

# AESB2440: Geostatistics & Remote Sensing

## Lecture 9: Geometric Terrain Analysis

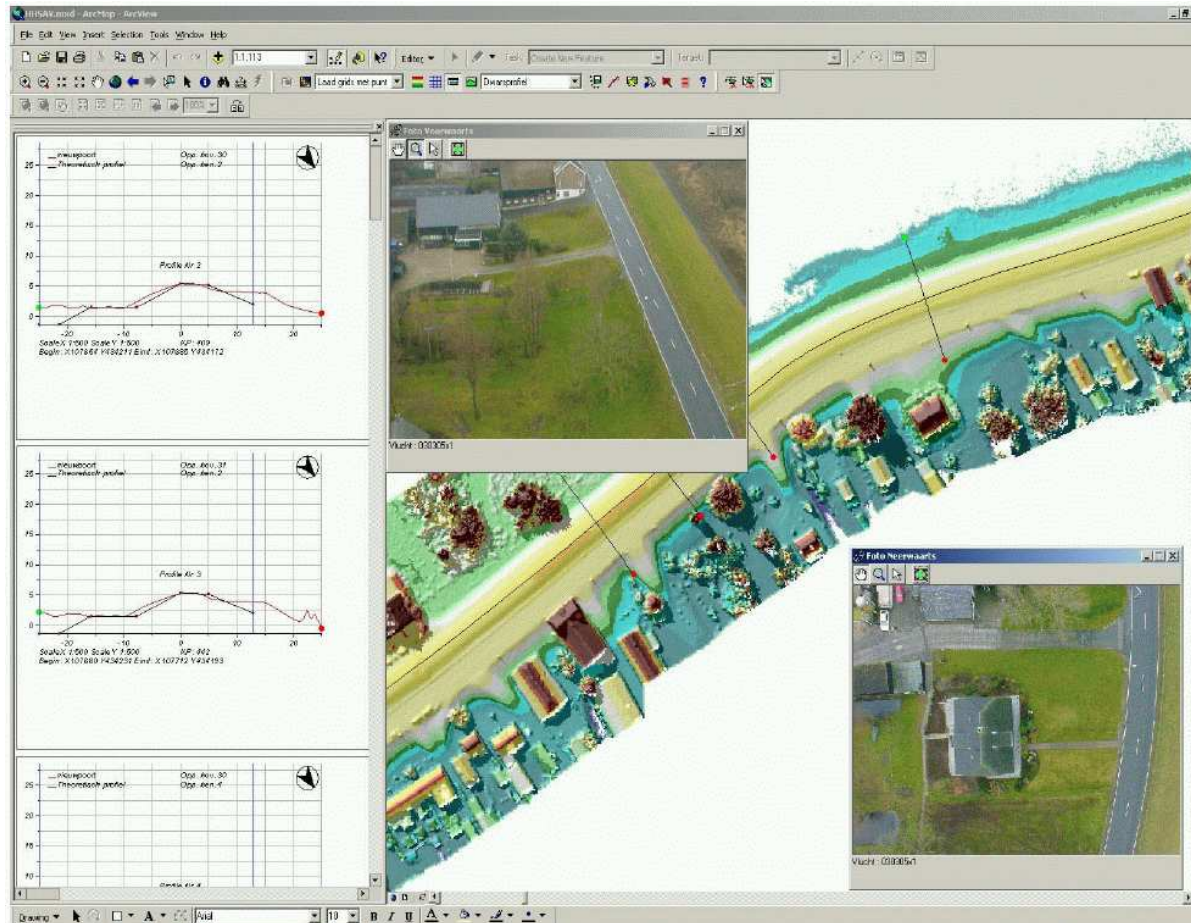
May 11, 2015

Roderik Lindenbergh

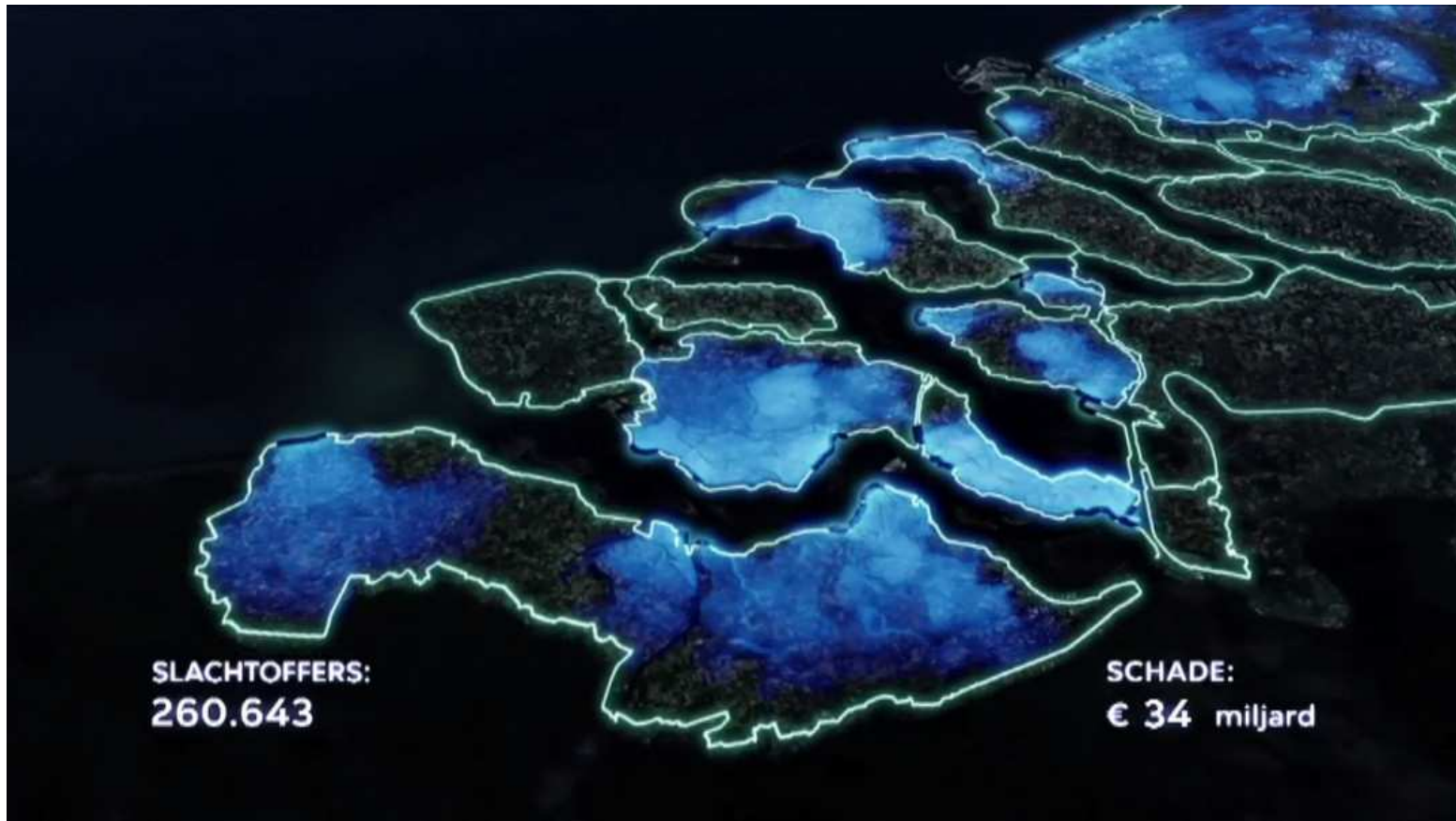
1

# Why geometric terrain analysis?

## Dike Inspection



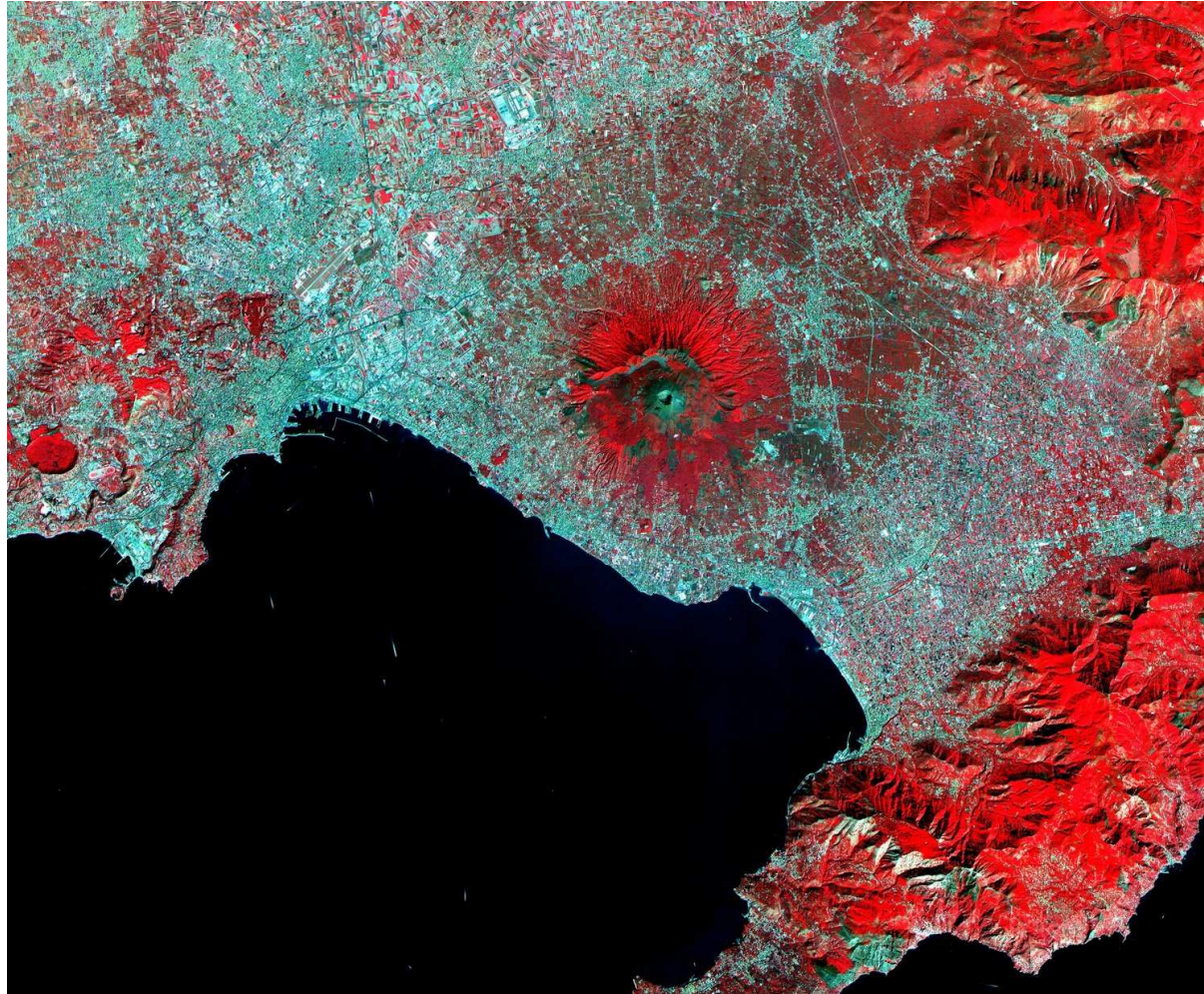
# Worst possible flooding



Source [http://www.vpro.nl/metropolis/speel.WO\\_VPRO\\_032787.html](http://www.vpro.nl/metropolis/speel.WO_VPRO_032787.html)



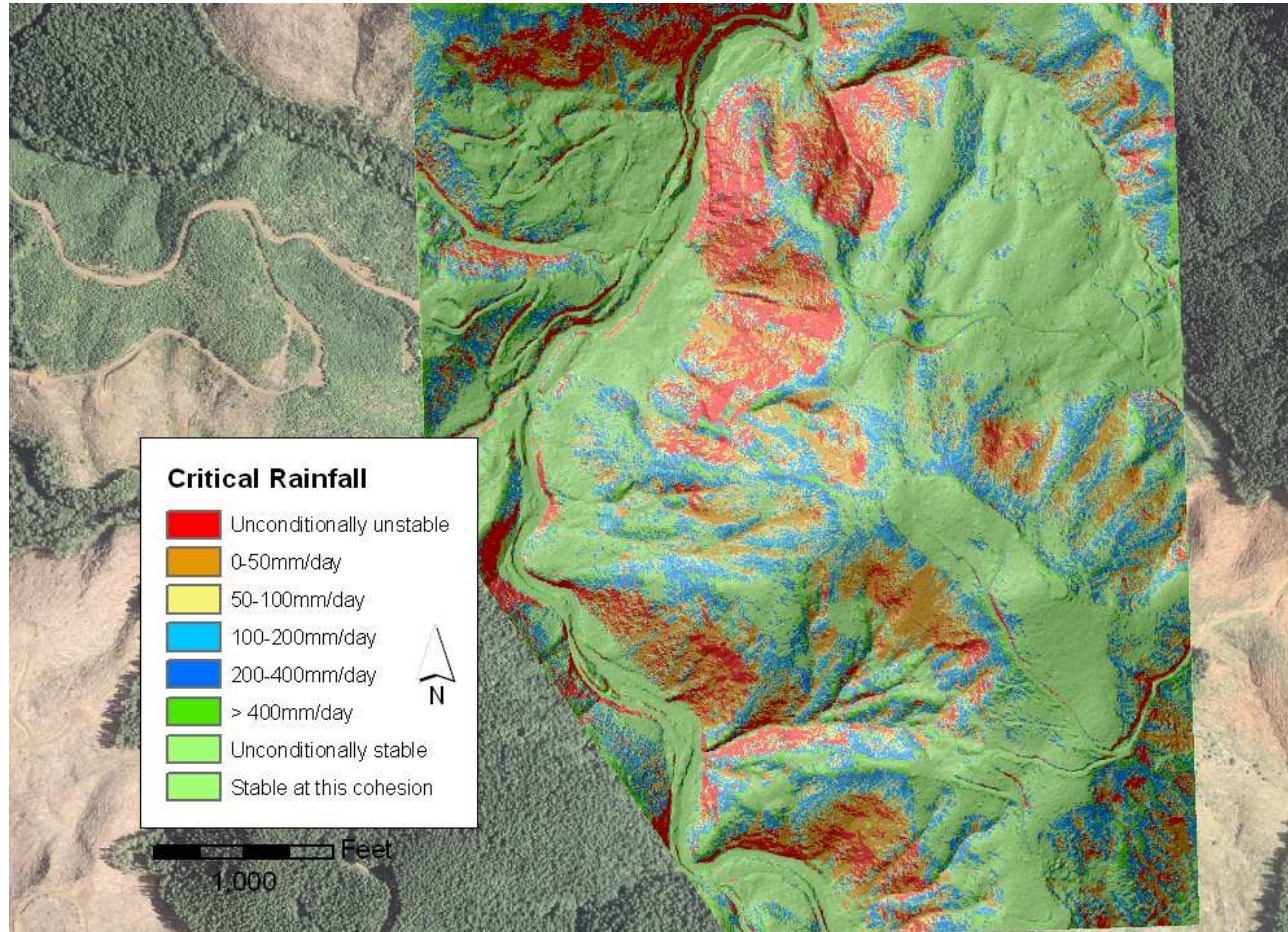
# Lava flow



Source [http://earthobservatory.nasa.gov/IOTD/view.php?id=1045&eocn=image&eoci=related\\_image](http://earthobservatory.nasa.gov/IOTD/view.php?id=1045&eocn=image&eoci=related_image)



# Slope stability



Source <http://gis.ess.washington.edu/areas/littlemountain/>

# Contents

## Terrain analysis

- Slope, aspect and orientation
- Relation with gradient
- Estimations from DEM data

## Drainage Networks

- D8 method
- Flow accumulation

# C. Slope, aspect and orientation

# Discrete Differential Geometry

How to obtain and use discrete equivalents of the notions of

**Gradient** - 'First derivative'

**Curvature** - 'Second derivative'

We consider three different representations of  $2\frac{1}{2}$  D surfaces:

1. Analytical function
2. **Regular grid/Raster** approximation
3. TIN approximation

[Technical Reference:

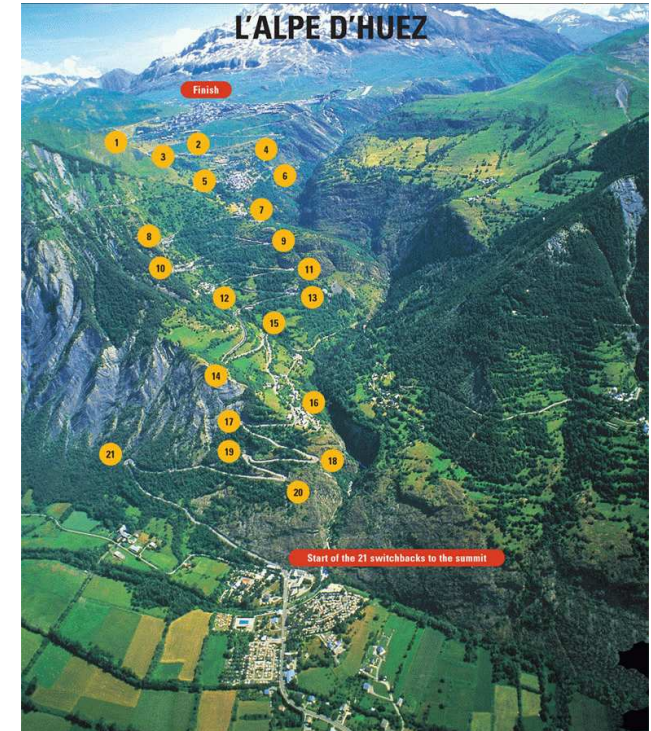
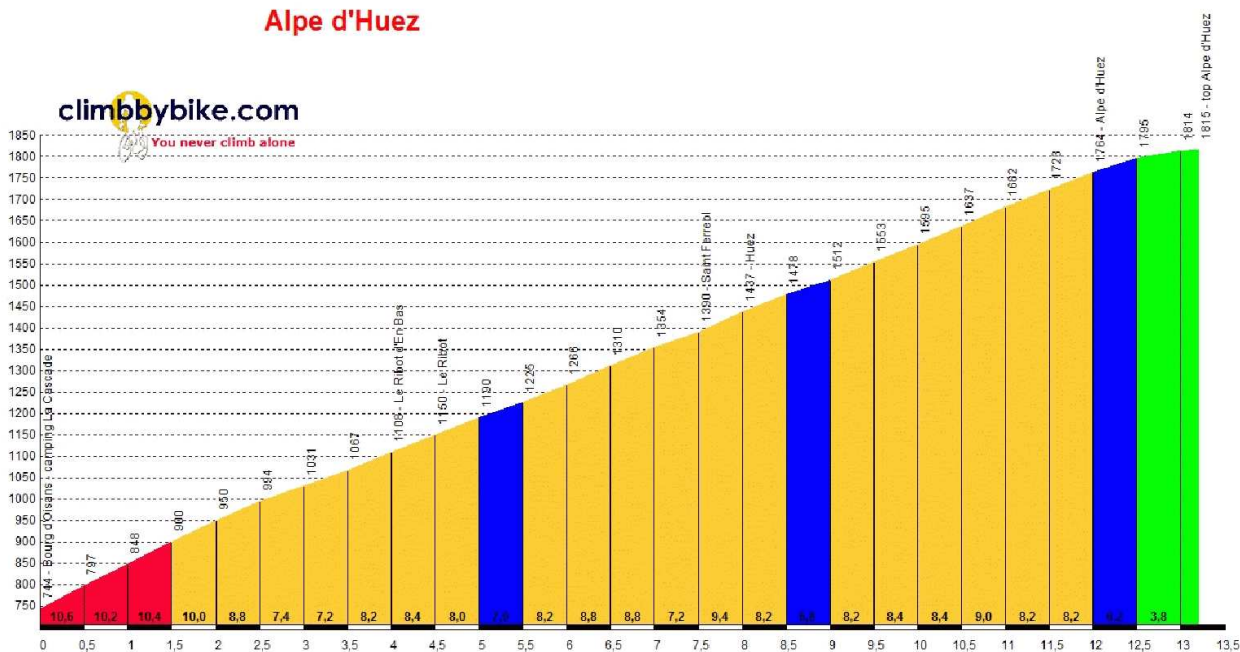
On the angular defect of triangulations and the pointwise approximation of curvatures

V. Borrelli, F. Cazals and J.-M. Morvan *Computer Aided Geometric Design*, 20(6), 2003, pp 319-341]

**Question:** what does 'discrete' means here?



# Slope - in 'real life'

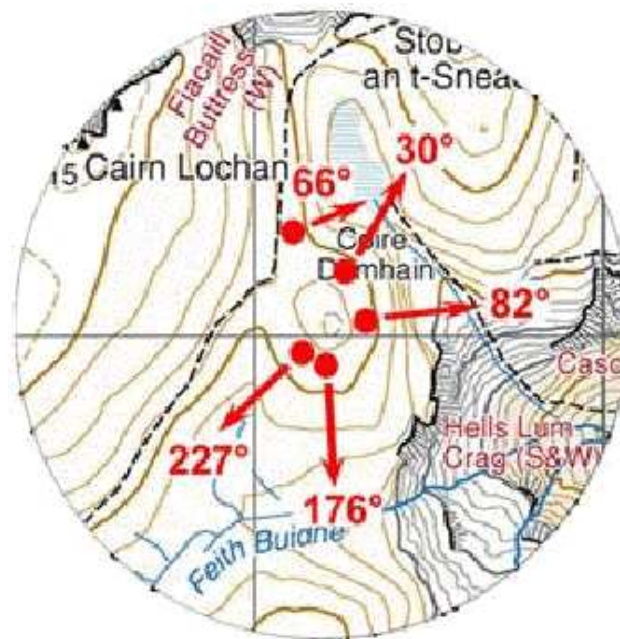


1D Slope - steepness of terrain at a point in a certain direction (e.g. in the direction of the road)

2D Slope - maximal steepness of terrain at a point

# Aspect - in 'real life'

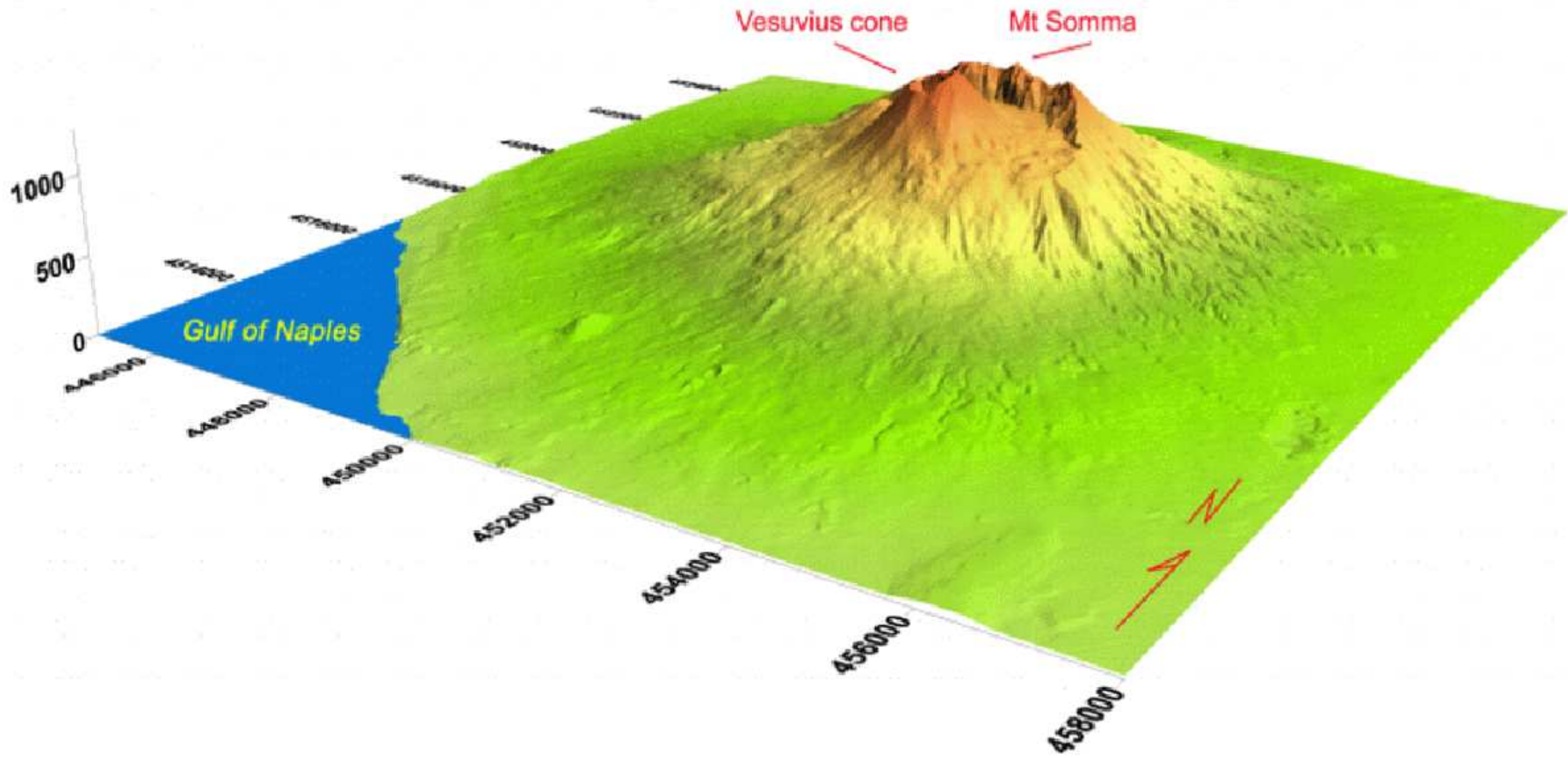
Aspect - direction of steepest slope



[Source: <http://ben-nevis.com/navigation/slope-aspect.php>]

**Question.** Relation with strike and dip?

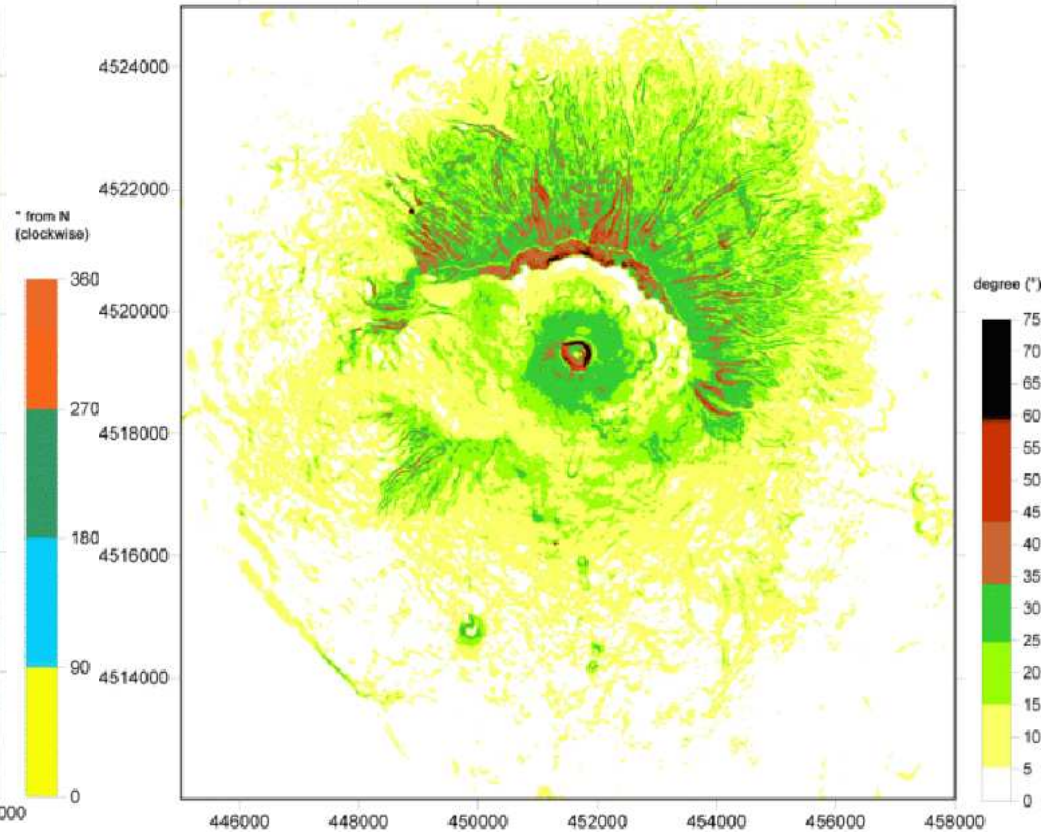
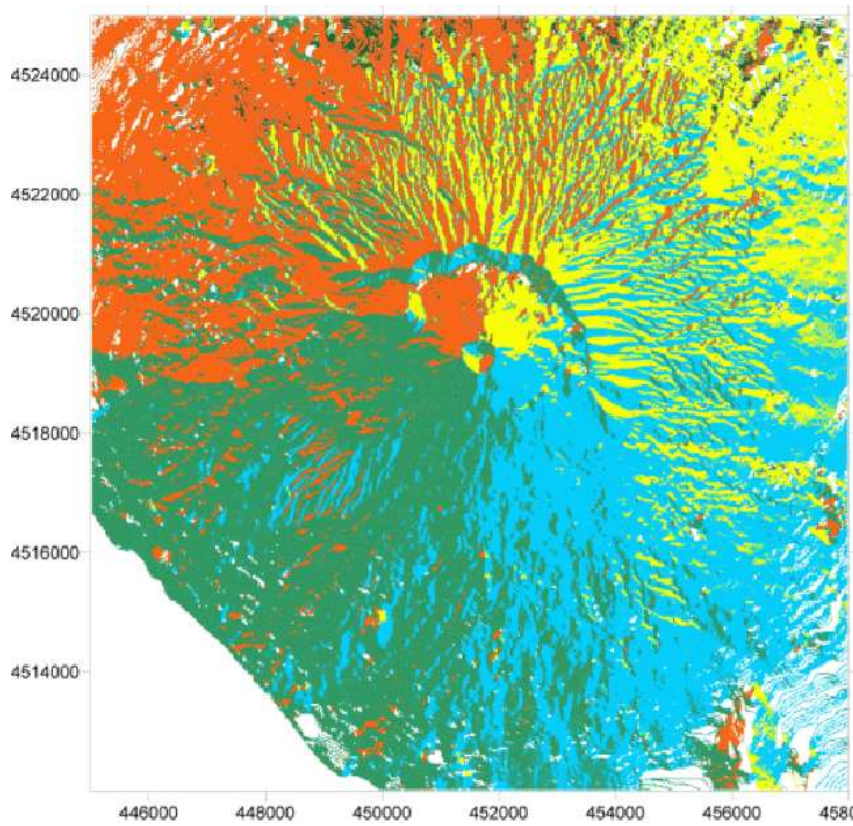
# Vesuv in 3D





# Vesuv, aspect

# slope





# Gradient and aspect

Let

$$\begin{aligned} f: \mathbb{R}^2 &\rightarrow \mathbb{R} \\ (x, y) &\mapsto f(x, y, z) \end{aligned}$$

the **gradient** of  $f$  at  $p$  is the vector

$$\nabla(f)(p) = \left( \frac{\partial f}{\partial x}(p), \frac{\partial f}{\partial y}(p) \right)$$

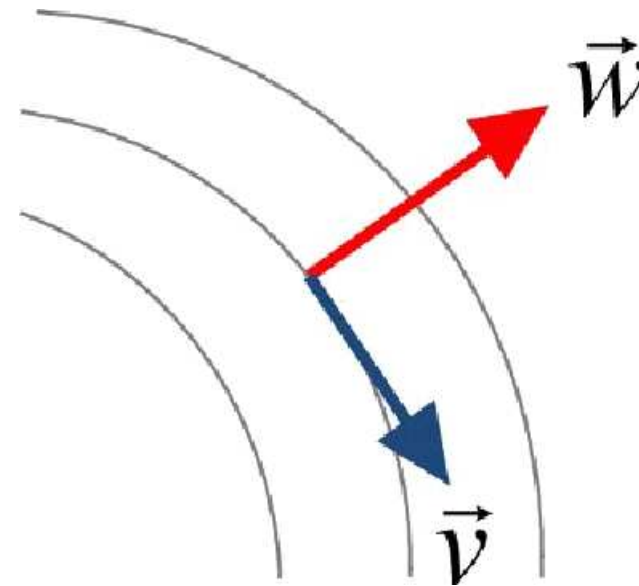
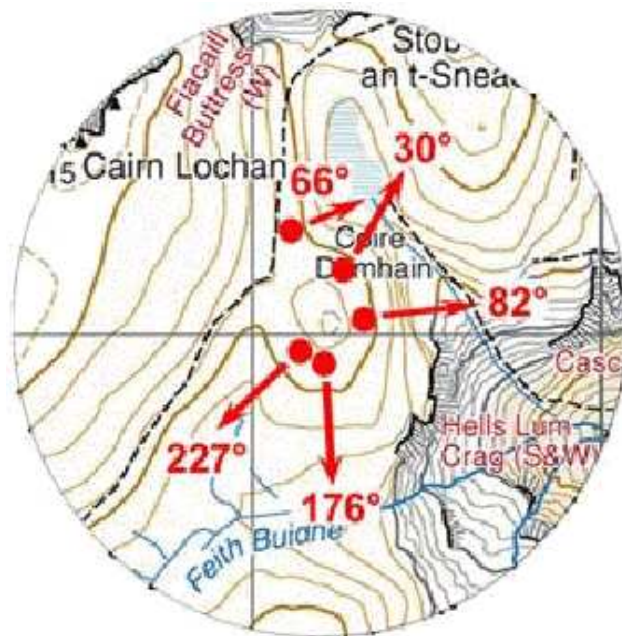
The gradient (vector) points in the direction of maximal change.

This direction between  $0^\circ$  and  $360^\circ$  is the **aspect**

The **slope** is the magnitude of maximal change, which can be quantified as the length  $\|\nabla f\|$  of the gradient vector.

# Contour lines

Isoline/Contourline: curves of constant elevation



**Theorem** The gradient vector is always perpendicular to the contour line

# Proof of Theorem

Let  $\nabla f(x, y)$  be the gradient at a point of the isoline  $f(x, y) = c$ .

Assume the isoline curve  $f_c$  can be smoothly parameterized by arc length as

$$x = x(s), \quad y = y(s)$$

The unit tangent vector to  $f_c$  at  $s$  is

$$T(s) = \left( \frac{dx}{ds} \right) \mathbf{i} + \left( \frac{dy}{ds} \right) \mathbf{j}$$

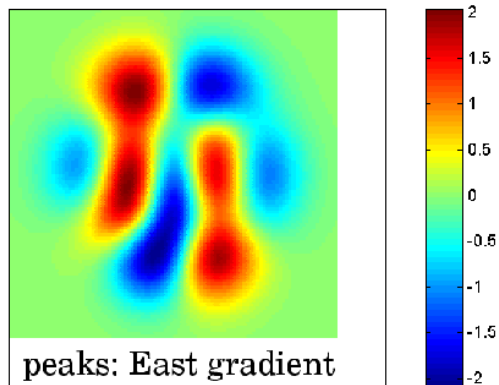
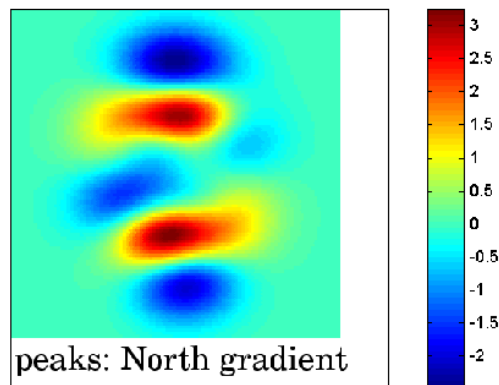
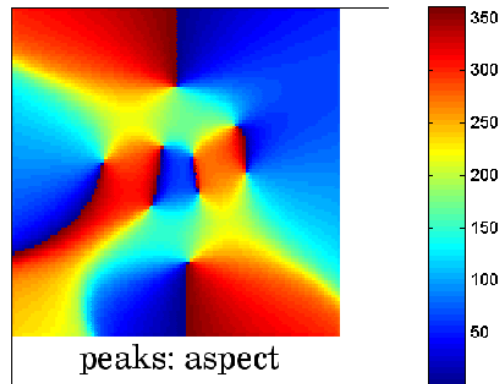
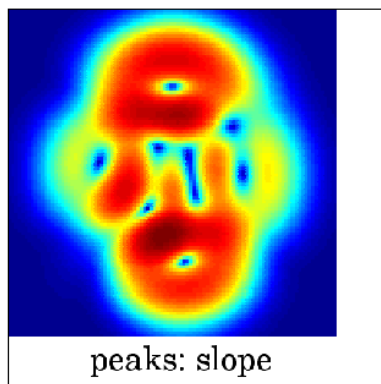
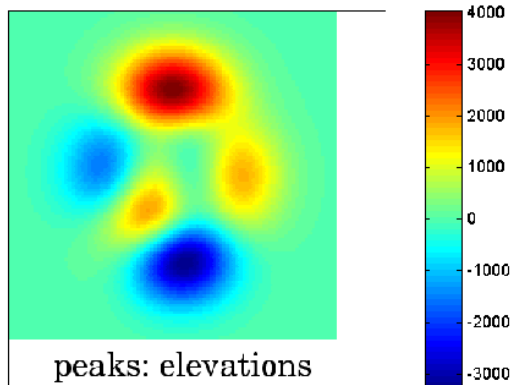
Differentiate the equation  $f(x, y) = c$  w.r.t.  $s$  using the chain rule:

$$\frac{\partial f}{\partial x} \frac{dx}{ds} + \frac{\partial f}{\partial y} \frac{dy}{ds} = 0$$

or

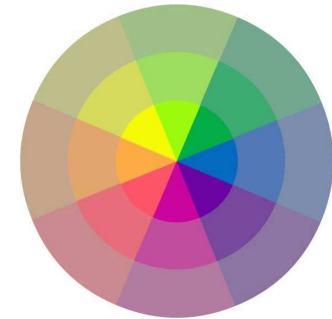
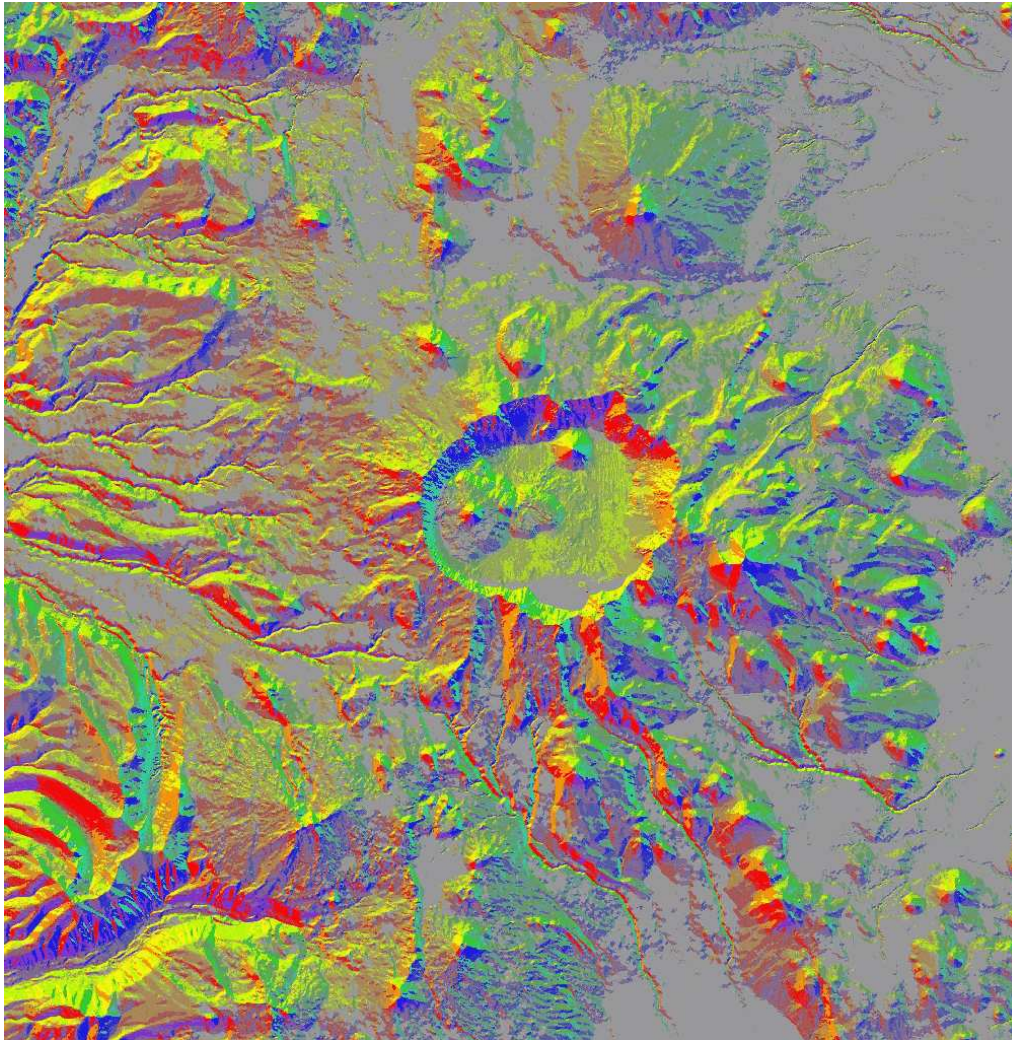
$$\left( \frac{\partial f}{\partial x} \mathbf{i} + \frac{\partial f}{\partial y} \mathbf{j} \right) \cdot \left( \frac{dx}{ds} \mathbf{i} + \frac{dy}{ds} \mathbf{j} \right) = \nabla f(x, y) \cdot T(s) = 0$$

[Proof from Calculus, H. Anton]<sub>15</sub>





# Aspect and Slope together



Source: <http://blogs.esri.com/Support/blogs/mappingcenter/archive/2008/05/22/aspect-slope-map.aspx>

# Gradient image

From elevation raster to gradient raster

range image       $d(\text{column}, \text{row})$   
gradient image     $g = \nabla d = (d_c, d_r)$

## Example

1	2	3	3	1
1	3	3	2	1
0	3	4	3	2
0	3	5	4	3

Range image

1	1	0	-2
2	0	-1	-1
3	1	-1	-1
3	2	-1	-1

Row gradient image

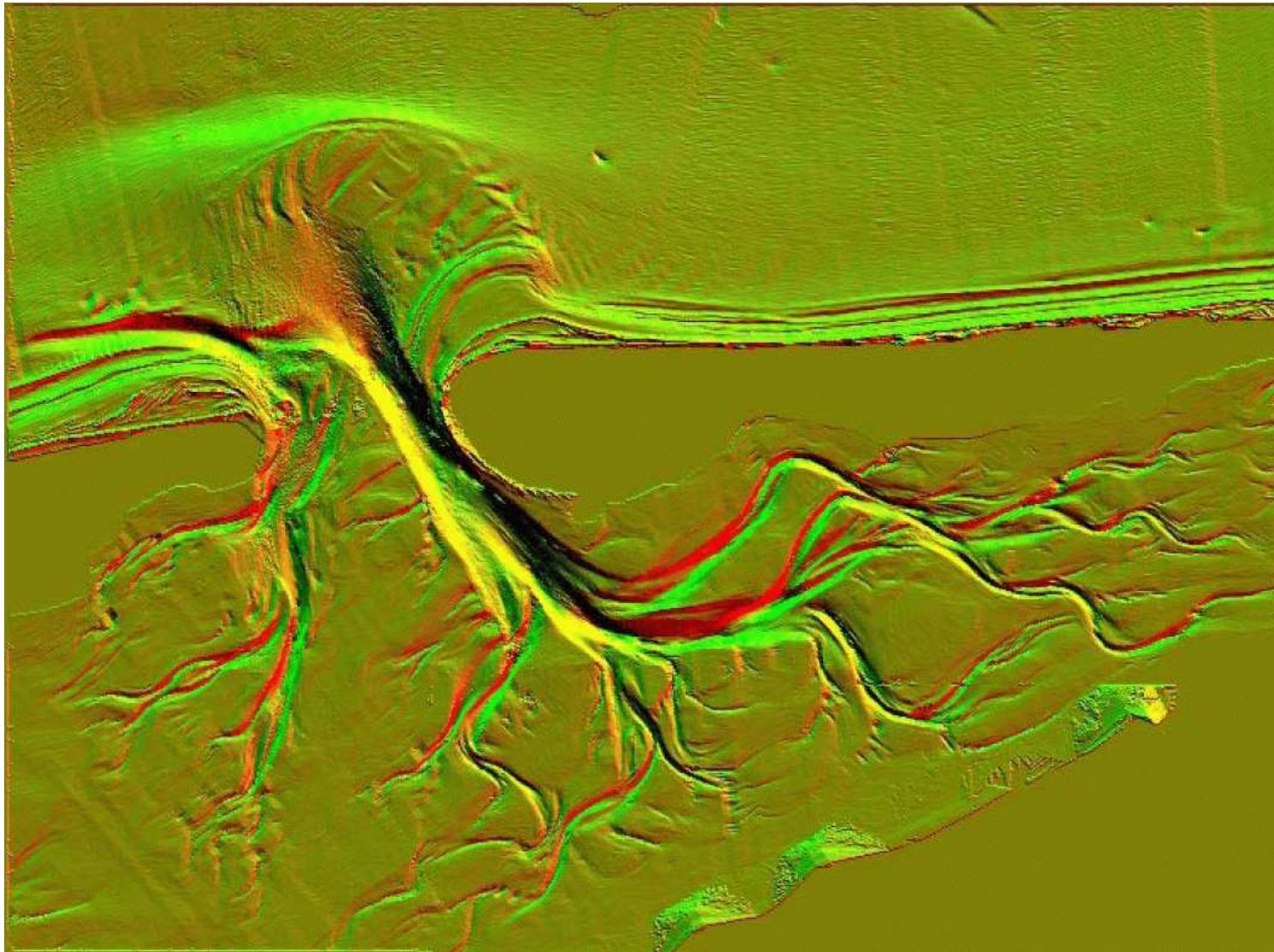
0	-1	0	-1	0
-1	0	-1	1	1
0	0	-1	1	1

Column gradient image

**Remark:** Note that here raster and image are equivalent notions

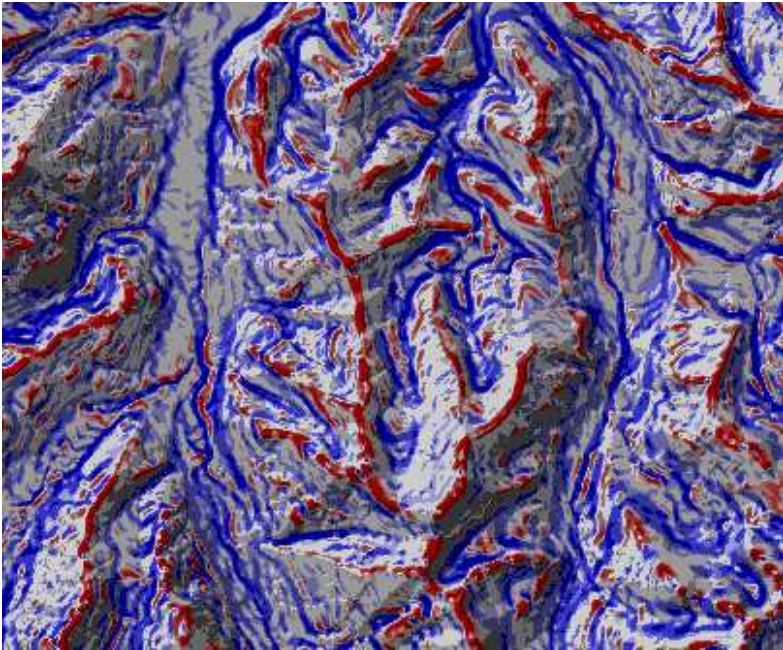


# Waddenzee: colored by x- and y-slope

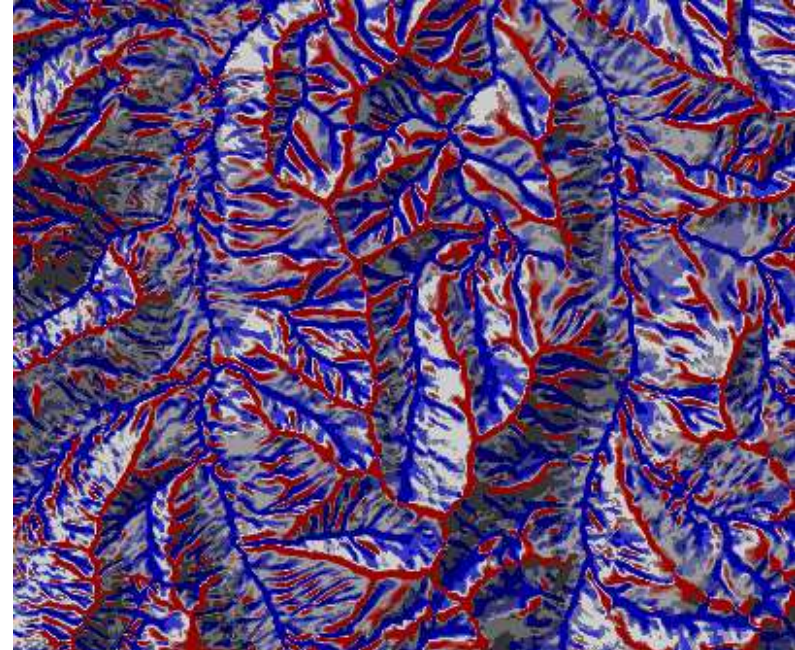




# Curvature: $\approx$ 2nd derivative



‘Profile Curvature’



‘Planar Curvature’

[Source: [http://www-vv.slu.se/fs/tatry/fin\\_rapp/FR\\_22.htm](http://www-vv.slu.se/fs/tatry/fin_rapp/FR_22.htm)]

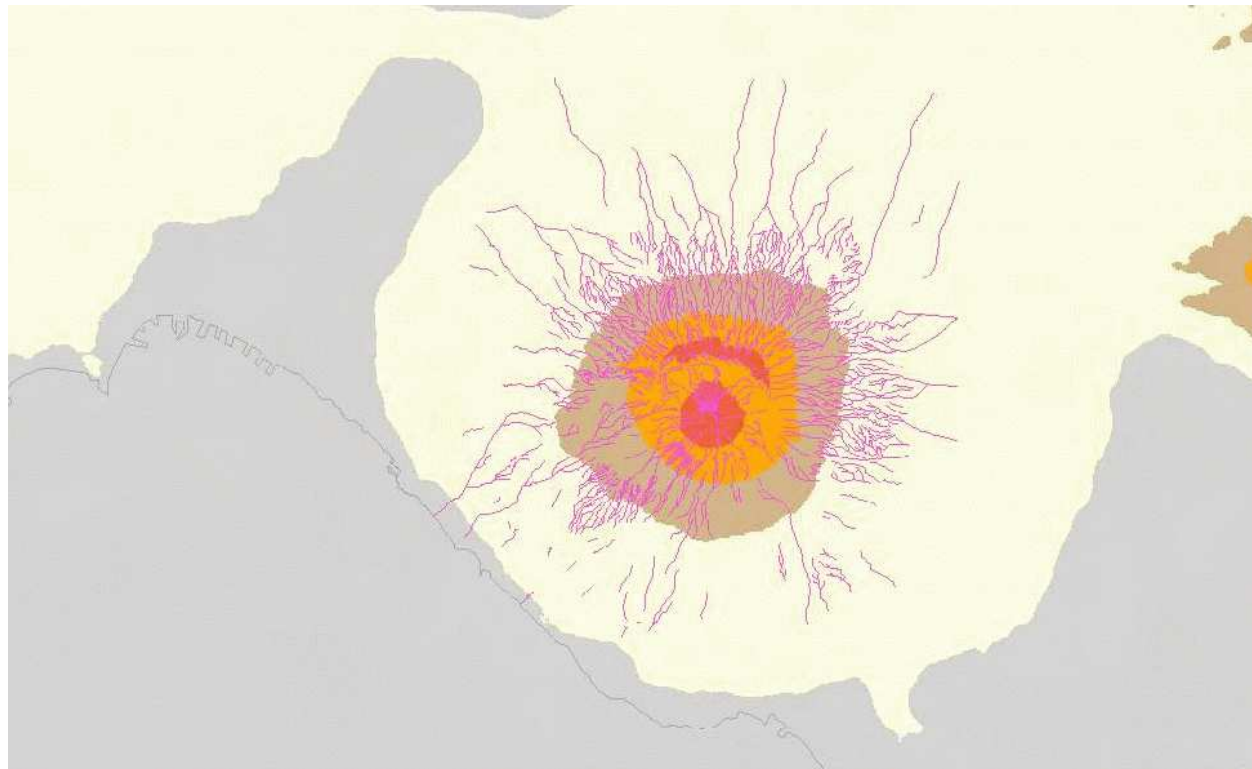
Profile curvature  $\sim$  ‘Shape of the relief in the direction of the slope’

Planar curvature  $\sim$  ‘Shape of the relief perpendicular to the slope’



# D. Drainage Networks

# Vesuv again



Source [http://ipf.ov.ingv.it:81/pmapper/map.phtml?config=siscam&dg=reg\\_sud,SR\\_Cam\\_45](http://ipf.ov.ingv.it:81/pmapper/map.phtml?config=siscam&dg=reg_sud,SR_Cam_45)

# Contents

## A. Watersheds and Drainage Networks

- Definitions
- Examples

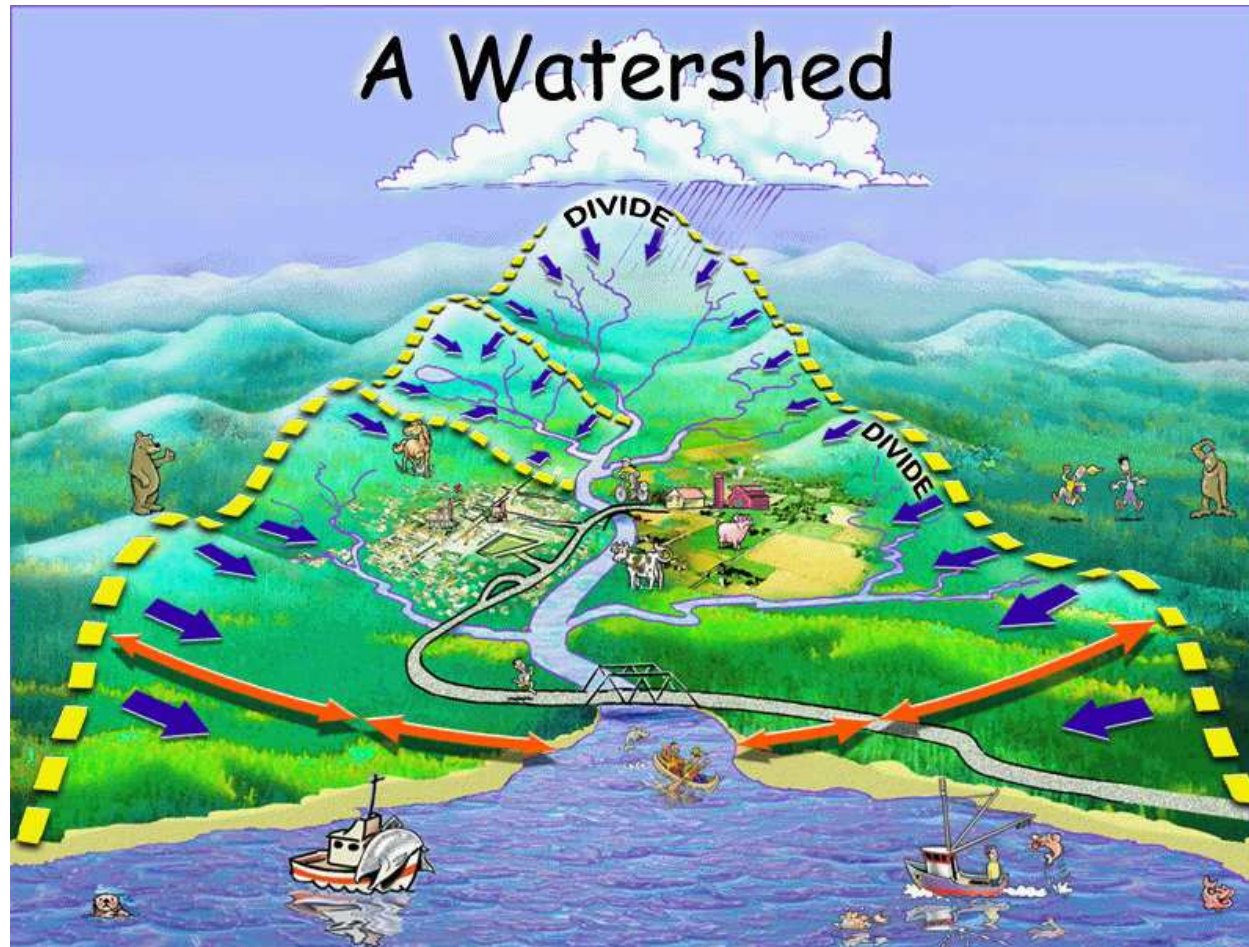
## B. D8 algorithm

- Drain direction
- Flow accumulation
- Pour-points
- Dealing with flat areas

## C. Hydrosheds

- Example of a drainage network product

# Watersheds



**Watershed:** area of land that captures water in a common water body, like river or lake



# River catchments

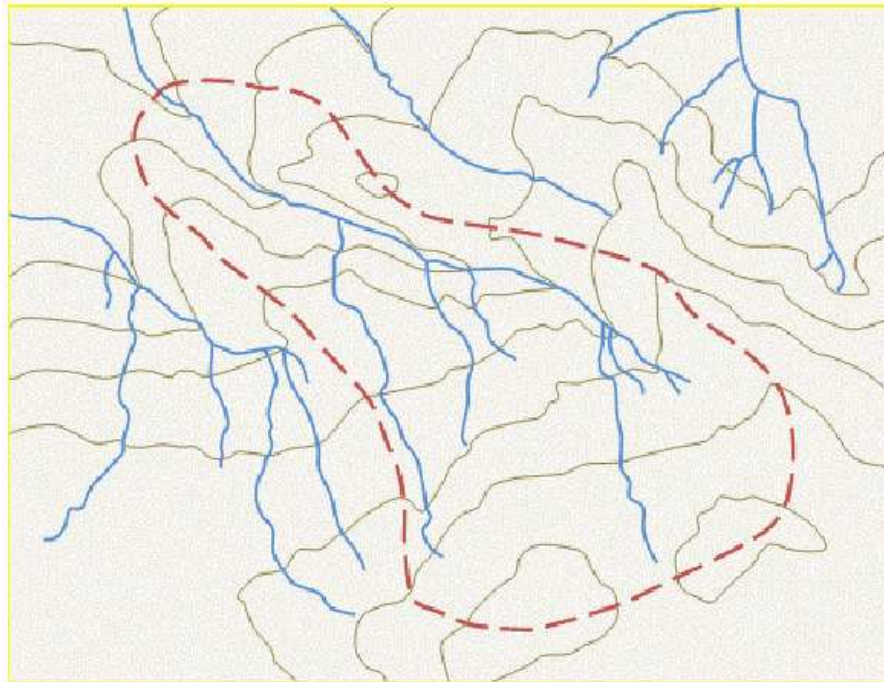
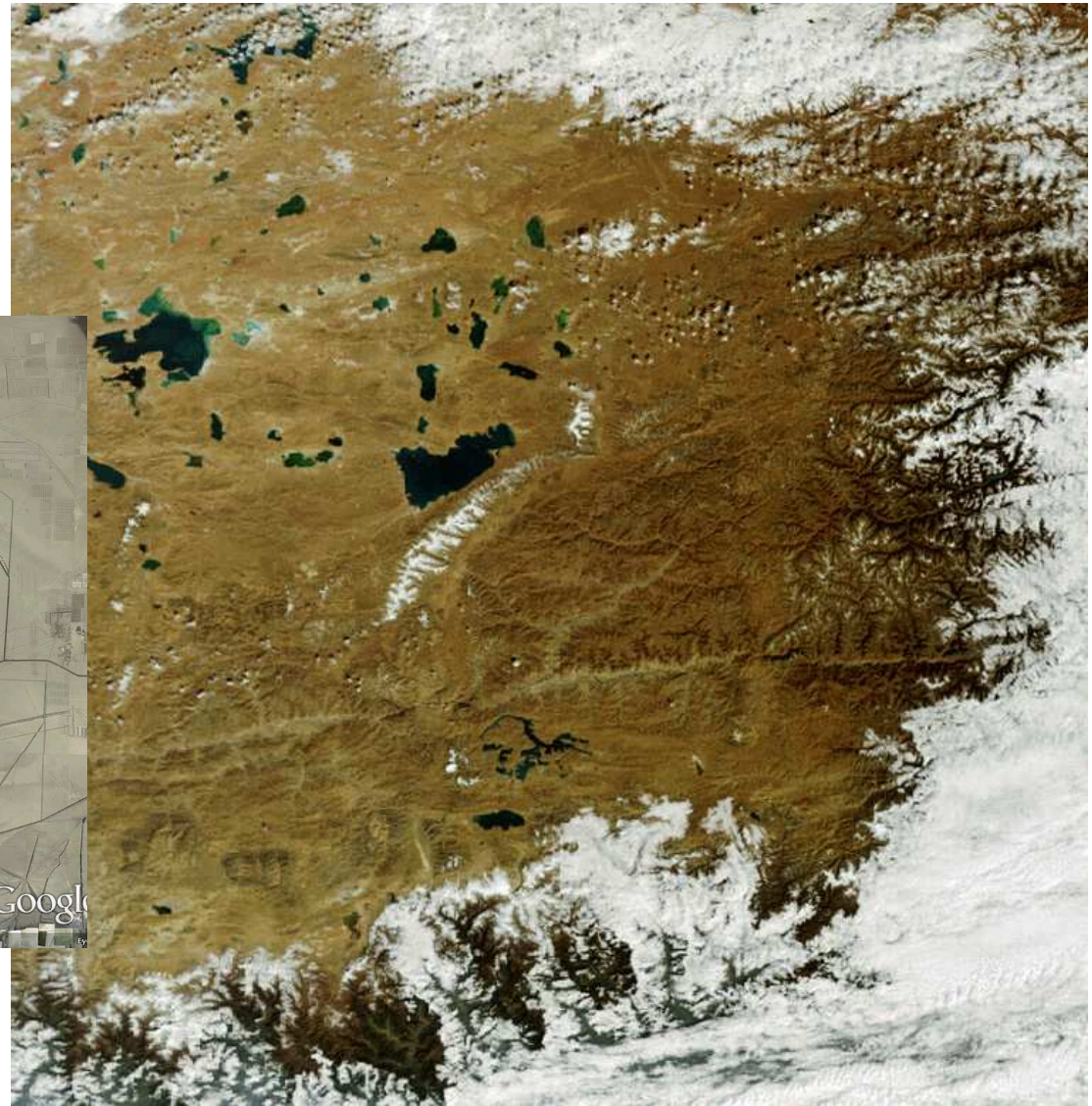


Figure 1.2 Example catchment boundary (— rivers, - - - catchment boundary, - - - contour lines)

Watersheds, catchments and basements are similar notions

# Endorheic lakes



**Question:** Why are endorheic lakes salty?

Source <http://www.eosnap.com/lakes/lake-namtso-north-of-the-nyainqentanglha-mountains-tibet/>

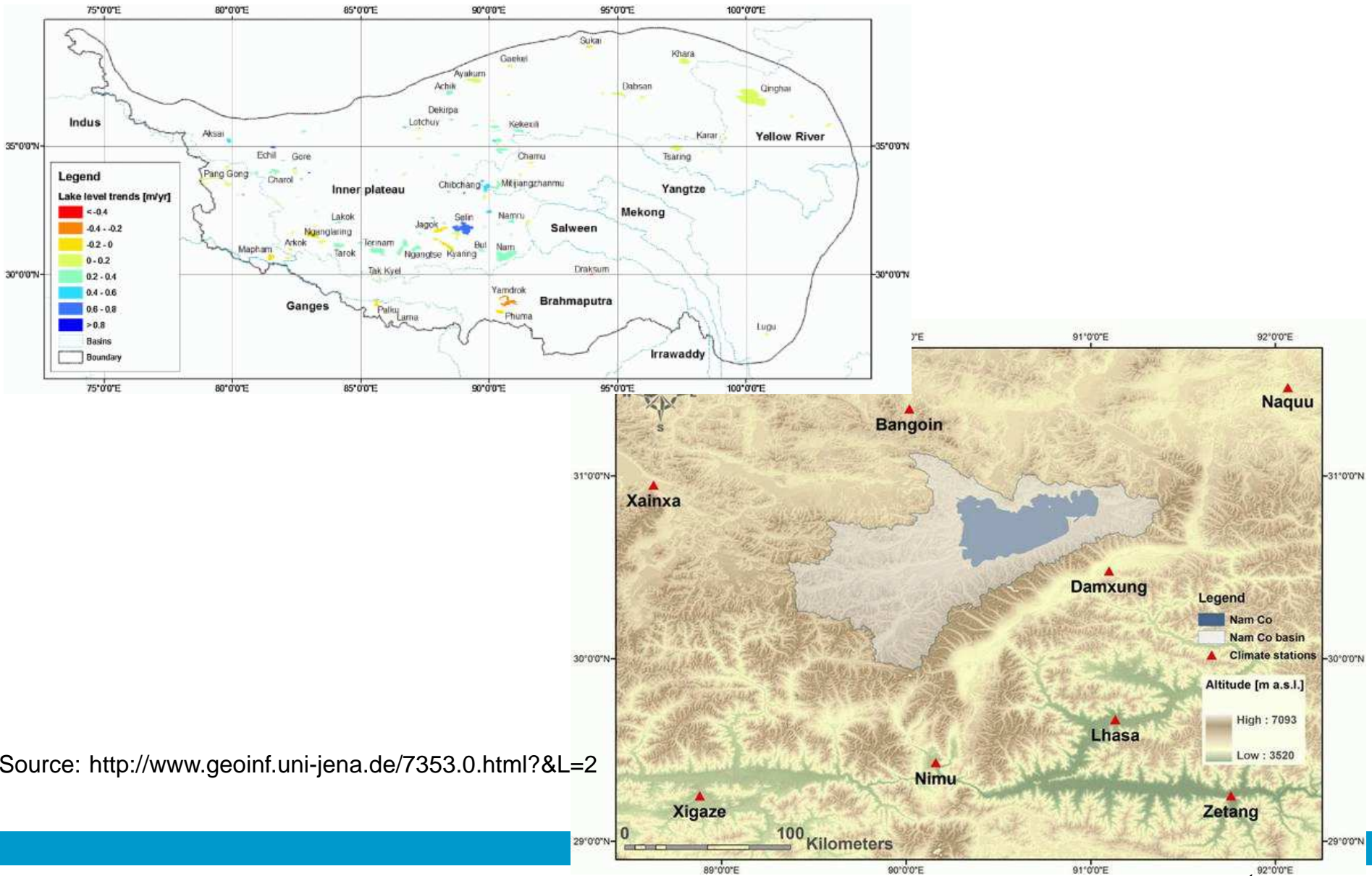
Source [http://meltdownintibet.com/e\\$\\_\\_\\$salt.htm](http://meltdownintibet.com/e$__$salt.htm)



# Tibet example

Source: ICESat derived elevation changes of Tibetan lakes between 2003 and 2009, V. Phan Hien et al.

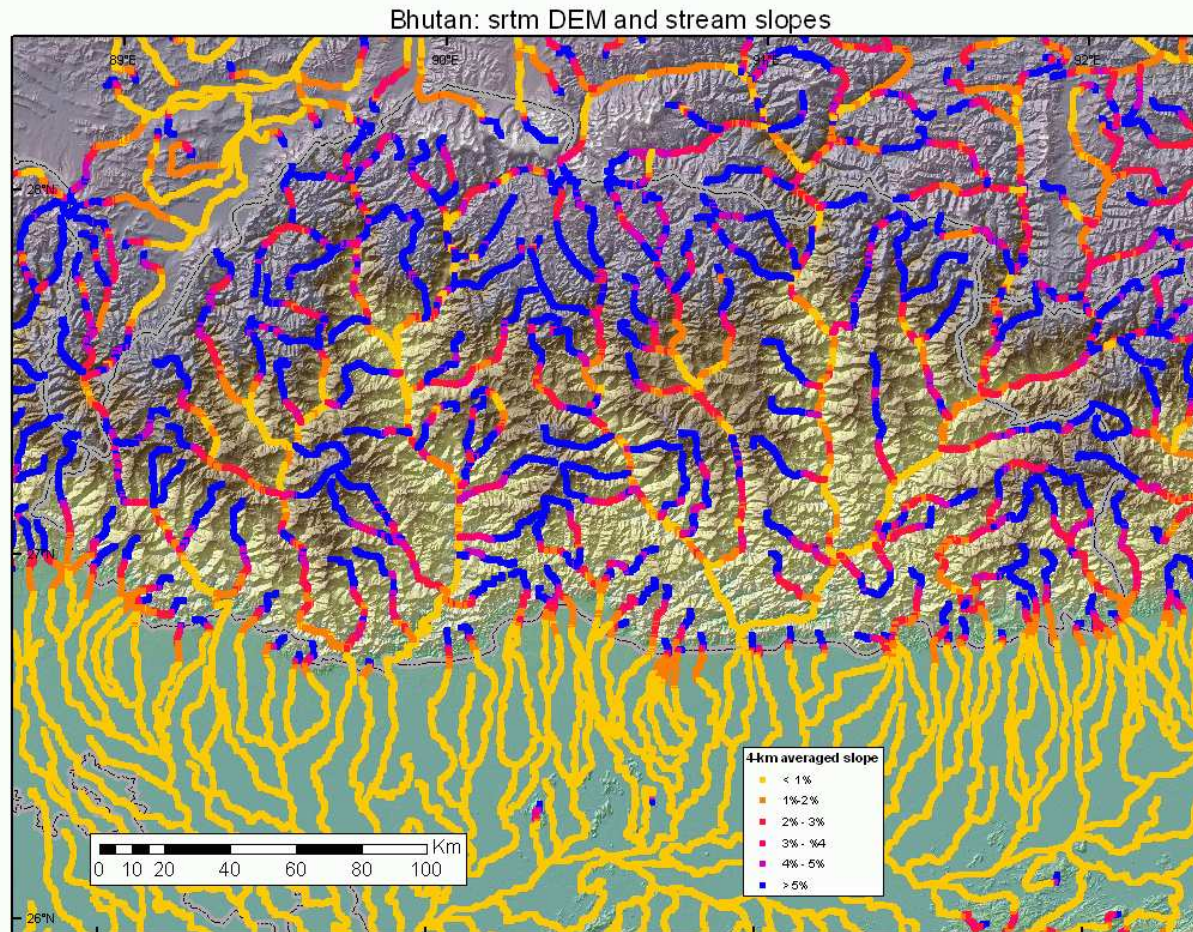
Int. Jnl. of Applied Earth Observation and Geoinformation, 17, pp. 12-22, 2012



Source: <http://www.geoinf.uni-jena.de/7353.0.html?&L=2>

# Bhutan Stream Slopes

Source: <http://gis.ess.washington.edu/areas/Bhutan/streamslopes/index.html>





# Geometrical wishes and applications

## Wishes:

1. Get catchment boundaries
2. Get stream locations

## Applications:

- Assessing flooding risks
- Irrigation
- Geological interests
- Assessing pollution risks
- Transport
- Water balance

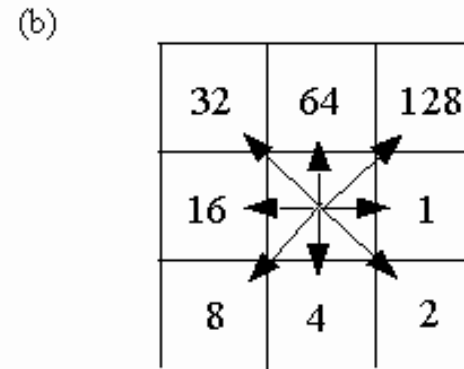
# D8 algorithm - Idea

Grid Elevations:

(a)

78	72	69	71	58	49
74	67	56	49	46	50
69	53	44	37	38	48
64	58	55	22	31	24
68	61	47	21	16	19
74	53	34	12	11	12

Direction Encoding:

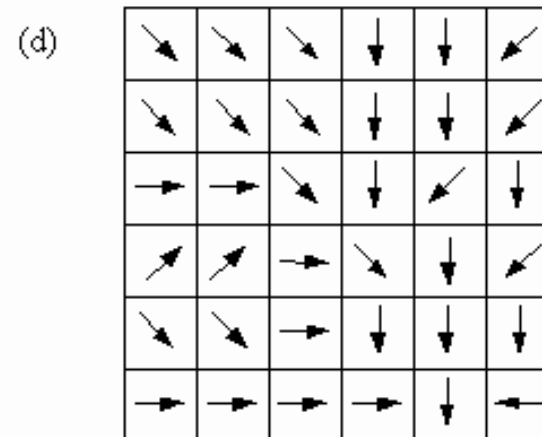


Encoded Grid Directions:

(c)

2	2	2	4	4	8
2	2	2	4	4	8
1	1	2	4	8	4
128	128	1	2	4	8
2	2	1	4	4	4
1	1	1	1	4	16

Visualized Grid Directions:



(Source: <http://www.nws.noaa.gov/oh/hrl/gis/data.html>)

# D8 algorithm - details

**Input:** DTM stored as regular grid

**Assumption:**

Water in each cell flows towards unique 8-neighboring cell, the cell in the direction of steepest descent

**Question:** are all cells on equal distance?

**Flow accumulation**, per cell:

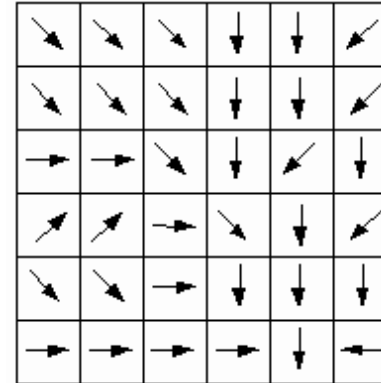
Sum of weights of all cells flowing to that cell.

# Flow accumulation - example

Elevations

78	72	69	71	58	49
74	67	56	49	46	50
69	53	44	37	38	48
64	58	55	22	31	24
68	61	47	21	16	19
74	53	34	12	11	12

Directions

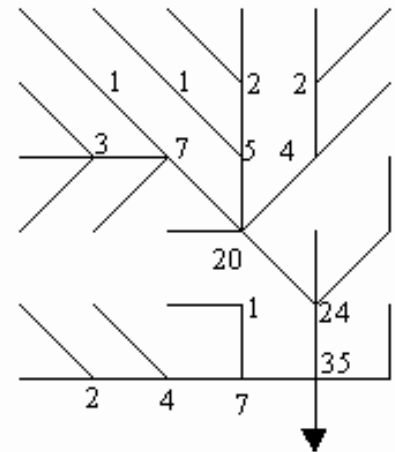


(a)

0	0	0	0	0	0
0	1	1	2	2	0
0	3	7	5	4	0
0	0	0	20	0	1
0	0	0	1	24	0
0	2	4	7	35	1

Accumulation

(b)



Drainage network

Example:  $35 = 7 + 24 + 1 + 3 \cdot 1$

(using equal weight for all cells)



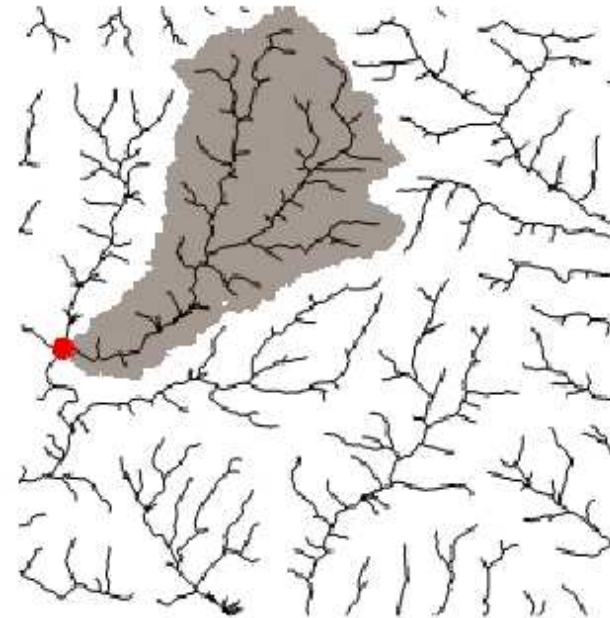
# Towards D8 watersheds

**Channel cells:** cells that drain a minimum threshold area

**Delineation of watersheds Input**

1. Flow direction grid
2. Pour-points

**Automatic pour-point selection:**  
Downstream end of each link in the drainage network



# D8 Watersheds

Accumulation  $\geq 3$

Labeling of stream links by integers

(a)

	1	1	1	1	
			1		
				1	
		1	1	1	

(b)

	1	1	2	3	
			4		
				4	
		5	5	6	

(c)

1	2	2	2	3	3
1	1	2	2	3	3
1	1	1	2	3	4
1	1	4	4	4	4
5	5	5	5	4	6
5	5	5	5	6	6

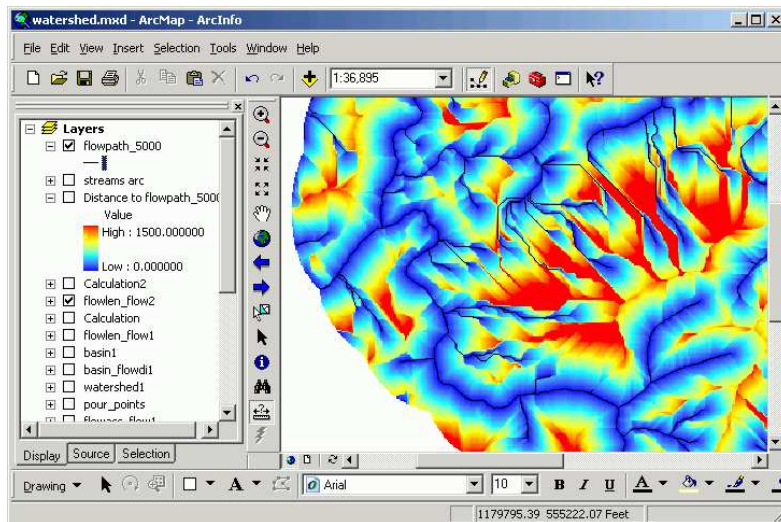
Division into subwatersheds

# Flow Lengths

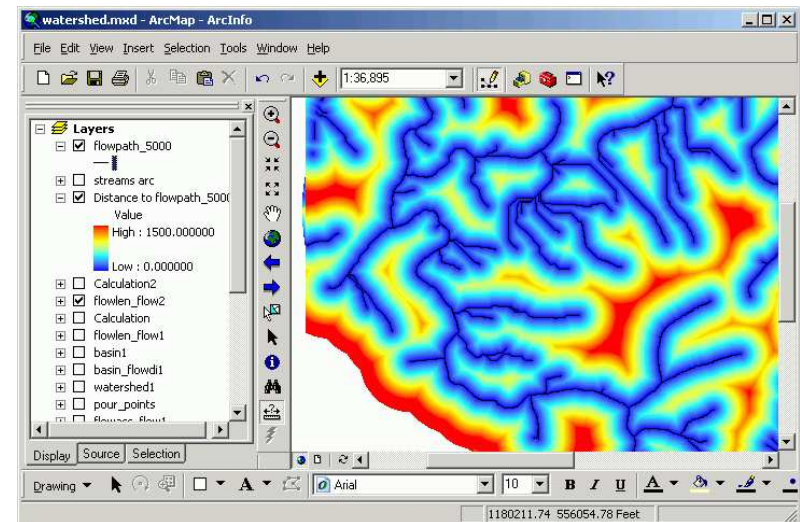
**Flow Length:**  
length of path from cell to outlet

6.66	6.24	5.83	5.41	5.83	6.24
6.24	5.24	4.83	4.41	4.83	5.24
5.83	4.83	3.83	3.41	3.83	3.41
6.24	5.24	3.41	2.41	2.00	2.41
4.41	3.41	3.00	2.00	1.00	2.00
4.00	3.00	2.00	1.00	0.00	1.00

Running example



Distance to stream following flow



Euclidean distance to stream

# Problem areas

Possible flow direction modifications:

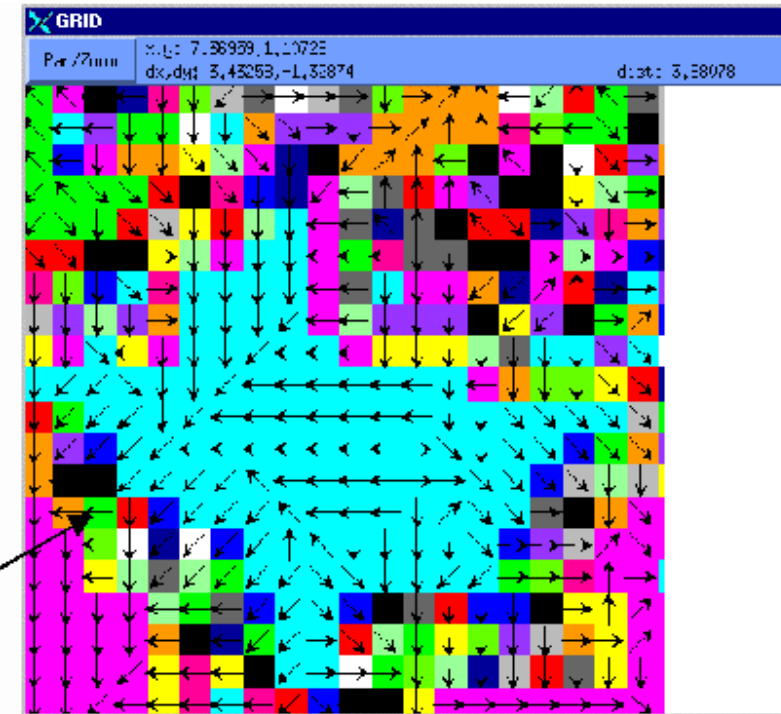
1) Route **lakes** to their outlets defined by

- rivers and streams, or by,
- lowest water edge elevation flowing away from the lake

2) Route **rivers** from their headwater to their tail waters. Route cells immediately adjacent to rivers into the rivers

3) Route **flat areas** to intersecting streams or the nearest routed cell

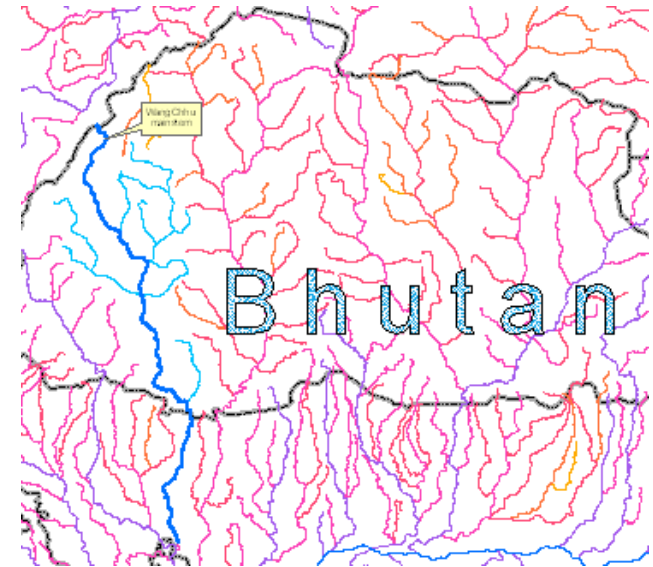
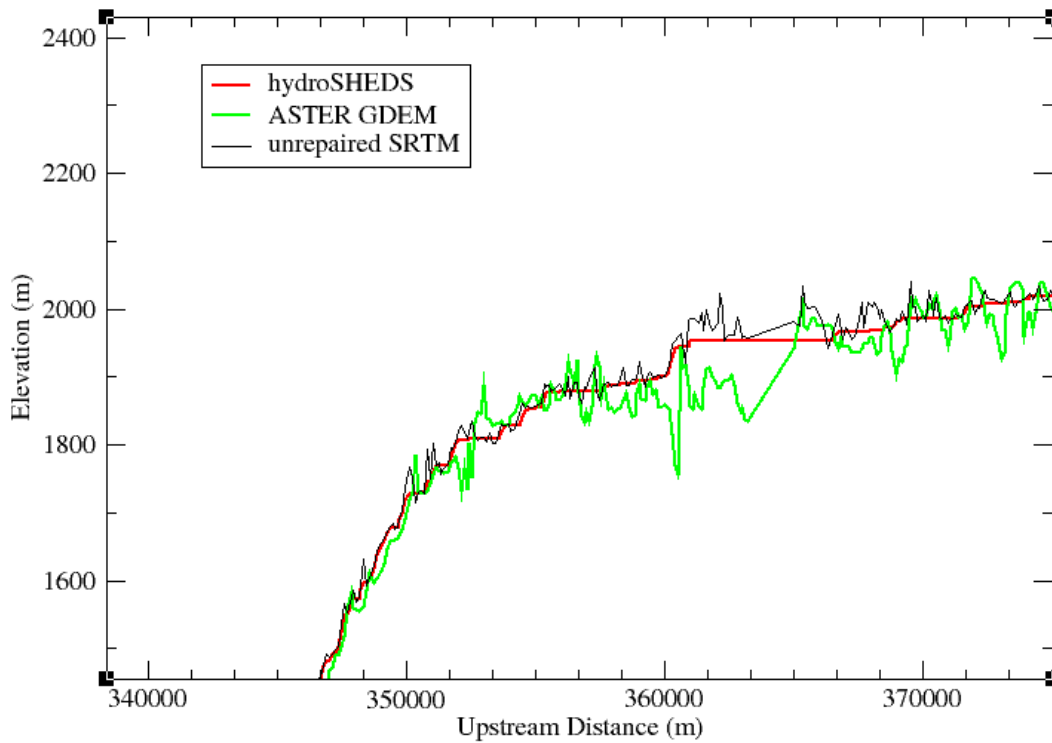
55 cells with no neighboring cell of lower elevation flow to this cell which is the lowest of all cells bordering the flat area





# Sinks & non-monotonicity

Main Stem Wang Chhu  
Comparing profiles



Source: <http://gis.ess.washington.edu/areas/Bhutan/streamslopes/index.html>

# Sink identification

**Sinks:** interrupt continuous flow across the DEM surface

Some procedures implemented by **Hydrosheds** to improve river topology extraction:

**Deepen river and lakes:** Force flow to stay inside

**Stream burning:** Enforce known location of rivers; Lower elevation values (e.g. by 12 m) along a river with a buffer of e.g. 500m from the river center to the surroundings.

**Sink filling:** Raise elevations in the sink until outflow is possible

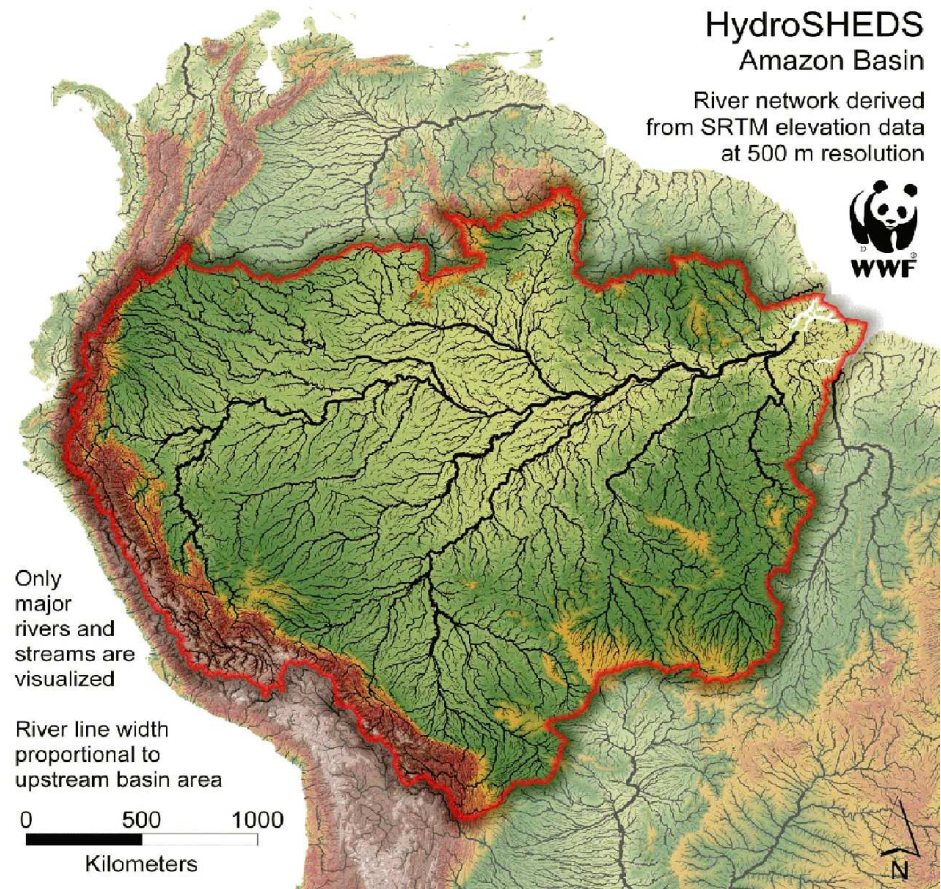
**Carving:** Remove rises in the river courses

# Hydrosheds

<http://hydrosheds.cr.usgs.gov/>

Hydrological data and maps based on Shuttle Elevation Derivatives at multiple Scales

- river networks
- watershed boundaries
- drainage directions
- flow accumulations

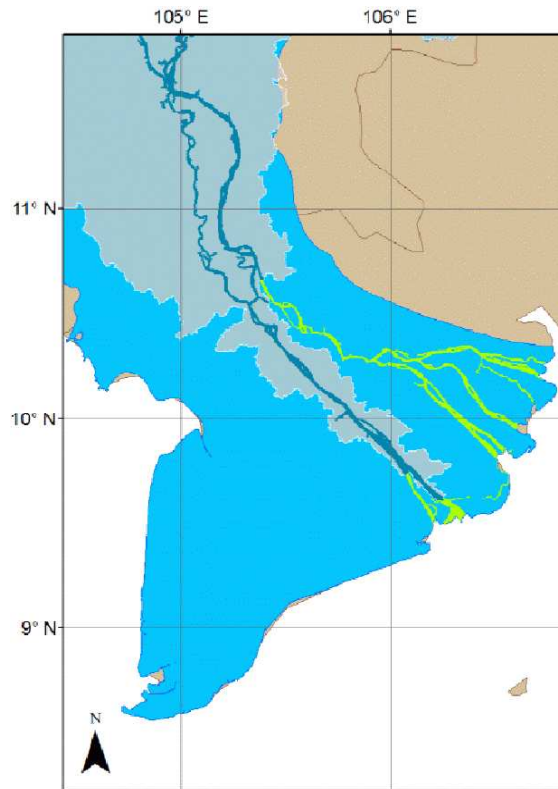




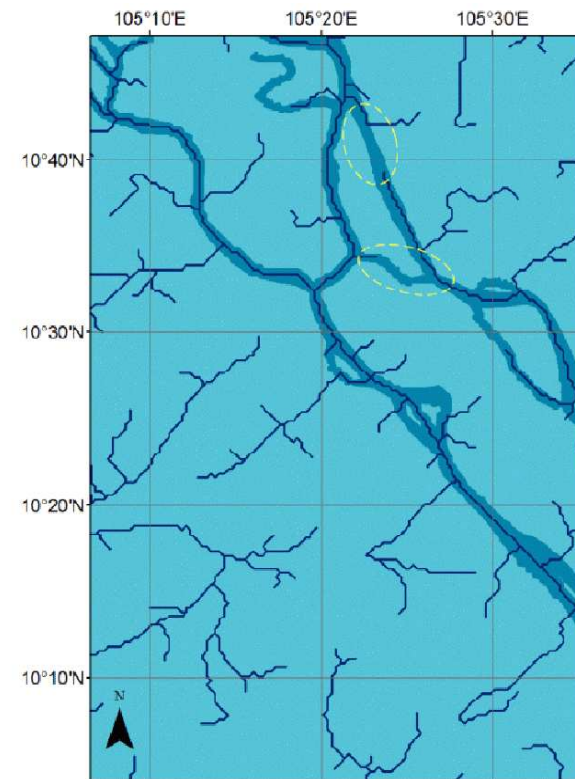
# Hydrosheds example, Mekong river

Source: MSc thesis, Marjolein Koudijs

**Project aim:** monitoring Mekong water level changes using ICESat elevations



**Figure 3.5:** USGS drainage basin compared with IWMI drainage basin, distributaries in the Mekong River Delta (green lines) are excluded by USGS.



**Figure 3.16:** Errors in USGS HydroSHEDS River Network (RIV) causes absence of the complete left branch in the Mekong River Delta.

# Conclusions

## Local terrain analysis

- Surface normal and roughness are obtained by local least squares fitting
- Slope and aspect are derived from gradients
- Ridge and valley points: second derivative

## Global terrain analysis

- Link local results (like gradients)
- Example: [Path way of water](#) is an important application of elevation data.
- Resolution issues and errors can spoil physical consistency

# Exercises