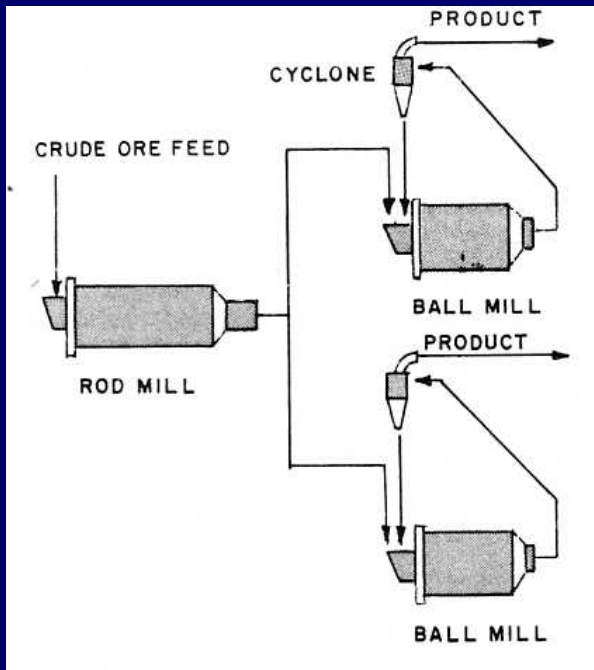


- Minimum equipment requirement (low investment)
- High pulp density. This is favourable when the mill product is leached, e.g. in uranium and gold-silver ores.
- Reduction ratio, n , is only small
- Size reduction to a coarse natural grain size, e.g. grinding of cemented sandy rock
- Flotation middlings are returned to the mill
- Particle size distribution is uncritical (over- and undersize can be tolerated)

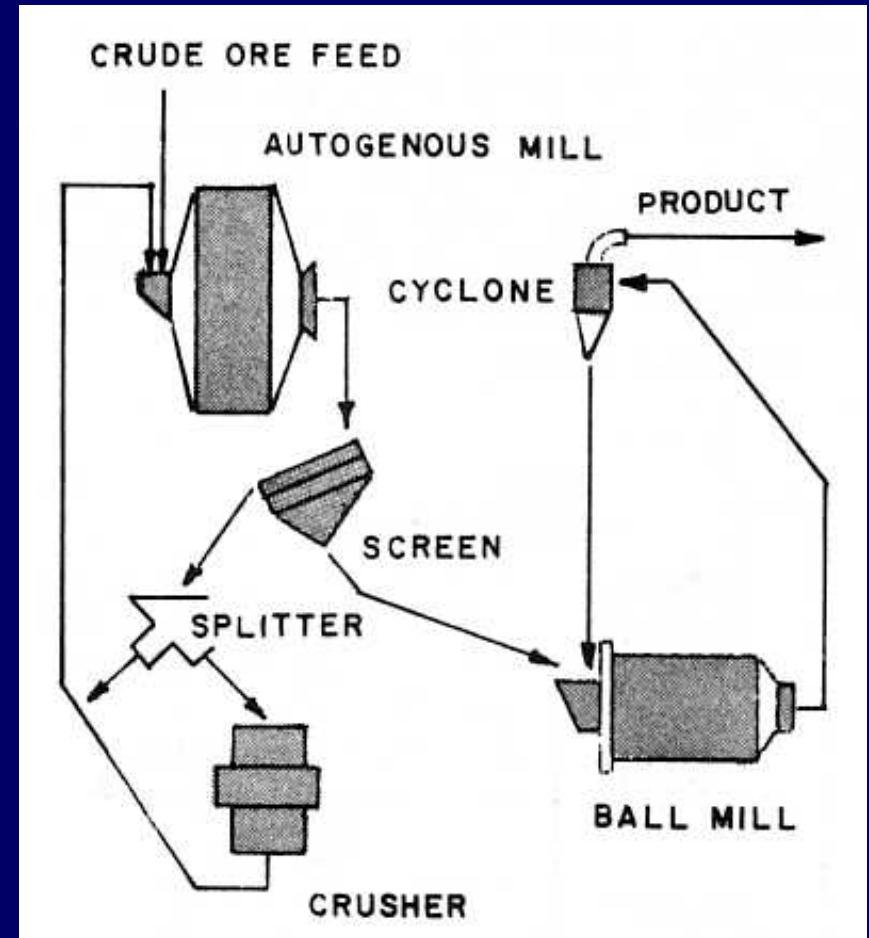
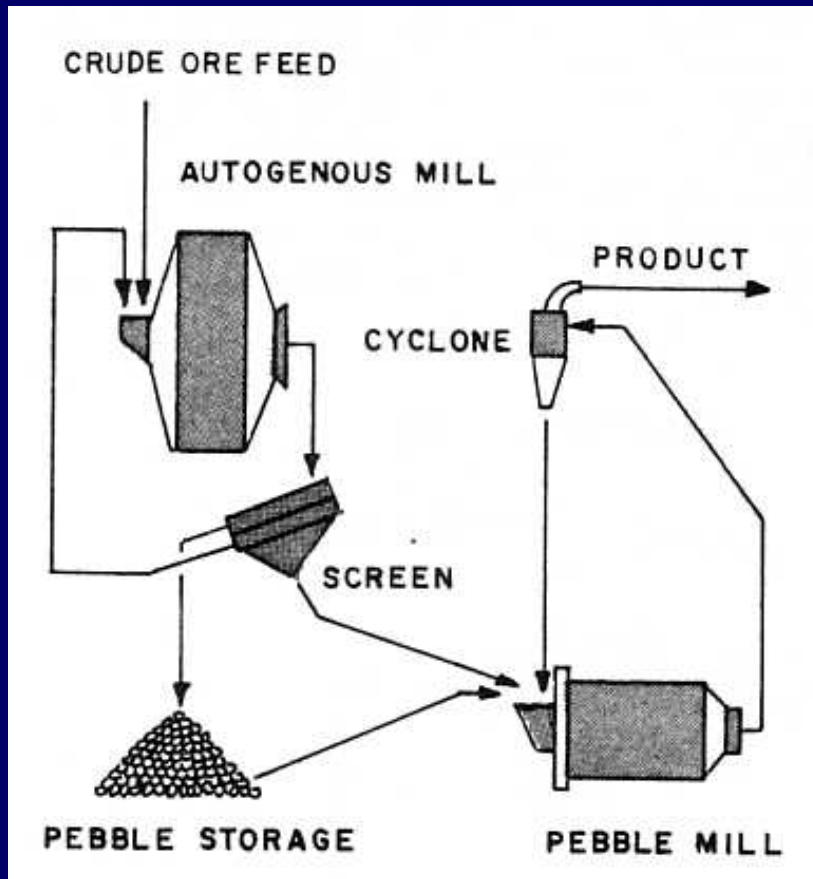
1 stage wet closed



2 stage closed circuit

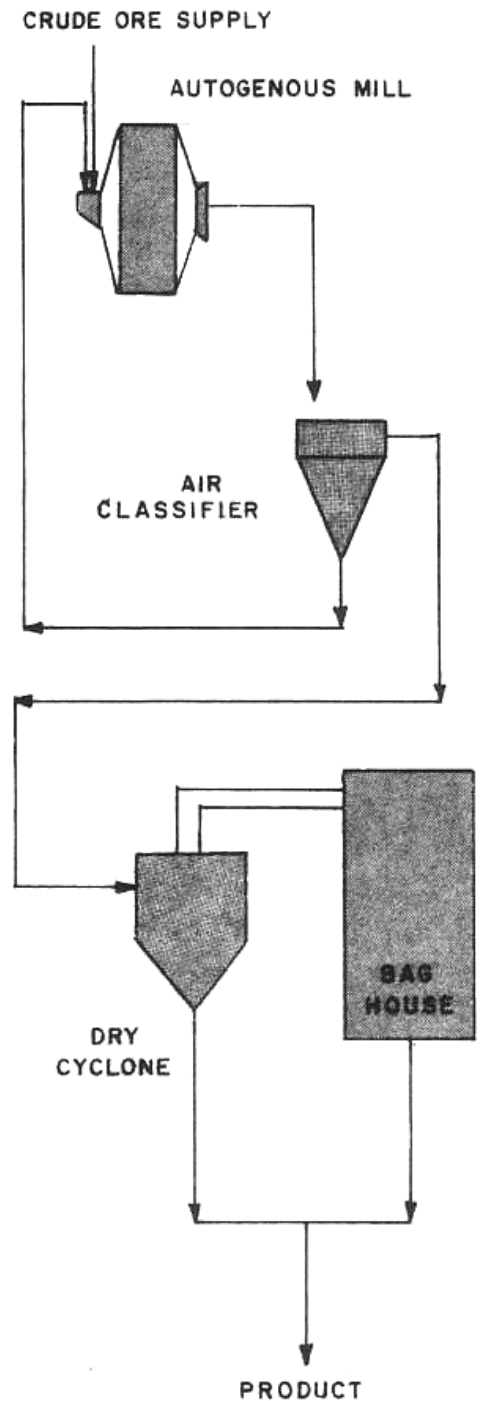
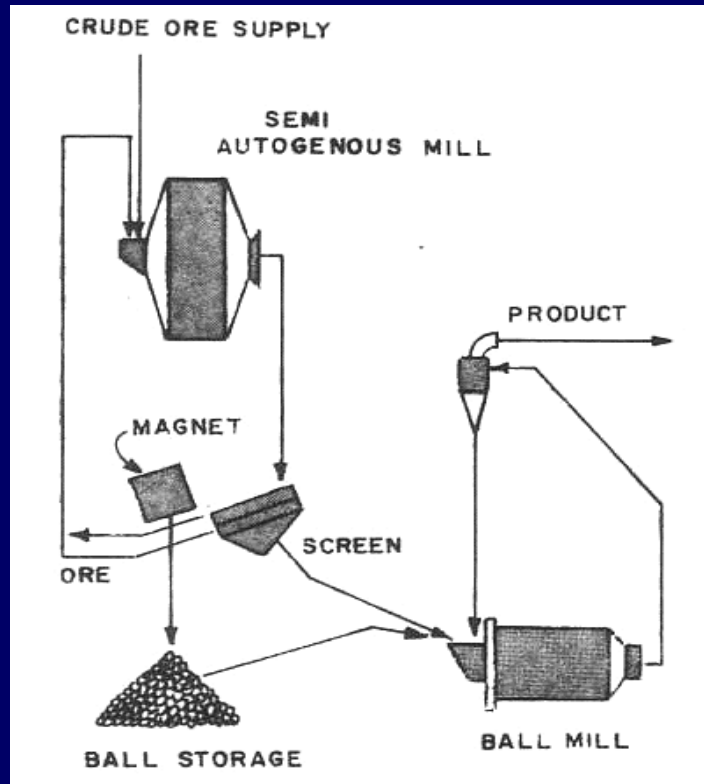


(S)AG mill circuits

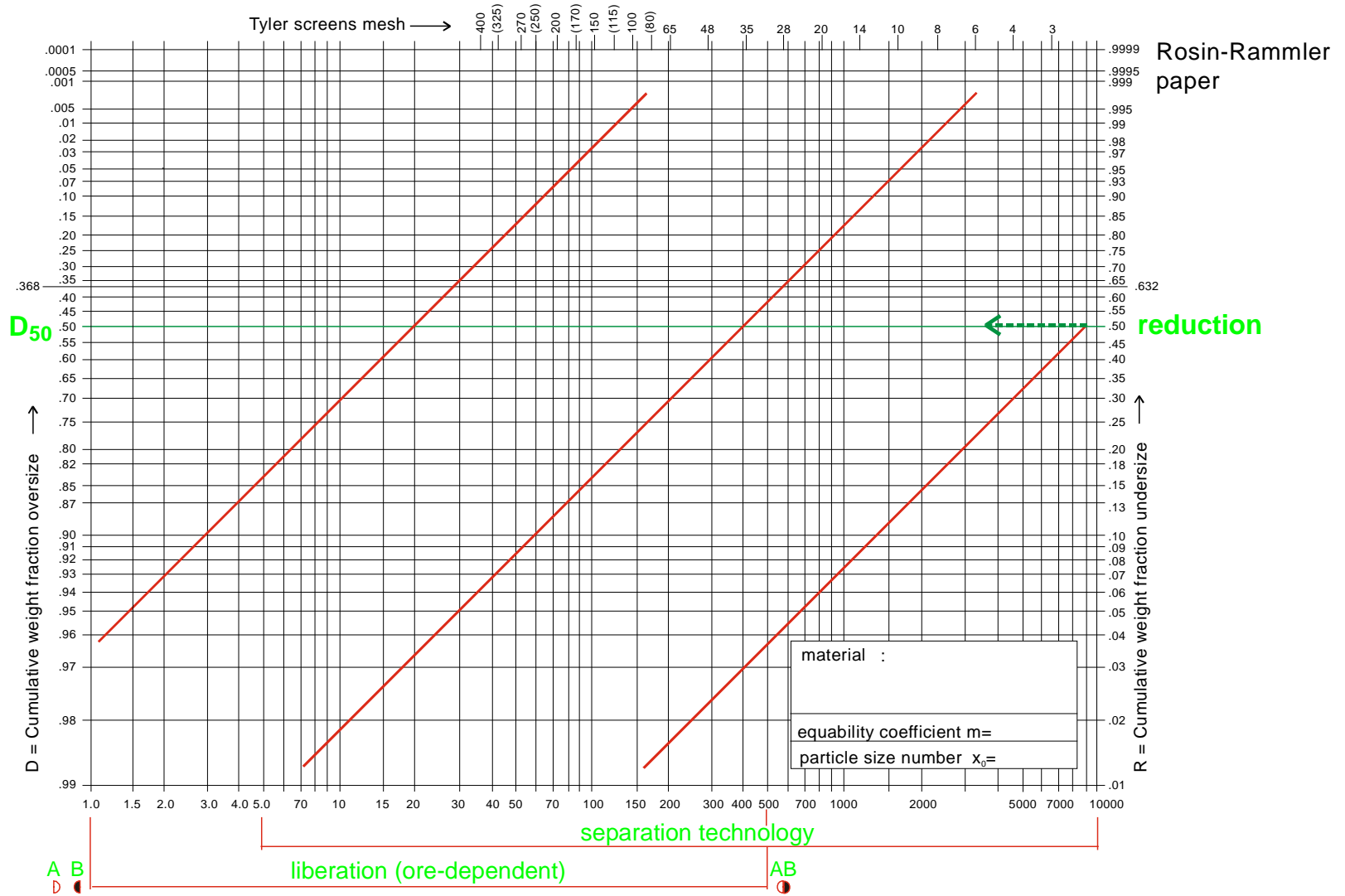


Dry circuits

Cement, iron ore: Often open circuit, 1 or 2 stage

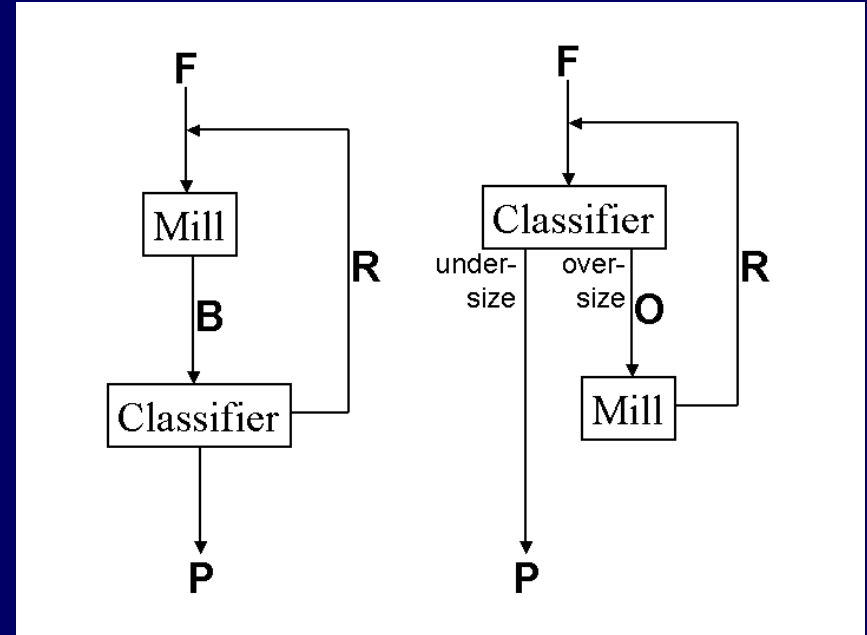
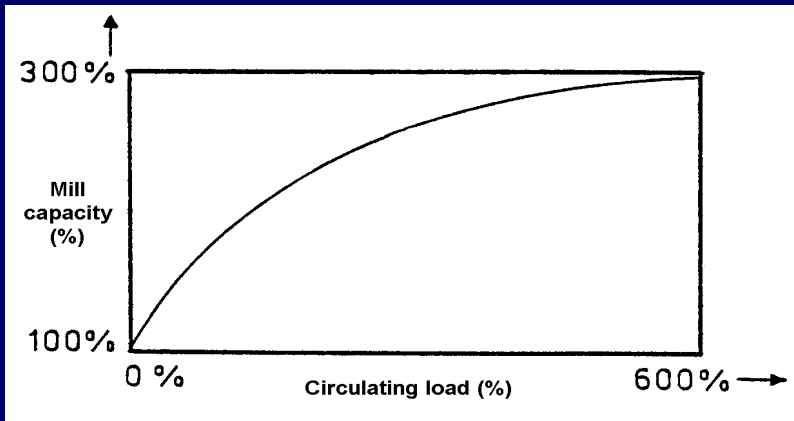


Size distribution

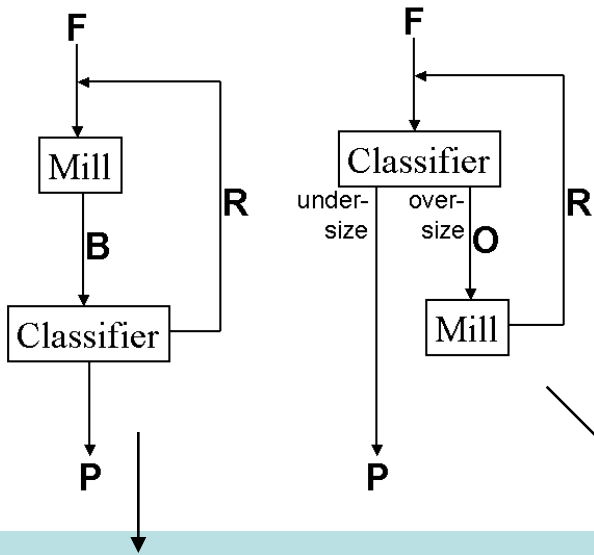


Circulating load

Avoid overgrinding,
Less energy use, more capacity



$$C = 100\% \frac{R}{F}$$



r = too coarse, E = classification efficiency
equilibrium:

$$R = \frac{r}{E} + \frac{r^2}{E^2} + \dots + \frac{r^n}{E^n}$$

$n \rightarrow \text{infinity}$ $R = \frac{r/E}{1 - r/E}$

Mass balance:

$$F = P$$

$$B = P + R$$

$$Bb_i = Pp_i + Rr_i$$

b_i, p_i, r_i = undersize fractions

$$F = P$$

$$O = R$$

$$Ff_i + Rr_i = Oo_i + Pp_i$$

$$C = 100\% \frac{R}{F} = 100\% \frac{p_i - b_i}{b_i - r_i}$$

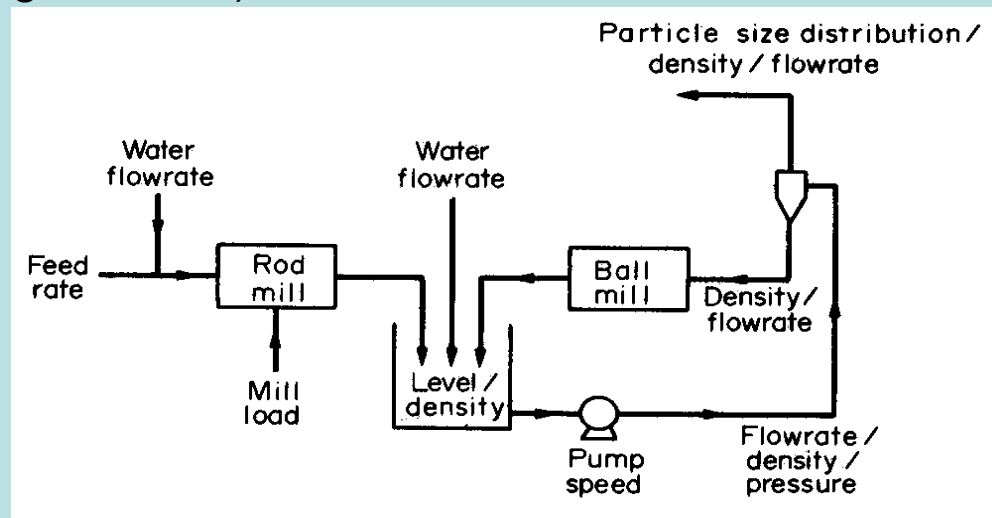
→ Mass difficult to determine

→ Steady state when all C 's for various mesh sizes are about the same

Process control

Control objectives:

- Maintaining constant product size at maximum throughput
- Maintaining constant feed rate within a limited product size range
- Maximise production per unit time in conjunction with downstream processing (e.g. flotation)



feed rate and **water addition** can be varied independently

→ variable speed feeders combined with weightometers

Process control

Grinding medium charge is controlled by monitoring power draw of the mill. When it drops, fresh grinding media must be added.

Flow rate and density can be monitored by magnetic flow meters and nuclear density gauges.

Sump level is monitored continuously.

Changes in feed rate → initiate a slow progressive change in which the final equilibrium represents the maximum product response

Changes in classifier water addition → immediate maximum response with only a relatively small equilibrium product response. Increasing water addition increases circulating load and sump level.

Control strategies

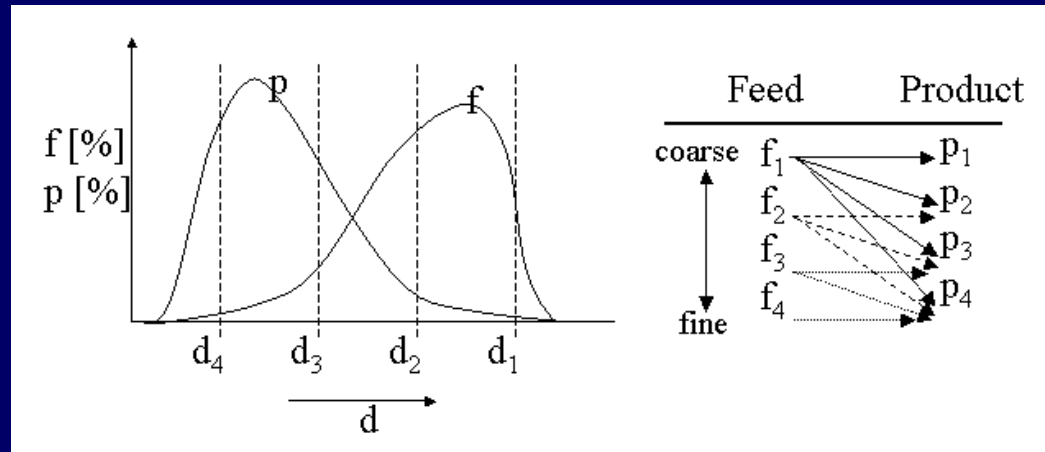
If **constant product size at constant feed rate** is required, only the classifier water addition can be manipulated, resulting in volumetric and density fluctuations in the cyclone overflow.

Maximum throughput at constant product size allows manipulation of both feed rate and classifier water → fixed product size set-point and a circulating load set-point just below the maximum tonnage constraint.

Two control strategies are applied:

1. Product size is controlled by ore feed rate, and circulating load by classifier water addition.
2. Product size is controlled by classifier water, and circulating load by ore feed rate.

Modelling



Matrix representation

Selection of particles for crushing. Each particle has a specific probability of being crushed during a grinding stage.

Breakage of the selected particles.

Often a third operation is considered (closed circuit grinding):
Classification of the particle population after crushing

$$\begin{bmatrix} x_{11} & 0 & 0 & 0 \\ x_{21} & x_{22} & 0 & 0 \\ x_{31} & x_{32} & x_{33} & 0 \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Feed Product

Grinding matrix **X** is composed of:

Matrix **S** describing **particle selection**

Matrix **B** describing **breakage function**

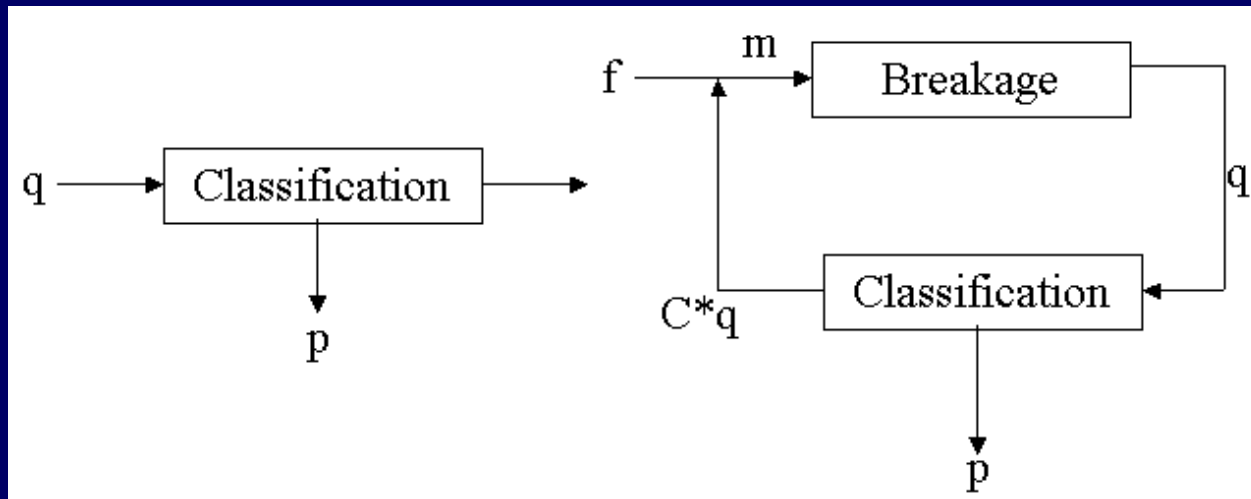
Matrix **C** describing **classification** of the population after each grinding stage

Selection and breakage:

$$\begin{bmatrix} b_{11} & 0 & 0 & 0 \\ b_{21} & b_{22} & 0 & 0 \\ b_{31} & b_{32} & b_{33} & 0 \\ b_{41} & b_{42} & b_{43} & b_{44} \end{bmatrix} \begin{bmatrix} s_1 & 0 & 0 & 0 \\ 0 & s_2 & 0 & 0 \\ 0 & 0 & s_3 & 0 \\ 0 & 0 & 0 & s_4 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} + \begin{bmatrix} 1-s_1 & 0 & 0 & 0 \\ 0 & 1-s_2 & 0 & 0 \\ 0 & 0 & 1-s_3 & 0 \\ 0 & 0 & 0 & 1-s_4 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \\ f_3 \\ f_4 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$

Classification (fines):

$$\begin{bmatrix} 1-c_1 & 0 & 0 & 0 \\ 0 & 1-c_2 & 0 & 0 \\ 0 & 0 & 1-c_3 & 0 \\ 0 & 0 & 0 & 1-c_4 \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} = \begin{bmatrix} p_1 \\ p_2 \\ p_3 \\ p_4 \end{bmatrix}$$



Matrix model

The overall matrix equation describing the grinding process becomes

$$\mathbf{p}_n = \mathbf{X}_n * \mathbf{f}$$

$$\text{with } \mathbf{X} = (\mathbf{I} - \mathbf{C}) * (\mathbf{B} * \mathbf{S} + \mathbf{I} - \mathbf{S}) * [\mathbf{I} - \mathbf{C} * (\mathbf{B} * \mathbf{S} + \mathbf{I} - \mathbf{S})]^{-1}$$

DEM = Discrete Element Modelling

