H7 (Fitts)

1) 165 m2 /day
2) 550 m /day
3) −0.0063 ft/day (0.19 cm/day). That $N < 0$ means that the leakage out the base of the aquifer exceeds the recharge in the top by this amount.
4) 95 ft2 /day (8.8 m2 /day)
5) Superposition is when solutions of a linear differential equation are added to each other, and the resulting solution is also a solution. This applies to linear differential equations. Many examples of superposition are described in the chapter, including a well in a uniform flow and wells with image wells near linear and circular boundaries.
6) There is a range of possible answers here. One possibility is to assume that the well is located 5 m ahead of the leading edge of the plume. Assuming that the origin of an x, y coordinate system is at the well, like in Figure 7.11, a point at the upper limit of the capture zone would have approximate coordinates of (−15.4, 4.5). The angle $\Theta = \arctan(y/x) = 2.86$ radians. Use Eq. 7.28 to solve for Q (m, day units): 1.7 m3 /day. As a check, we could examine the ultimate width of this capture zone far upstream from the well, using Eq. 7.30: ±5.1 m. This seems reasonable, since the y coordinates of the capture zone at the plume are ±4.5.
7) The solution neglects recharge and leakage – all flow is from lateral flow. Close to the well this may be reasonable, but on a larger scale recharge and leakage contribute a significant amount of flow and should not be ignored. The capture zone area is finite when there is finite recharge/leakage into the aquifer. Without accounting for recharge/leakage the area and extent of the capture zone is infinite.
8) A) Vertical thickness is 30 m (domain top elevation - bottom elevation). K = 1 m/day, T = 30 m2 /day.
   b) $q_x \simeq 0.0121$ m/day.
   c) $\partial h/\partial x \simeq -0.0121$.
   d) 0.0121 m/day.
   e) 100.6 m3 /day
   f) The well Q is 100 m3 /day, which essentially the same as computed in the previous step, allowing for inaccuracy in measuring the width of the capture zone.
9) The sketch map would look like Figure 7.15 with a real well and an image well with opposite discharge. The model equation would be $h^2 = Q/(\pi K) * \ln(r1/r2) + C$, where Q is the well discharge, r1 is the distance from the real well and r2 is the distance from the image well.
10) 1980 ft /day (184 m2 /day)
11) A) 0.273 m2 /day
    b) 0.49 m3 /day
    d) 0.0092 m/day
    e) 73,600 N/m2
12) 199.6 m
13) −3.7 m
14) C = 0.28 ft (8.5 cm), hd = −2.2 ft (-67 cm).

15) Only cases (b) and (d) are appropriate for flow net analysis.
16) 160 m3 /day
17) 120 m3 /day
18) A) Most of the head loss is in the clayey till, because its conductivity is about 67 times lower than that of the fine sand.

    b) 230 ft3 /day (6.5 m3 /day)

    c) 0.09 ft/day (2.7 cm/day)

    d) Assumptions made: (1) the clayey till is homogeneous with isotropic K, (2) the other materials are also homogeneous and isotropic with the conductivities listed, (3) flow is two-dimensional in the vertical plane. Item (1) is probably the most important source of uncertainty. To reduce the uncertainty associated with (1), you could conduct K tests on the clayey till compacted as it will be when constructed. Horizontal K values would be most appropriate for this analysis since flow is mostly horizontal. To reduce uncertainty associated with (2), K tests of these materials could be made. To reduce uncertainty associated with (3), investigate hydrogeologic conditions at the two abutments at the ends of the dam, and check the continuity of conditions along the dam.
19) A) Q = 0.67 m3 /day.

    b) Q = 1.85 m3 /day.

    c) Q = 0.84 m3 /day.

    d) With the model of part (c), the flow originates closer to the dam in the reservoir and exits closer to the dam in the tailwater, because there is little resistance to vertical flow. The model of part (b) has flow originating and exiting farther from the dam because the vertical resistance to flow exceeds the horizontal resistance.

    e) With the geometry of this situation, there is more horizontal than vertical flow. Thus, changing the horizontal K has a greater impact on the Q than changing the vertical K does.
20) xd = AK/2N = (0.4*1.5)/(2*0.007) = 42.9 ft (13.1 m).
21) A = -1.0, h^2 = -x+400.
22) A) K = 2.88 m/day.

    b) h0 =√(152.8) = 12.4 m.
23) This depends on how the model was constructed and in particular what far-field boundary conditions were used. I created a model with head-specified boundaries far away from the area of the wells, and set the heads at these to 12.36 m, the computed far-field head. The model matches the solution computed by hand very closely.

H9 (Fitts)

1) The derivative dy/dx at point a is the slope of the tangent line (dotted) at that point. A finite difference approximation of the derivative at a is the slope of a line connecting two points on the curve near a. The slope $\Delta y/\Delta x$ of the line from b to c (dashed) is a finite difference approximation of the derivative at a. A derivative is a measure at a point, but a finite difference is a measure over a finite interval.
2) If there is no source/sink in the cell, $Q_s = 0$ and the equation becomes $h = 1/6*[h(x+) + h(x-) + h(y+) + h(y-) + h(z+) + h(z-)]$. This states that the head at the central node equals the average of heads at the six neighboring nodes. This result is consistent with the three-dimensional mean value theorem, which applies to solutions of Laplace's equation.
3) Since for solutions of Laplace's equation, $\nabla^2 h = 0$, this equation can be rearranged to give the same result as in the previous problem $h = 1/6*[h(x+) + h(x-) + h(y+) + h(y-) + h(z+) + h(z-)]$.
4) $h_3 = 11.09$ ft (3.38 m), $h_2 = 11.42$ ft (3.48 m).
5) The model-predicted head will likely be lower than actual head. Generally model cells are much larger than the well radius. The finite difference method assumes a linear variation in head (constant gradient) between nodes, but near a well the variation in head is actually logarithmic, with steeper gradients closer to the well. The linear variation in a finite difference model misses this behavior, underestimating the drawdown in a pumping well and draw-up in an injection well.
6) $h = 380.42$ m.
7) $h = (K(x+)\, h(x+) + K(x-)\, h(x-) + K(y+)\, h(y+) + K(y-)\, h(y-) + Q_s) / (K(x+) + K(x-) + K(y+) + K(y-))$
   The values of $K(x+)$, $K(x-)$, $K(y+)$, and $K(y-)$ are calculated based on equations like Eq. 9.9.

12) $q_{b2} = 0.0109$ m/day, $Q = 13.5\ m^3$/day.
13) $h_n = ¼ (h_o + h_p + h_q + h_r)$
14) a) $T = K_1 h$ for ($h \le b_1$), $T = K_1 b_1 + K_2 (h - b_1)$ for ($b_1 \le h \le (b_1 + b_2)$),
    $T = K_1 b_1 + K_2 b_2$ for ($h \ge (b_1 + b_2)$).
    b) $\Phi = K_1 b_1^2 + K_1 b_1 b_2 + K_2 b_1 b_2 + K_2 b_2^2 + C_3$ for ($h = b_1 + b_2$) and
    $\Phi = ½ K_1 b_1^2 + K_1 b_1 b_2 + ½ K_2 b_2^2$ for ($h = b_1 + b_2$),
    $C_3 = -½ K_1 b_1^2 - K_2 b_1 b_2 - ½ K_2 b_2^2$
15) $\nabla^2 (½ K h^2) = \nabla^2 \Phi = -N$
16) There are two wells, both extracting water from the aquifer. There is a string of three imper-meable barrier line elements in the center of the plot, and a string of three line-sinks in the lower right of the plot. In addition, there is a uniform flow from upper left to lower right.
17) When the head is high enough that aquifer is confined, $\Phi = Th - ½ Kb^2$.
18) $K = 1/(½h_p^2 - b h_0 - ½b^2) * Q/(2\pi) \ln(r_{p \to p}/ r_{i \to p})$
    $= 1/(½*18^2 - 35*39 + ½*35^2) * 2310/2\pi \ln(0.25/118)$.