Exam - AESB2440, Geostatistics & Remote Sensing

2C-Zaal 2, August 12, 2015, 14.00 - 17.00

This exam consists of 45 questions

You start with 10 points. Every correct answer is worth 2 points.

Always clarify your answer.

It is allowed to use a simple calculator without memory.

Good luck!

Probabilities and events

A coin is tossed four times. Recall that a single coin toss results in head or tails with equal probability.

1. Determine the sample space Ω of tossing the coin four times.

Describe the following events A, B and D in terms of the outcomes ω_i of Ω and give the probabilities of each event.

2. A = { head is thrown exactly twice }.
3. B = { head is thrown at least twice }.
4. D = { the first throw results in a head }.
5. What is the complement C of D?
6. Determine $P(A \cap B^C)$.

Rainfall

The average rainfall per month in Delft is 9.22 cm. Assume the monthly rainfall X is normally distributed with a standard deviation of 2.83 cm.

7. Explain how a normal distribution with mean $\mu = 9.22$ and standard deviation $\sigma = 2.83$ is transformed to the standard normal distribution.
8. What is the probability that Delft receives not over 7cm of rain next month? Express your answer as an evaluation of the standard normal distribution.
9. What is the variance and standard deviation of the random variable $12X$?
10. What is the probability that Delft receives at least 1m of rain over the next year? Again, give your answer as an evaluation of the standard normal distribution.

Uniform distribution

The random variable X has a uniform distribution on the interval $(-3, 1)$, that is $X \sim U(-3, 1)$.

11. Sketch the probability density function and cumulative distribution function of X in one plot.
12. What is the mean of the random variable $Y = 2X - 1$?
13. What is the mean of the random variable $Z = X^2 + 3$?

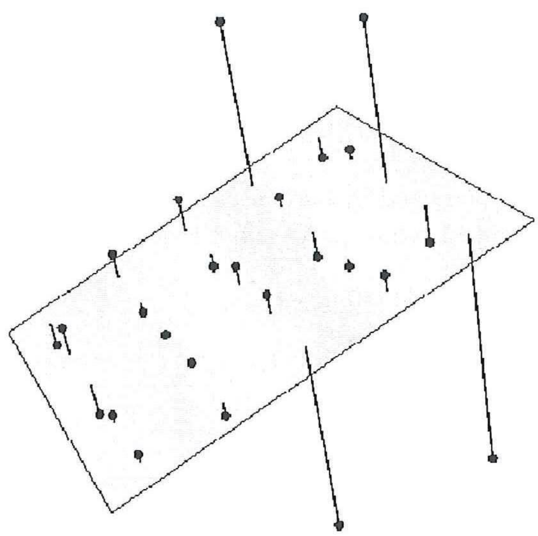

Figure 1: Plane fitted to a set of 25 observations. Four of the observation are outliers.

Ransac for plane fitting

Consider the observations in Figure 1. There are 4 outliers and 21 correct observations (inliers). Ransac is a method that identifies outliers by evaluating many runs.

14. What is the minimum number of points needed in $I\!R^3$ to fix a plane?
15. What is the probability that all points are inliers if you randomly select this minimum number of points in Figure 1?
16. Explain how one run of RANSAC would work in the case of Figure 1.
17. Assume that 100 runs are evaluated for the case of Figure 1. Which run would be considered most successful?
18. Would such a run be unique? Why (not)?

Least Squares

time in minutes	1'	2'	4'	5'
distance in km.	2	3	7	11

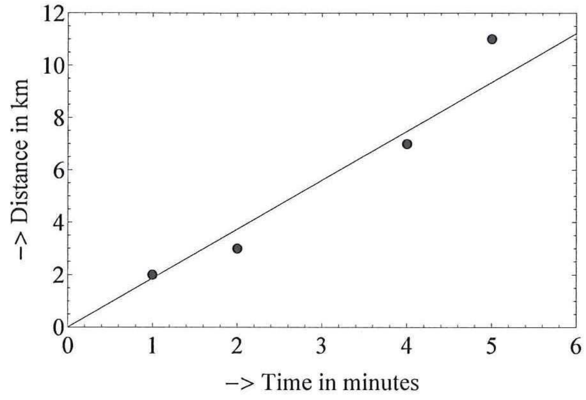

Figure 2: Measured distance in km.

Hilde measured the distance covered by her car at four instances as indicated in Figure 2. She wants to detemine the average driving speed using least squares.

19. How many parameters should Hilde estimate?
20. What is the vector of observations $\mathbf{y}$?
21. Give the model matrix A.
22. Determine the least squares solution $\hat{\mathbf{x}}$.
23. Give the vector of adjusted observations $\hat{\mathbf{y}}$.
24. What do the adjusted observations represent?
25. Give the vector of residuals $\hat{\mathbf{e}}$
26. In which direction are the errors in the observations minimized?

Hildes car made claims that after 2' allready 4 km was covered. Hilde doesn't agree but decides to add $(2', 4km)$ as an additional measurement to the table in Figure 2.

27. How should the linear model of observations be adapted to incorporate this additional measurement?
28. Determine the change in the least squares average driving speed as a consequence of adding this additional measurement.

The standard deviation of the distance measurements is $\sigma = .5km$.

29. Explain how the Monte Carlo method can be used to get insight in the quality of the least squares estimation of the average driving speed.

Deterministic Interpolation

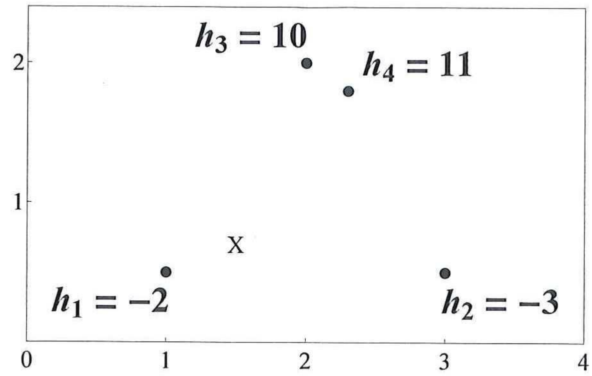

Figure 3: Four height observations: $h_1 = -2$, $h_2 = -3$, $h_3 = 10$, $h_4 = 11$.

Figure 3 shows four height observations. Emilie wants to estimate the height at the cross.

30. Give two different ways to triangulate the four locations in Figure 3.
31. Which of the two ways is the Delaunay triangulation? Why?

Weights	w_1	w_2	w_3	w_4	height
1. Nearest Neighbor					
2. Triangular					
3. Inverse Distance, power = 2					
4. Inverse Distance, power = 19					

Table 1: Table of weights

Enter your answers on the following four questions in (a copy of) Table 1. It is not necessary to use formulas, but explain your answers. Your answers may be approximate.

32. Indicate in the table which weight each observation gets in case of interpolation according to *Nearest neighbor* interpolation. Also give a corresponding estimate of the height.
33. Same question for *Triangular interpolation.*
34. Also give weights and height estimate for an *Inverse distance interpolation* with power 2.
35. Finally, give weights and height estimate for an *Inverse distance interpolation* with power 19.
36. Imagine each of the four methods in Table 1 is applied to generate an interpolated map at a dense grid, based on the observations in Figure 3. Order the resulting maps according to their smoothness.

Stochastic Interpolation

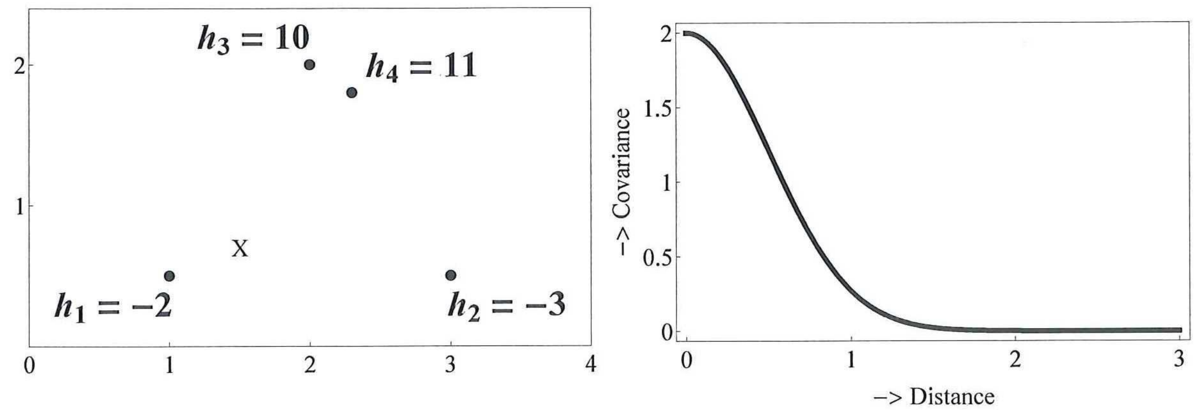

Figure 4: **Left.** Four height observations: $h_1 = -2$, $h_2 = -3$, $h_3 = 10$ and $h_4 = 11$. **Right.** Covariance function $f(x) = 2e^{-2x^2}$.

The correlation between the observations in the left figure is described by the covariance function on the right.

37. What are the range and sill of the covariance function $f(x)$ in Figure 4?
38. Which observations in Figure 4 are uncorrelated according to covariance function $f(x)$?
39. How should the number -2 in $f(x)$ be changed to ensure that the four observations in Figure 4 are strongly correlated?
40. What is the (approximate) Kriging-the-mean mean of the four observations?

Arthur wants to use Ordinary Kriging (OK) for interpolation at the cross in Figure 4.

41. Give the OK redundancy matrix (with approximate entries) for an interpolation at the cross.
42. Give the OK proximity vector (with approximate entries) for an interpolation at the cross.
43. Does the OK proximity vector change when the location of the cross is changed?
44. What are the (approximate) weights that each observation gets for an OK interpolation at the cross? What condition does the sum of the weights fulfill?
45. What determines the OK variance when the intepolation location is moved around in Figure 4, left?