Exam - AESB2440, Geostatistics & Remote Sensing

2C-Zaal 2, July 2, 2015, 14.00 - 17.00

This exam consists of 45 questions

You start with 10 points. Every correct answer is worth 2 points.

Always clarify your answer.

It is allowed to use a simple calculator without memory.

Good luck!

Probabilities

Consider the experiment in which people are asked on which day of the week they are born.

1. What is the sample space?
2. Assign a probability to each day of the week.
3. What is the probability that three people are all born outside the weekend?

Events

Let A and B be events in a probability space. $P(A) = 0.4$ and $P(B) = 0.5$. Furthermore, $P(A \cap B) = 0.1$.

4. Compute the probability $P(A \cup B)$.
5. Determine the probability of the event $A^C \cup B$.

Uniform distribution

The random variable X has a uniform distribution on the interval $(-1, 2)$, that is $X \sim U(-1, 2)$.

6. Sketch the probability density function of X. What is its integral?
7. Sketch the cumulative distribution function of X.
8. What is the mean of X?
9. What is the mean of the random variable $Y = 3X + 7$?
10. What is the mean of the random variable $Z = X^2 + 3$?

Running track

Henk sets out the track for a 100-meter running match. He does so by taking 100 steps. The length of each step has a normal distribution $\phi(x)$ with mean $\bar{x}$ and standard deviation $\sigma = 0.10m$. The lengths of the steps are independent.

11. What are the mean and standard deviation of the standard normal distribution?
12. Where is the maximum of the derivative of the cumulative distribution function of $\phi(x)$?
13. Explain how a normal distribution with mean $\bar{x}$ and standard deviation $\sigma = 0.10m$ is transformed to the standard normal distribution.
14. If $\bar{x} = 0.95m$, what is the probability that the length of one step is 1 m or more? Give your answer as an evaluation of the standard normal distribution.
15. What is the variance and the standard deviation of the random variable $100\bar{x}$?
16. If $\bar{x} = 0.97m$, calculate the probability that the length of the track is below $95m$. Again, give your answer as an evaluation of the standard normal distribution.

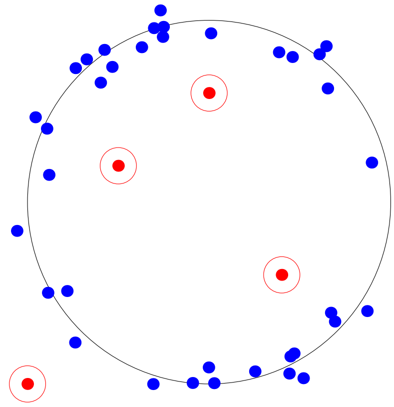

Figure 1: Circle fitted to some observations. Four outliers are indicated by small circles.

Ransac for circle fitting

Consider the observations in Figure 1. There are 4 outliers and 36 correct points (inliers).

17. Why is ordinary least squares not a robust method?
18. What is the minimum number of points needed in $\mathbb{R}^2$ to fix a circle?
19. What is the probability that all points are inliers if you randomly select this number of points in Figure 1?
20. Explain how Ransac should be used to identify and remove the outliers in Figure 1.

Least Squares

Hint: The inverse of a 2×2 matrix:

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^{-1} = \frac{1}{\mathrm{D}} \begin{pmatrix} a_{11} & -a_{12} \\ -a_{21} & a_{22} \end{pmatrix}, \text{ with D} = a_{11}a_{22} - a_{12}a_{21}$$

The height of three points $i = 1, 2, 3$ is denoted h_i. A surveyor measures the following three height differences in meter,

$$\begin{aligned} h_{12} &= h_2 - h_1 &= 57m, \\ h_{13} &= h_3 - h_1 &= 69m, \\ h_{23} &= h_3 - h_2 &= 21m \end{aligned}$$

The height of point 1 is known to be zero, $h_1 = 0$. The surveyor wants to determine the height of the other two points in her office using least squares. That is, she wants to estimate $\hat{\mathbf{x}} = (h_2, h_3)^T$

21. What is the vector of observations $\mathbf{y}$?
22. Give the model matrix A.
23. Determine the least squares solution $\hat{\mathbf{x}}$.
24. Give the vector of adjusted observations $\hat{\mathbf{y}}$.
25. What does the vector of adjusted observations represent?
26. Give the vector of residuals $\hat{\mathbf{e}}$.
27. What does the vector of residuals represent?

The surveyor decides to measure the height difference h_{23} again.

28. How should the linear model of observations be adapted to incorporate this additional measurement?

The standard deviation of the height difference measurements is $\sigma = 5m$.

29. Explain how the Monte Carlo method can be used to get insight in the quality of the estimated heights of the points.

Deterministic Interpolation

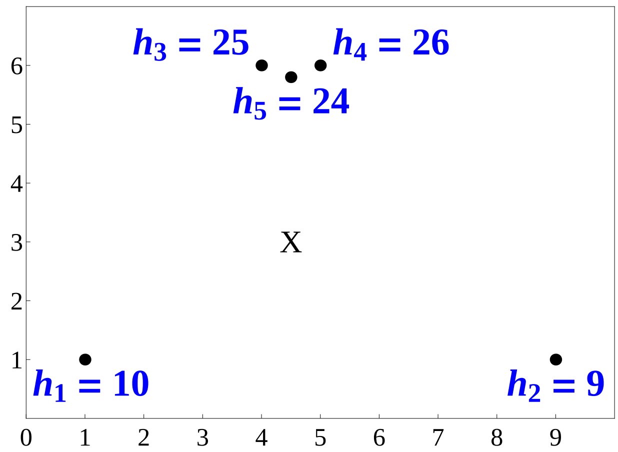

Figure 2: Five height observations: $h_1 = 10$, $h_2 = 9$, $h_3 = 25$, $h_4 = 26$ en $h_5 = 24$.

Figure 2 shows five height observations. Marco wants to estimate the height at the cross.

30. Is triangle Δ_{145} a triangle of the Delaunay triangulation of the five height locations in Figure 2? Why?

31. What is the convex hull of the five height locations?

Weights	w_1	w_2	w_3	w_4	w_5	height
1. Nearest Neighbor						
2. Triangular						
3. Inverse Distance, power = 1						
4. Inverse Distance, power = 2						

Table 1: Table of weights

Enter your answers on the following four questions in (a copy of) Table 1. It is not necessary to use formulas, but explain your answers. Your answers may be approximate.

32. Indicate in the table which weight each observation gets in case of interpolation according to *Nearest neighbor* interpolation. Also give a corresponding estimate of the height.

33. Same question for *Triangular interpolation*.

34. Also give weights and height estimate for an *Inverse distance interpolation* with power 1.

35. Finally, give weights and height estimate for an *Inverse distance interpolation* with power 2.

36. Imagine each of the four methods in Table 1 is applied to generate an interpolated map at a dense grid, based on the observations in Figure 2. Order the resulting maps according to their smoothness.

Stochastic Interpolation

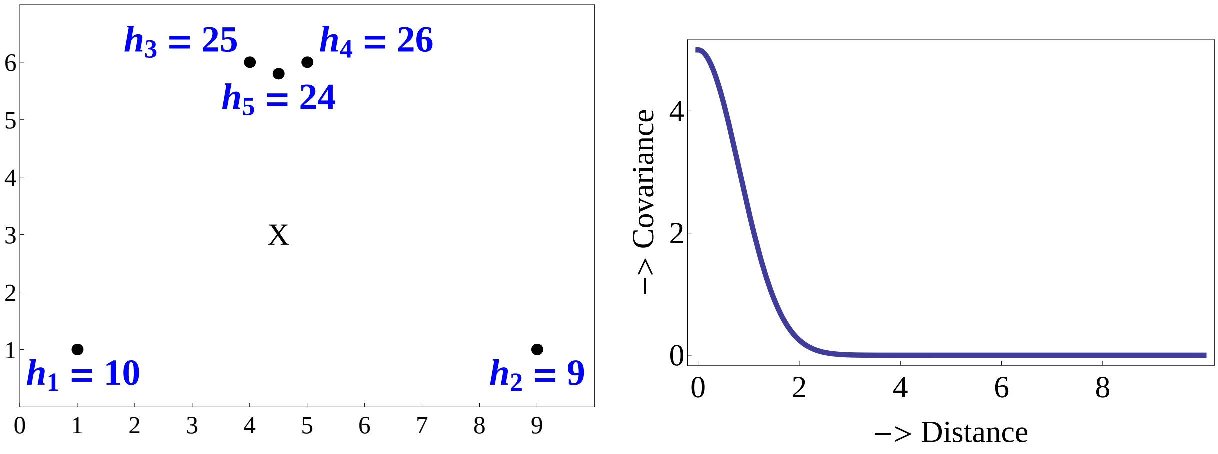

Figure 3: **Left**. Five height observations: $h_1 = 10$, $h_2 = 9$, $h_3 = 25$, $h_4 = 26$ en $h_5 = 24$. **Right.** Covariance function $f(x) = 5e^{-\frac{3}{4}x^2}$.

The correlation between the observations in the left figure is described by the covariance function on the right.

37. What are the range and sill of the covariance function $f(x)$ in Figure 3?
38. Which observations in Figure 3 are correlated according to covariance function $f(x)$?
39. How should the number $\frac{3}{4}$ in $f(x)$ be changed to remove all correlation between the five observations in $f(x)$?
40. What is the (approximate) Kriging-the-mean mean of the five observations?

Lina wants to use Ordinary Kriging (OK) for interpolation at the cross in Figure 3.

41. Give the OK redundancy matrix (with approximate entries) for an interpolation at the cross.
42. Does the OK redundancy matrix change when the location of the cross is changed?
43. Give the OK proximity vector (with approximate entries) for an interpolation at the cross.
44. What are the (approximate) weights that each observation gets for an OK interpolation at the cross? What condition does the sum of the weights fulfill?
45. Where is the OK variance minimal and where maximal when the cross is moved around in Figure 3, left?