AESB2440: Geostatistics & Remote Sensing

Lecture 4: LIDAR & Triangular Interpolation

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Overview

LIDAR

- Techniques and Principles
- Airborne, terrestrial and mobile
- AHN: Actueel Hoogtebestand Nederland

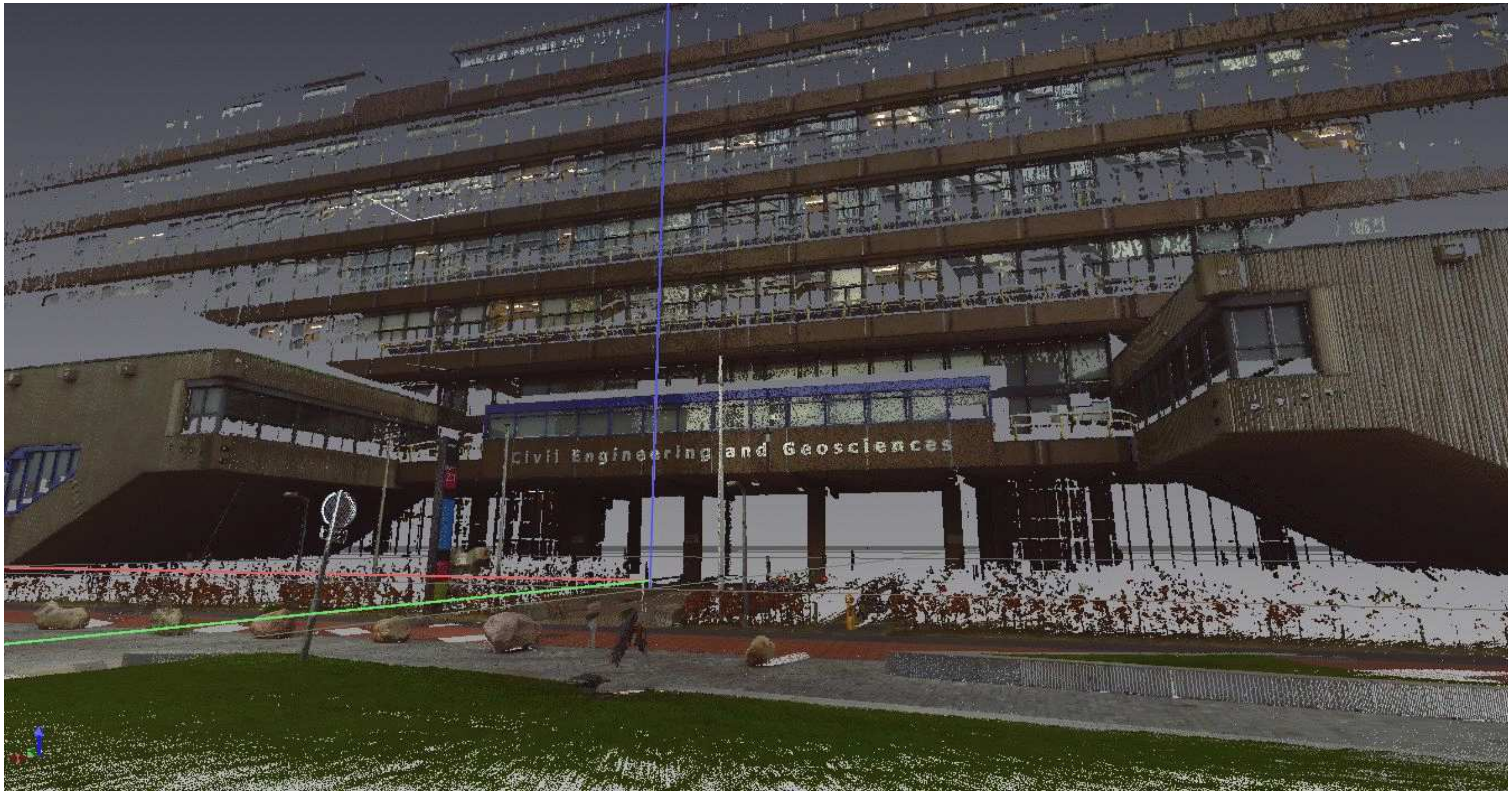
Triangular Interpolation

- Interpolation and extrapolation
- Convex hull
- Nearest neighbors
- Voronoi diagram
- Delaunay triangulation
- TINs

Interpolation properties

- Realistic results
- Robustness
- Weights
- Computational efficiency;
- Quality description

What's this?



A. Sensors: LIDAR

LIDAR

LiDAR Light detection and ranging.

$$R = \frac{1}{2} \cdot c \cdot t$$

R: range from laser (and receiver) to object

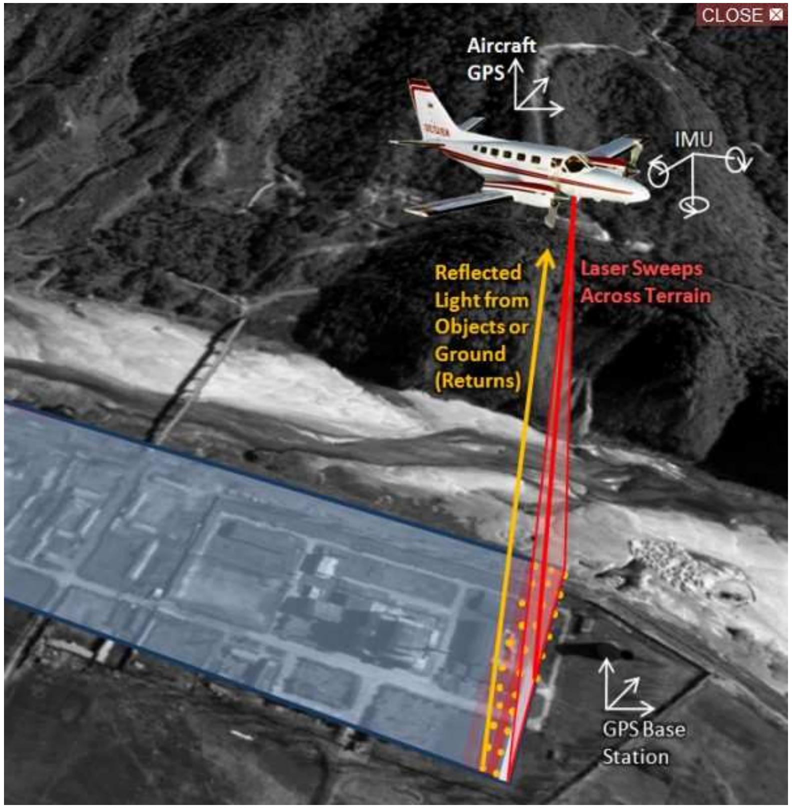
c: speed of light

t: two way travel time

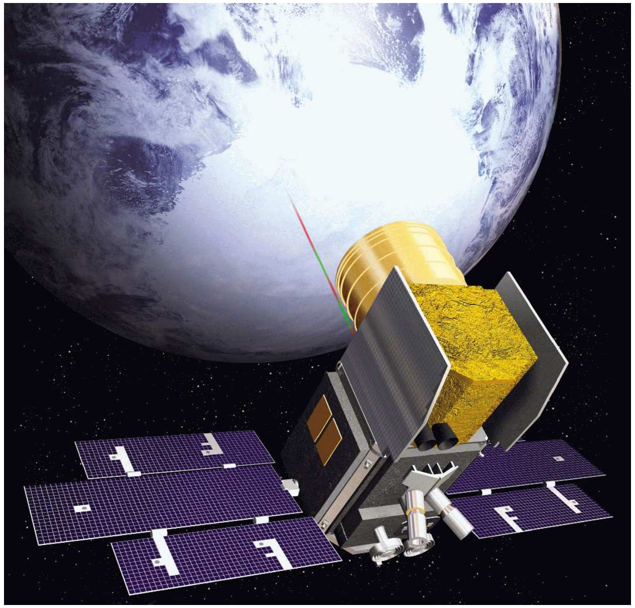
Combine range signal with

- Position laser
- Attitude (orientation) laser

$\Rightarrow$ 1 georeferenced XYZ point



From tripod to satellite



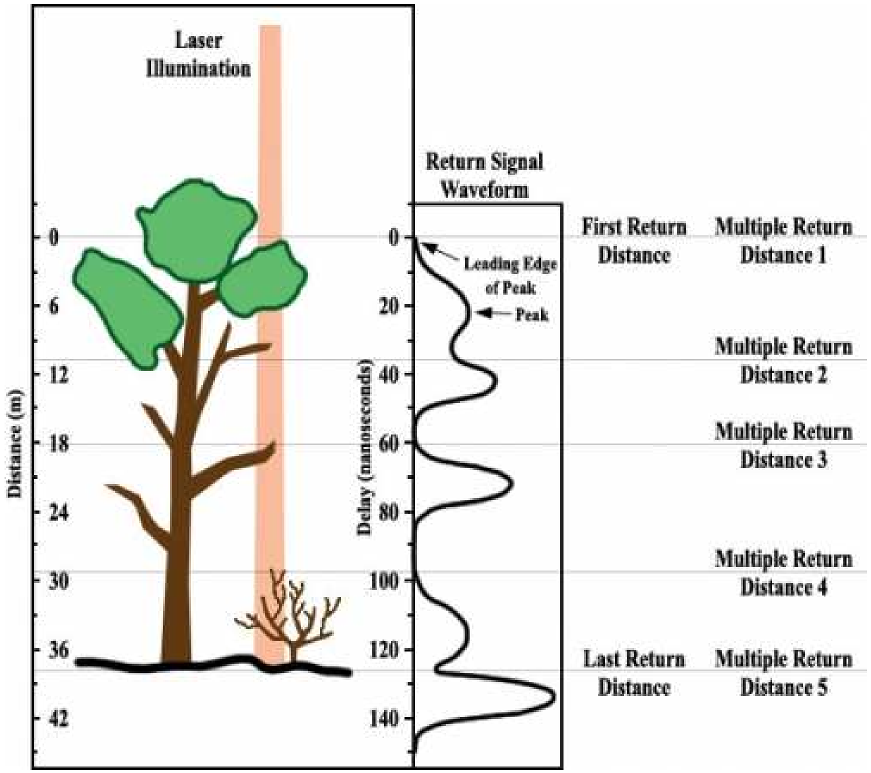
Range determination

Possibilities for (automated) range determination:

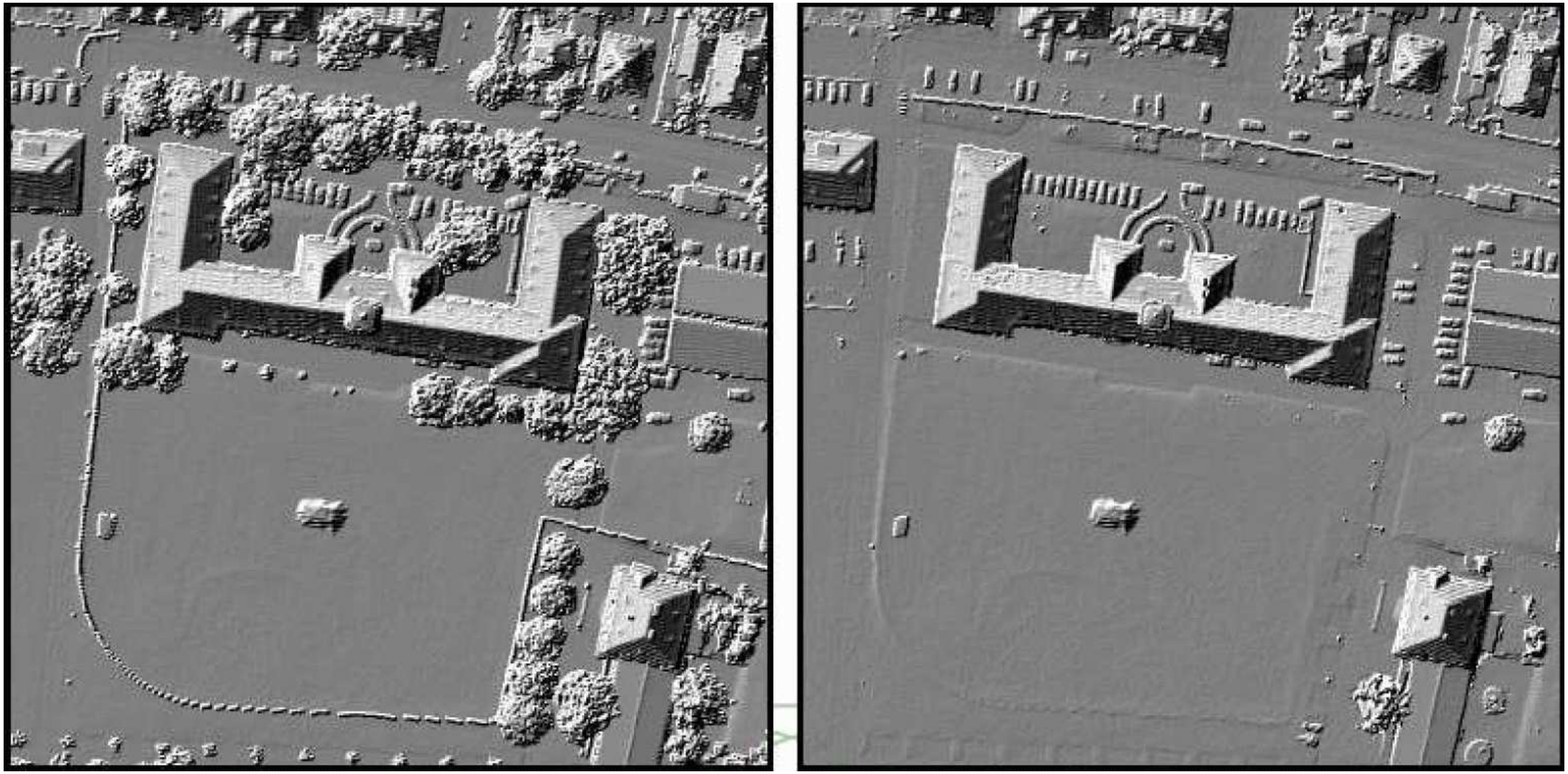
- Signal median
- Signal maximum
- First significant signal

Alternatives

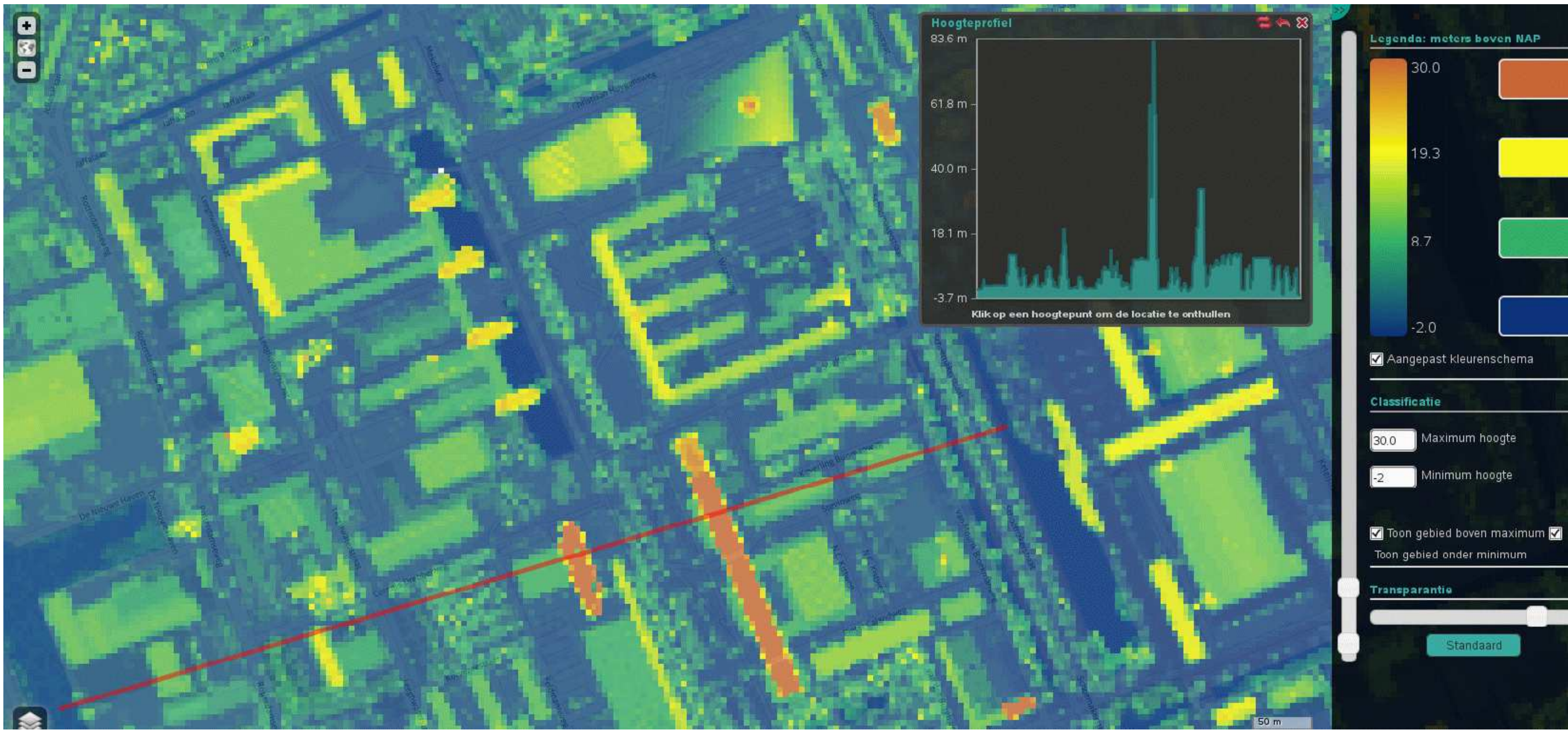
- Multiple echoes
- Full waveform



First and Last Recording



Actueel Hoogtebestand Nederland



AHN viewer: http://ahn.geodan.nl/ahn/

AHN properties

AHN: Dutch National Laser archive

Organization: Rijkswaterstaat (Dutch Public Works Dept.)

	AHN 1	AHN 2	AHN 3
Acquisition	1996-2004	2007-2012	in progress
Point Density	1 pt/m^2, or 1pt/16 m^2	8-20 pts/m^2	
Accuracy	5 cm	5 cm	
Precison	15 cm st.dev.	5 cm st.dev.	

Products:

1. Laser points, decomposed into class terrain and other
2. Grids: 0.5 m (AHN 2), 5 m, 25 m & 100 m (AHN 1)

AHN 2: 135.200.000.000 elevations 0.5 m grid

Applications, AHN

Tree inventory:
http://www.boomregister.nl

Archeology
Celtic Fields in the forest

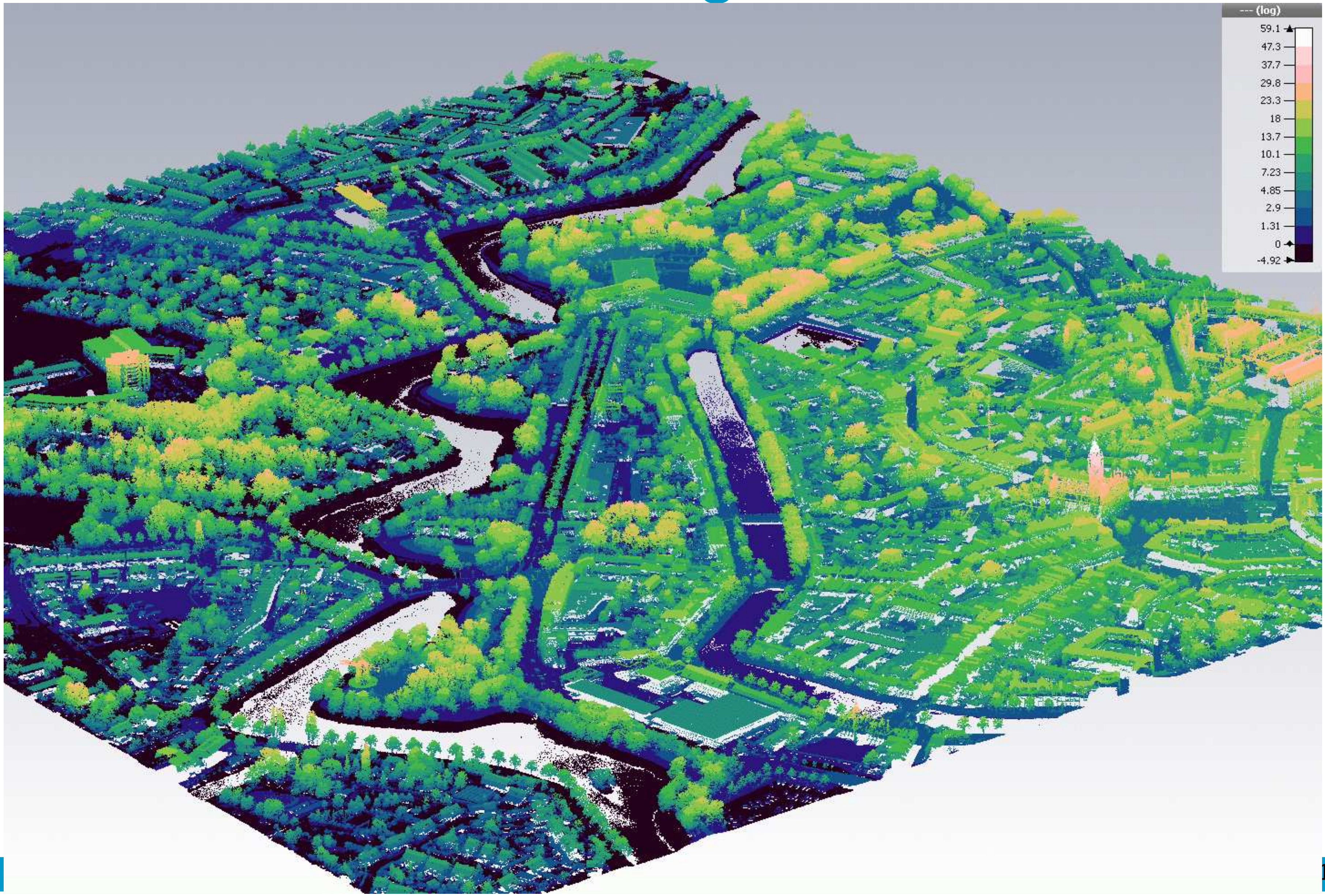
Physical Geography:
Ancient floodchannel mapping

Sustainable development:
Solar panel potential

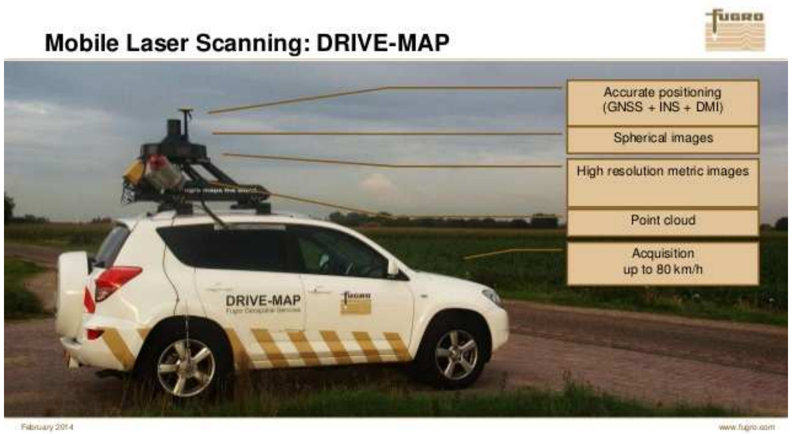
Free download from

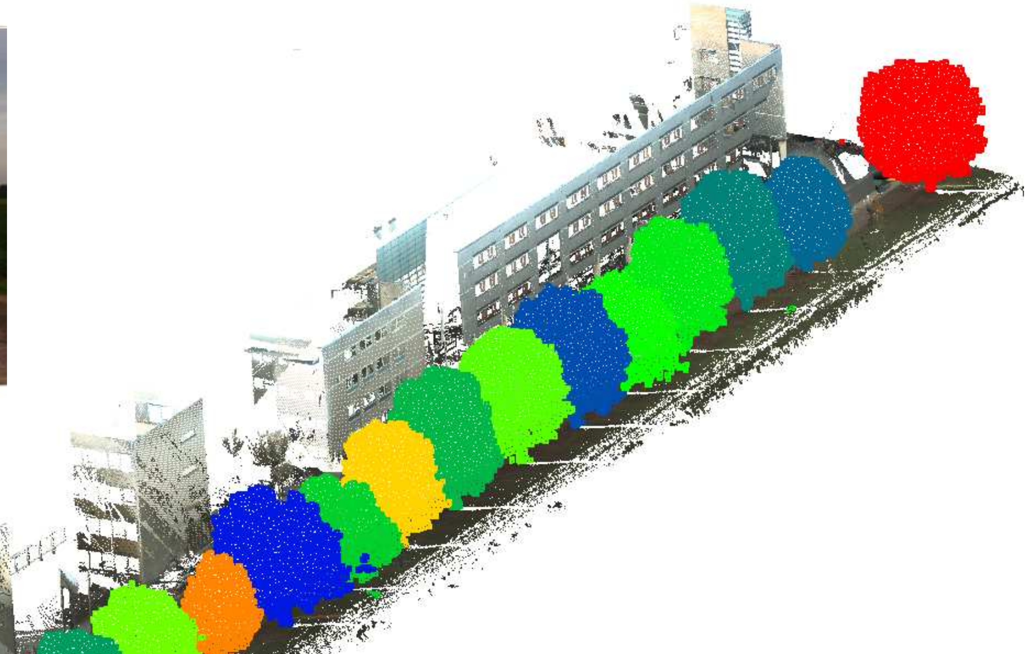
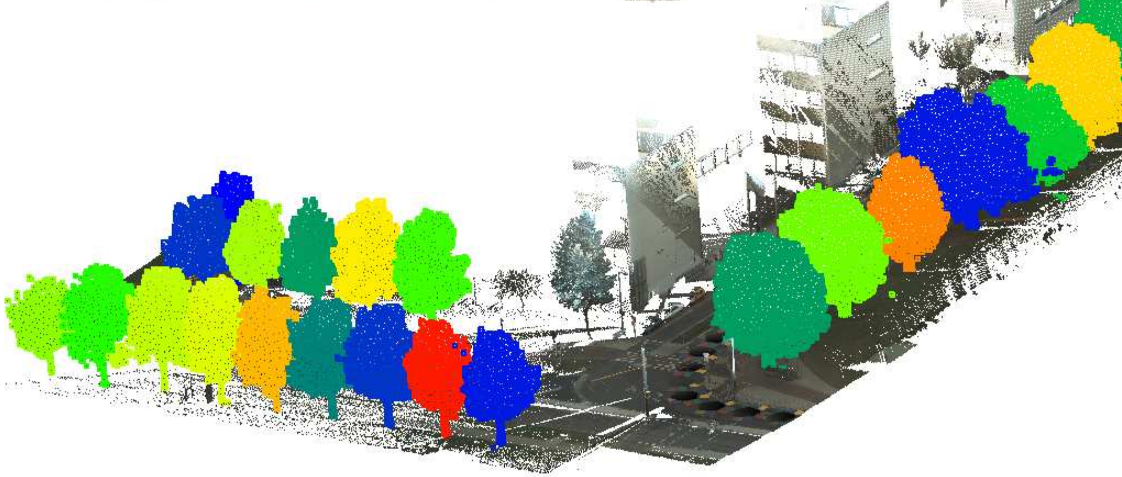
- http://www.pdok.nl
- and via QGIS PDOK plugin

AHN over Middelburg



Laser Mobile Mapping



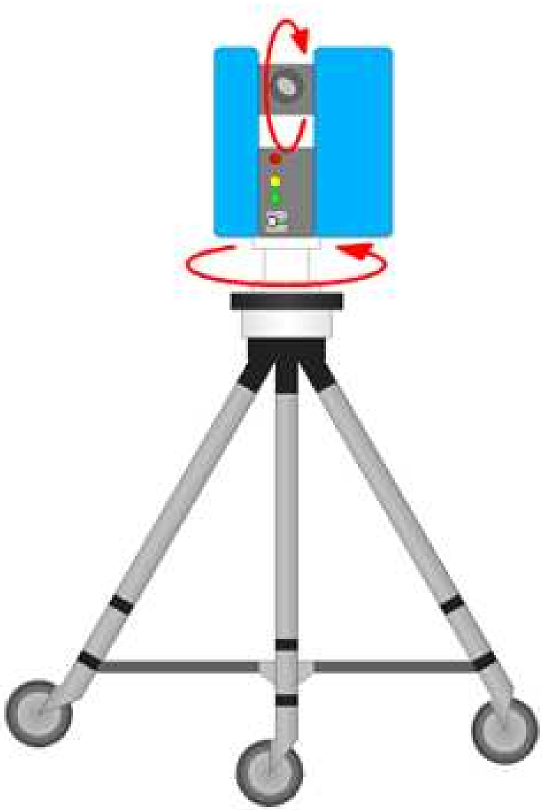



Source http://http://www.fugrogeospatial.com

Source http://www.slideshare.net/ICC-RS/de-waarde-van-3d-metingen

Trees Image: Jinhu Wang (Tu Delft)

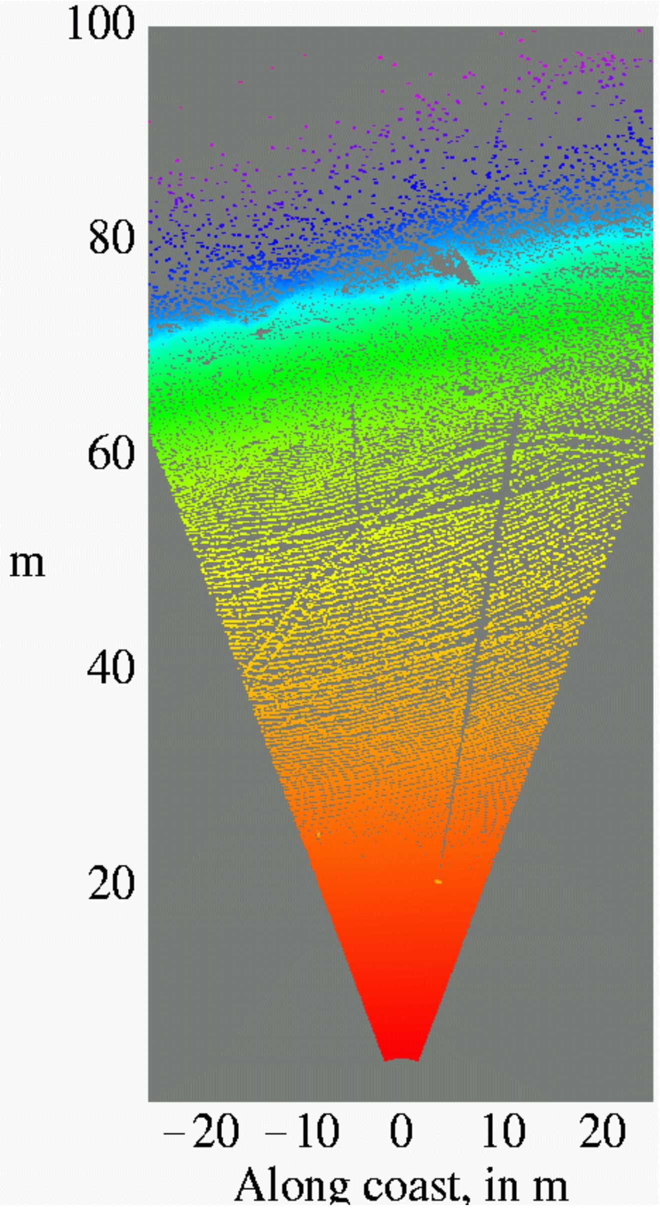
Terrestrial Laser Scanning.



Phase scanners:
modulated light wave

Pulse scanners: time of flight

Footprint size: mm

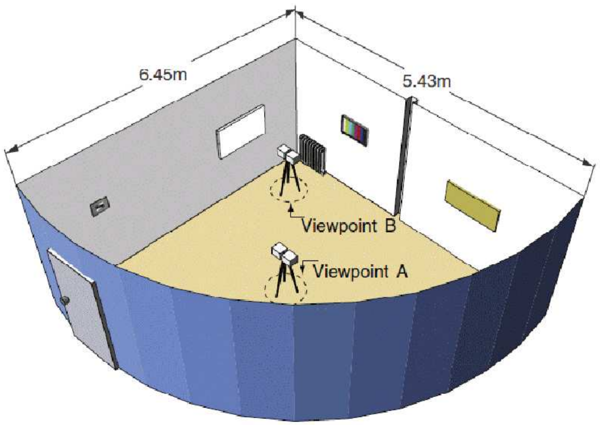


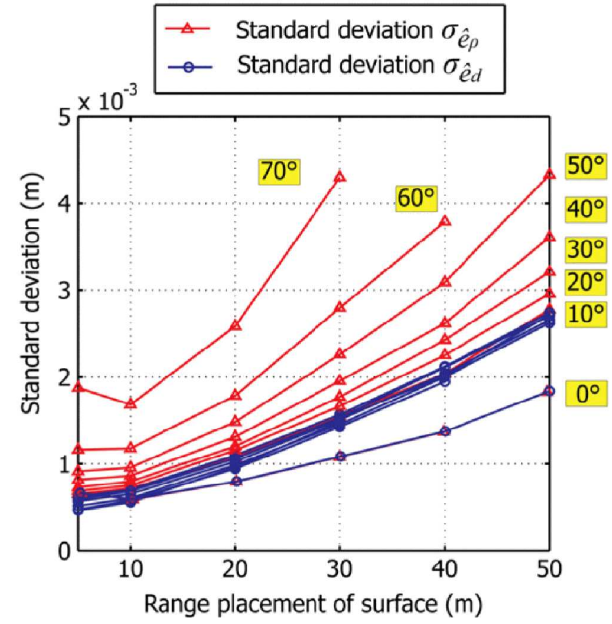
Static: TLS Range Image

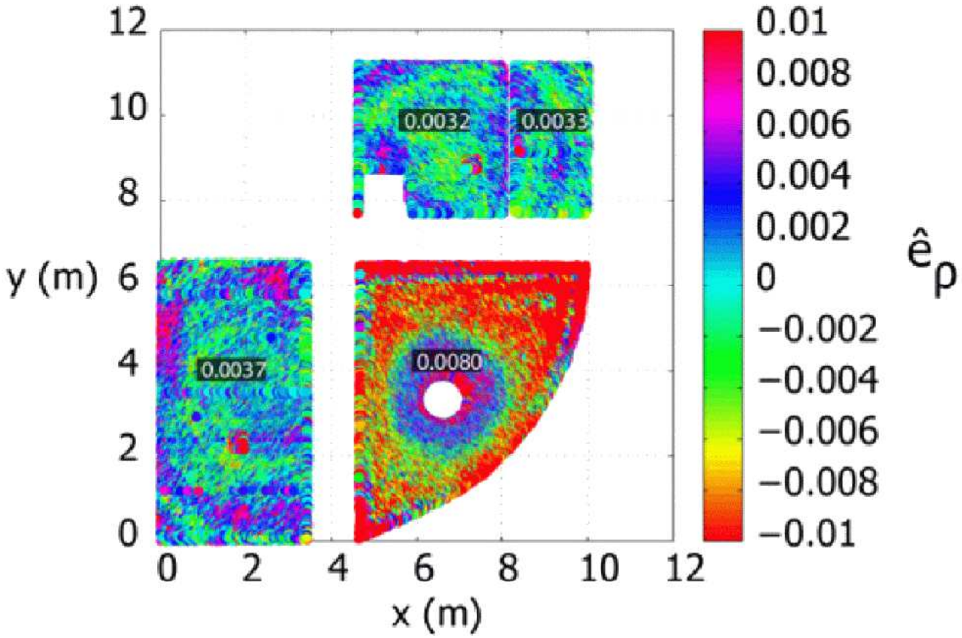
Spherical coordinate system, centered at scanner

Here: intensity image; Alternative: range image

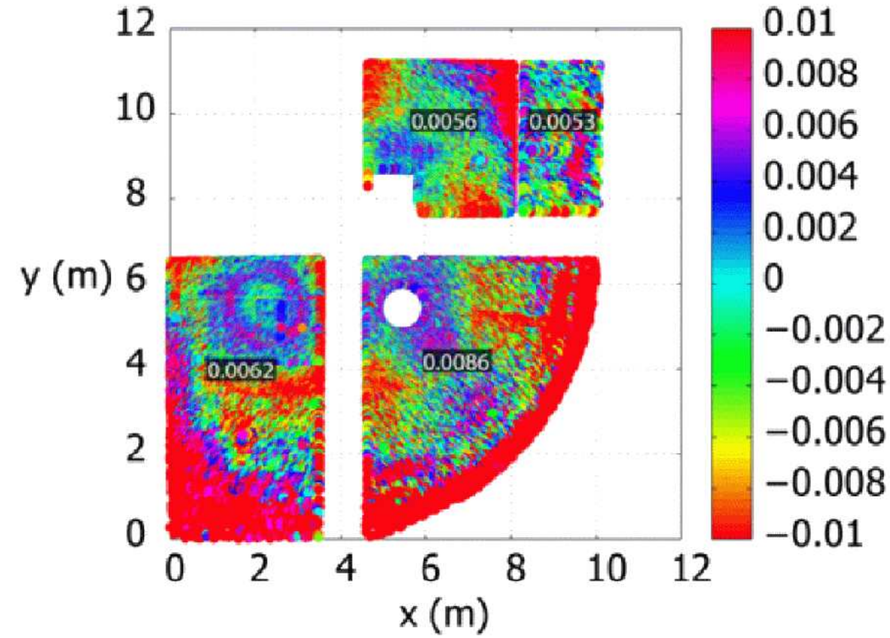
Varying noise levels





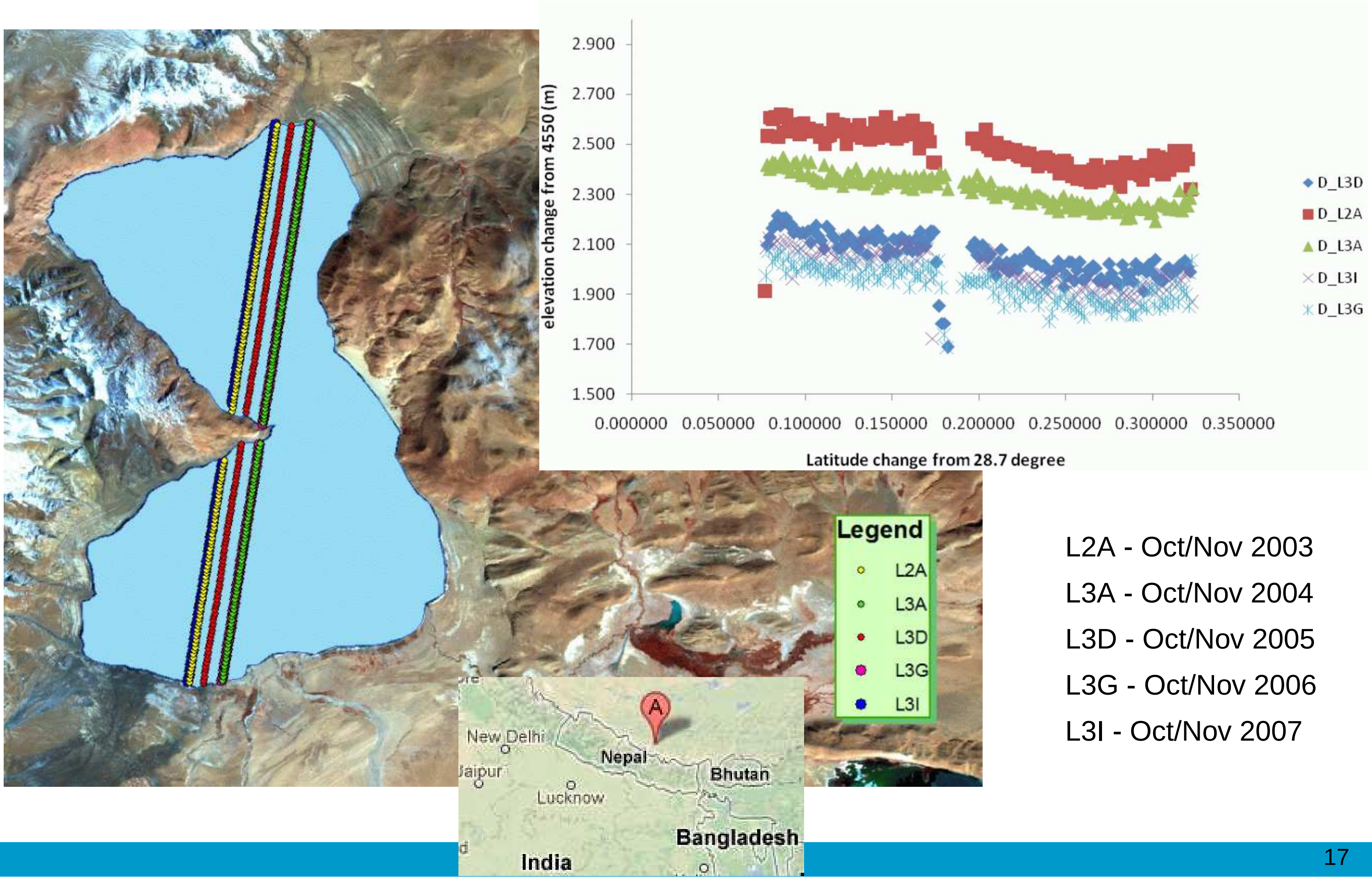


Viewpoint A



Viewpoint B

ICESat lake level changes at Pelku Tso



L2A - Oct/Nov 2003

L3A - Oct/Nov 2004

L3D - Oct/Nov 2005

L3G - Oct/Nov 2006

L3I - Oct/Nov 2007

Neighborhood Watch

References

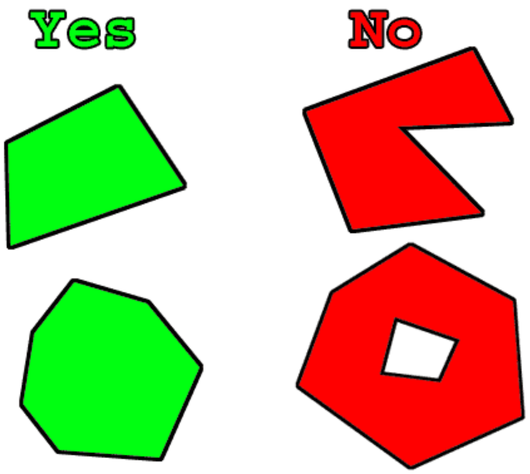
MSc level book:
Mark de Berg, Otfried Cheong, Marc van Kreveld, Mark Overmars,
Computational Geometry: Algorithms and Applications,
Springer-Verlag, 3rd edition, 2008.

Voronoi diagrams:
Read Chapter 7, Intro + 7.1, Definition and Basic Properties,

Delaunay triangulations:
Read Chapter 9, Intro + 9.1, Triangulations of Planar Point Sets, + 9.2, The Delaunay Triangulation,

Both chapters are downloadable from Blackboard.
Book website: http://www.cs.uu.nl/geobook/

Convex hull

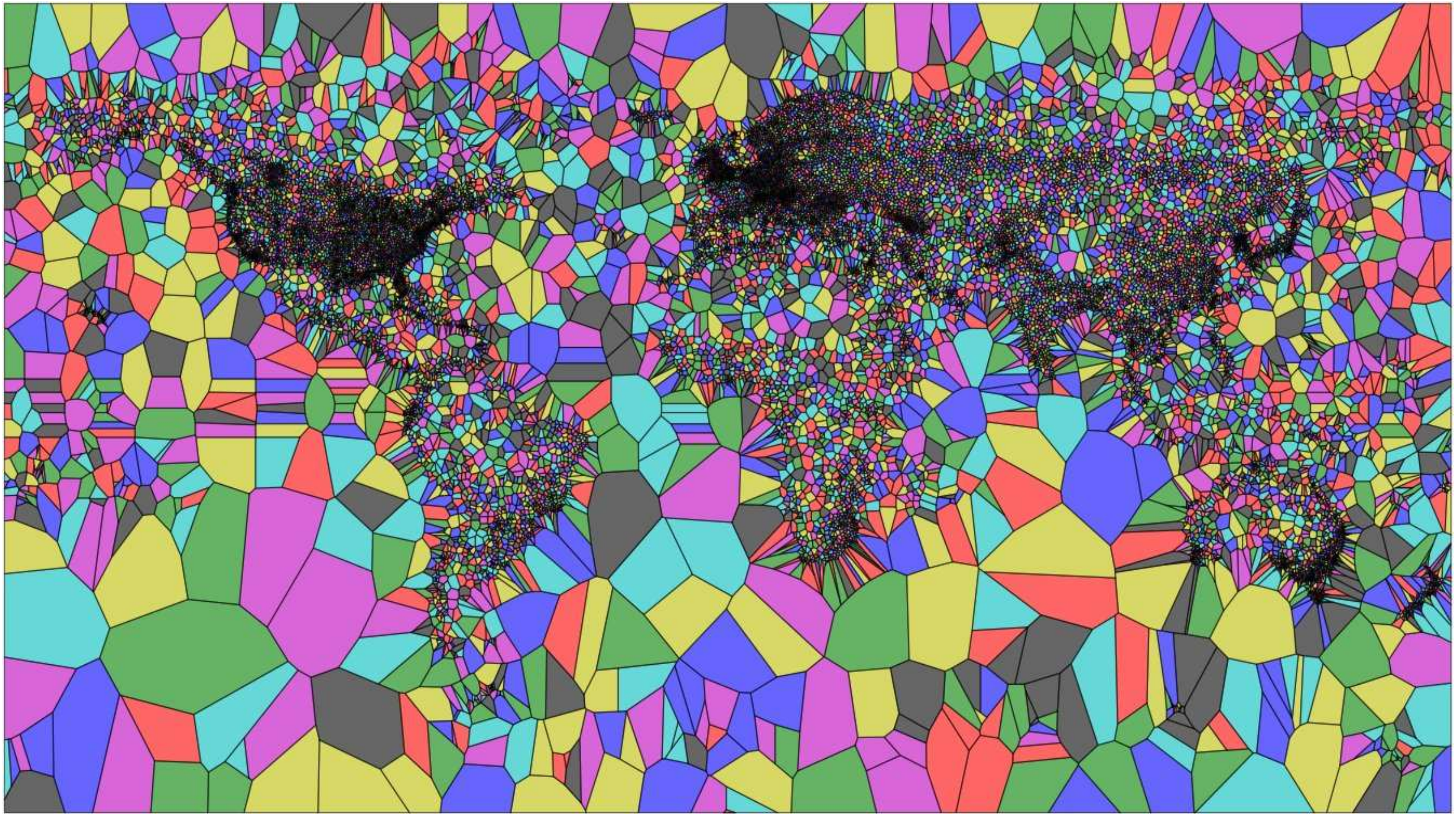


Definition.
A subset S of the plane is convex if for any two points $p, q \in S$ the line segment pq is contained in S as well. The convex hull of S is the smallest convex set that contains S.

Question: definition in 3D?

Remark: Difference between interpolation and extrapolation often defined as estimating values within or without the convex hull of the observations.

Closest Weather station



Source: http://sofser.blogspot.nl/2011/08/voronoi-diagram-for-geo-data-infochimps.html

Voronoi diagram (Euclidean)

Given is a set

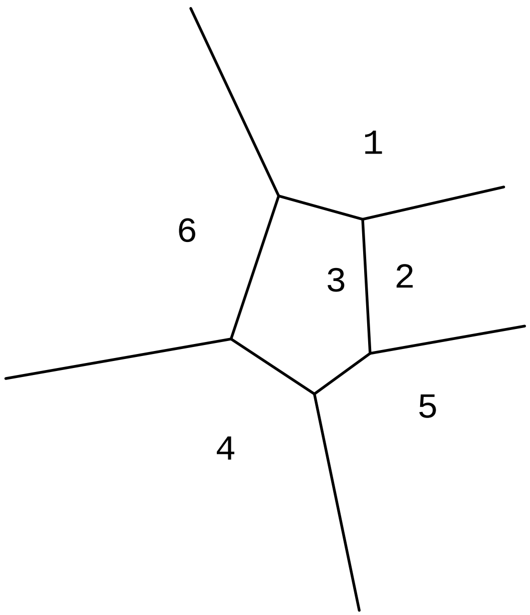
$$S = \{p_1, \ldots, p_n\}$$

of n distinct positions in $\mathbb{R}^2$.

The Voronoi cell $V(p_i)$ of p_i consists of all points most close to p_i.

The Voronoi diagram of S is the subdivision of the plane in cells $V(p_i)$.

Voronoi diagrams consist of cells, edges and vertices.



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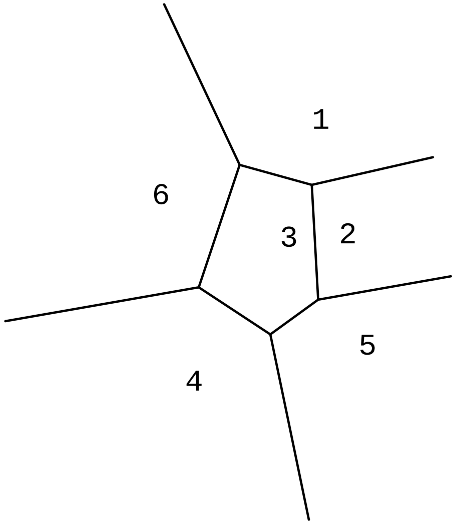
Bisectors

Voronoi cells are bounded by bisectors $b(p_i, p_j)$.

Every bisector $b(p_i, p_j)$ is the intersection of two half planes

$$b(p_i, p_j) = h(p_i, p_j) \cap h(p_j, p_i)$$

The half plane $h(p_i, p_j)$ are those points that are not further from p_j then from p_i.



Claim. $V(p_i) = \bigcap_{j \neq i} h(p_i, p_j)$.
Or, in words: each Voronoi cell can be constructed as an intersection of half-planes

Question:
How many different half-planes exist for a set of, say, $n = 1.000.000$ points?

Empty circle criterion + boundary points

Recall:

$$S = \{p_1, \ldots, p_n\}$$

is a set of n distinct positions in $\mathbb{R}^2$.

[Empty circle criterion]
i) $q \in \mathbb{R}^2$ is a Voronoi vertex of VD(S) $\Leftrightarrow$ q is the center of an S-empty circle.

[Participating bisectors]
ii) The bisector of p_i and p_j defines and edge in VD(S) $\Leftrightarrow$ there exists $q \in \mathbb{R}^2$ and an S-empty disk $C_S(q)$ that has p_i and p_j on its boundary.

[Boundary points]
iii) $V(p_i)$ is unbounded $\Leftrightarrow$ p_i on the boundary of the convex hull of S.

See also:
http://web.informatik.uni-bonn.de/I/GeomLab/VoroGlide/index.html.en

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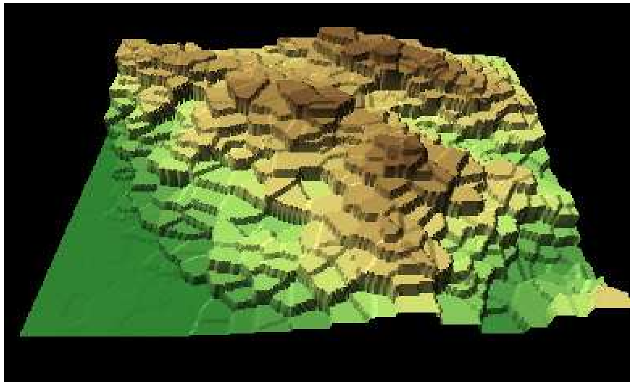
Nearest neighbor

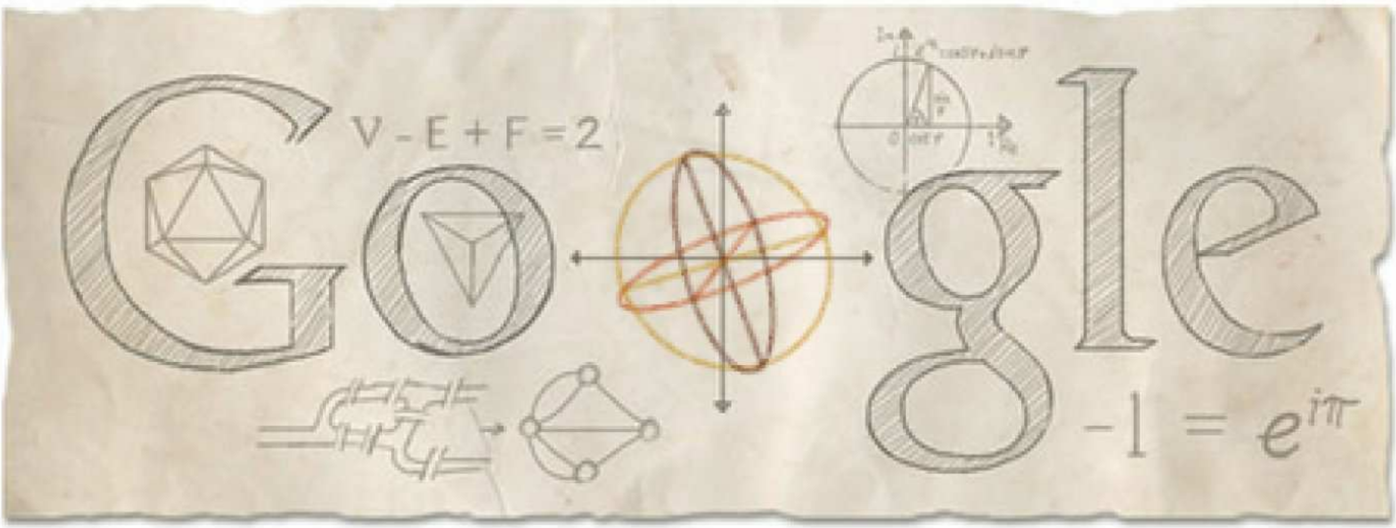
Interpolation Method 3.

1. Given is a set $S = \{p_1, \ldots, p_n\}$ of n positions with corresponding heights $h_1, \ldots, h_n$ and an estimation position p_0.
2. Determine p_i s.t. $p_0 \in V(p_i)$.
3. $h_0 = h_i$.



Questions

- Weights?
- Problematic cases?
- Disadvantages?
- Generalizations?



Google

Euler's formula

Consider a planar, embeddable graph without intersections.

v   # vertices
e   # edges
f   # faces

Theorem [Euler's Formula]
v - e + f = 1

Proof. Nineteen different proofs can be found here:
http://www.ics.uci.edu/ eppstein/junkyard/euler/

Corollary.

1. Number of Voronoi vertices is at most 2n-5.
2. Number of Voronoi edges is at most 3n-6.
3. Voronoi diagram "has the same size" as the number of points

Point cloud triangulation

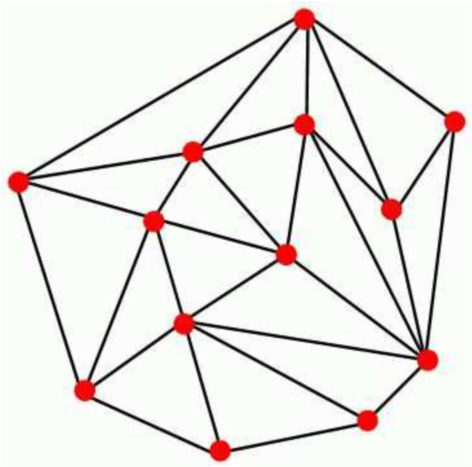
Let $S = \{p_1, \ldots, p_n\}$ be a set of points in a plane.

A maximal planar subdivision is a subdivision of the plane such that any edge added would intersect an existing edge.

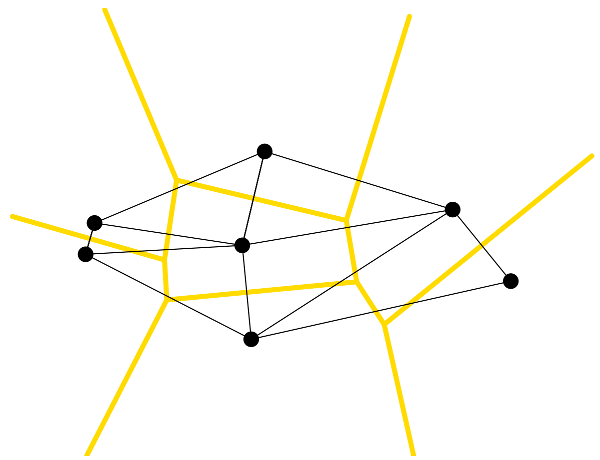
A triangulation of S is a maximal planar subdivision with vertex set S.

Corollary: Every face except the one outside the convex hull is indeed a triangle

Question: why?



Delaunay triangulation.



A) Voronoi Diagram
$\longrightarrow$ Delaunay Triangulation:

- Draw an edge between points p_i and p_j, whenever,
- p_i and p_j share an edge in the Voronoi diagram

B) Delaunay Triangulation $\longrightarrow$ Voronoi Diagram

1. For each triangle:
2. Draw a circle through its three corner points
3. Determine the center of that circle.
4. The circle centers are exactly the Voronoi vertices
   (indeed they are on equal distance of....)
5. Connect two circle centers by a Voronoi edge whenever the two circles have two common corner points.

Delaunay Properties

The resulting triangulation of the convex hull of S is the Delaunay Triangulation. Note the duality:

Voronoi diagram		Delaunay triangulation
Voronoi cell	$\leftrightarrow$	vertex/position/point
edge	$\leftrightarrow$	edge
Voronoi vertex	$\leftrightarrow$	triangle

Why Delaunay is popular.

From properties Voronoi diagrams:
A triangulation $\mathcal{T}$ of S is Delaunay $\Leftrightarrow$
The circumcircle of every triangle is S-empty.

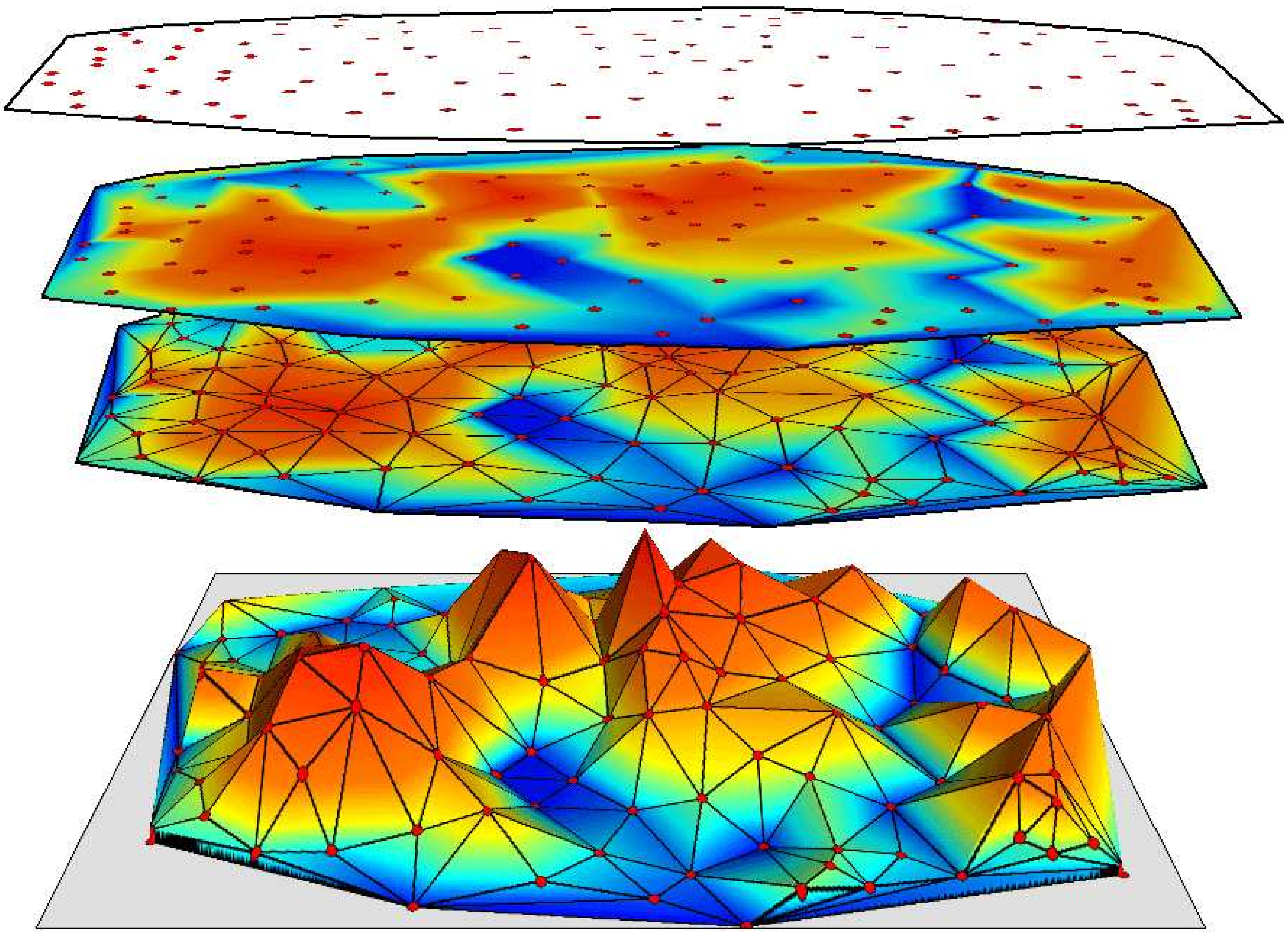
Popularity reason 1:
The Delaunay triangulation maximizes the minimum angle over all triangulations of S.

The *minimum angle* of a triangulation is the smallest angle of all 3 angles of all triangles

Popularity reason 2:
A Delaunay triangulation is relatively fast to build

The so-called computational complexity is O($n \log n$)

Triangular Interpolation



From observations towards a Triangulated Irregular Network (TIN)

Triangular Interpolation step by step

Input: n height observations

$$(x_1, y_1, h_1), (x_2, y_2, h_2), \ldots, (x_n, y_n, h_n)$$

Wish: obtain height estimates at $2D$ grid points c_1 to c_N.

Step 1: Determine the Delaunay triangulation $\mathcal{D}$ of the height positions $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$

Step 2: For each grid point c_u, $u = 1 \ldots N$

- Determine the triangle $\Delta_{ijk} \in \mathcal{D}$ that contains c_u
- Vertices of Δ_{ijk} are observations $(x_i, y_i), (x_j, y_j)$, and (x_k, y_k).
- All other observations get weight 0 for interpolation at c_u
- (Positive) weights for h_i, h_j and h_k are obtained from the triangular weight formula (next slide)

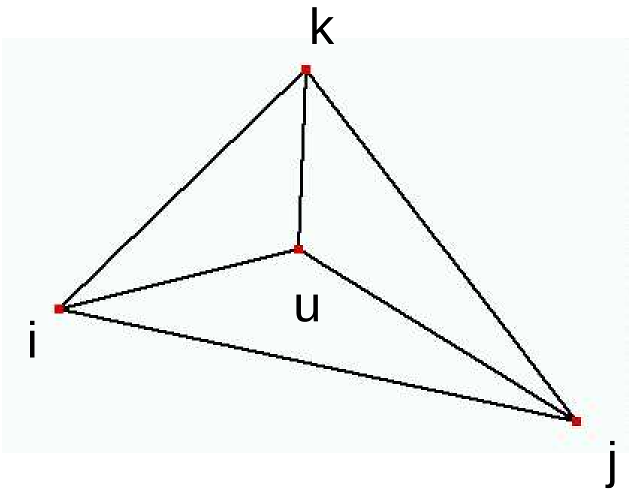
Triangular weight formula

Interpolation Method 3.

See http://www.fhi-berlin.mpg.de/grz/pub/preusser/TriFills.html

Weights:

$$\hat{h}_u = \frac{A_{ujk}}{A_{ijk}} \cdot h_i + \frac{A_{iuk}}{A_{ijk}} \cdot h_j + \frac{A_{iju}}{A_{ijk}} \cdot h_k$$

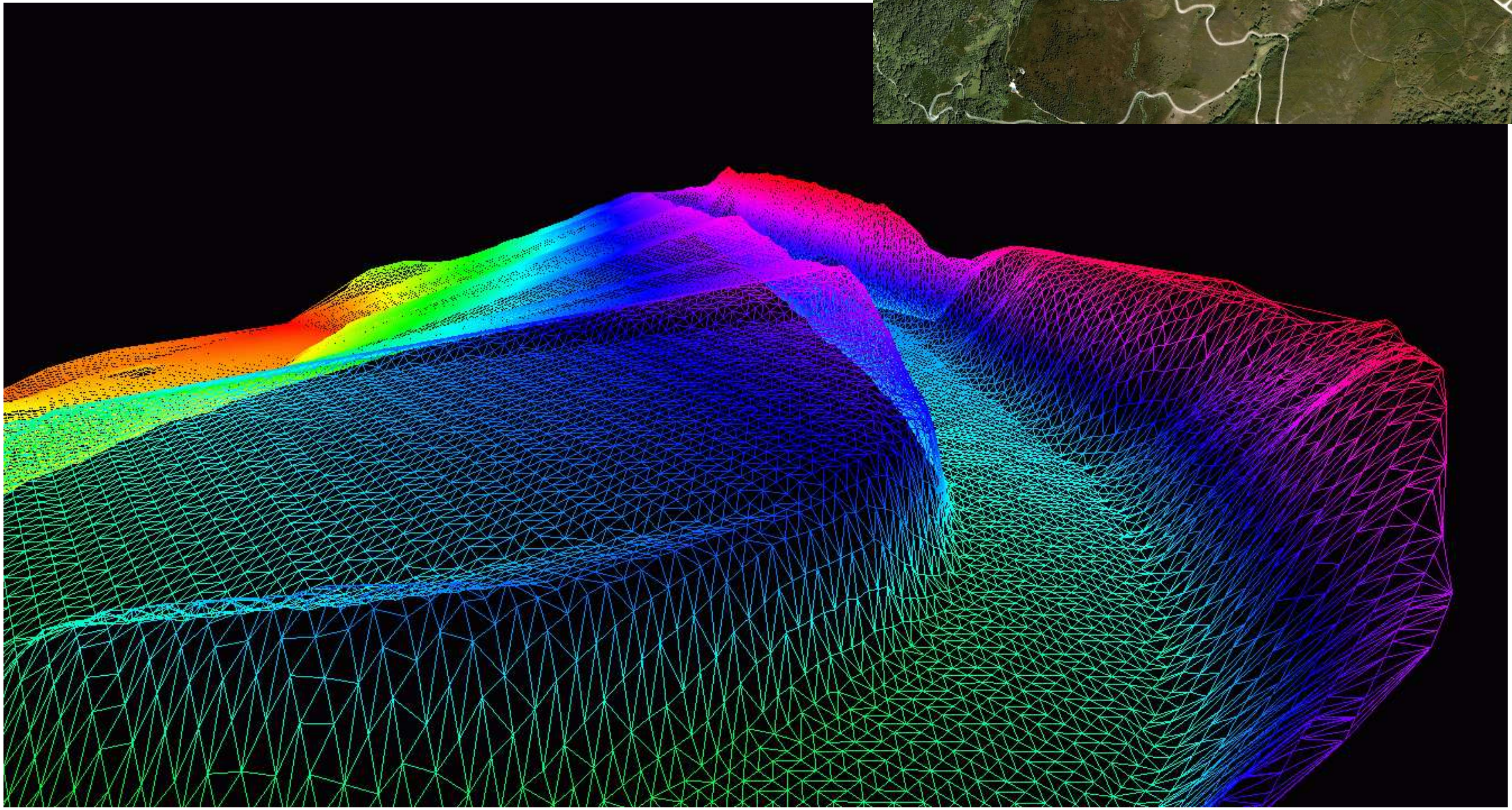


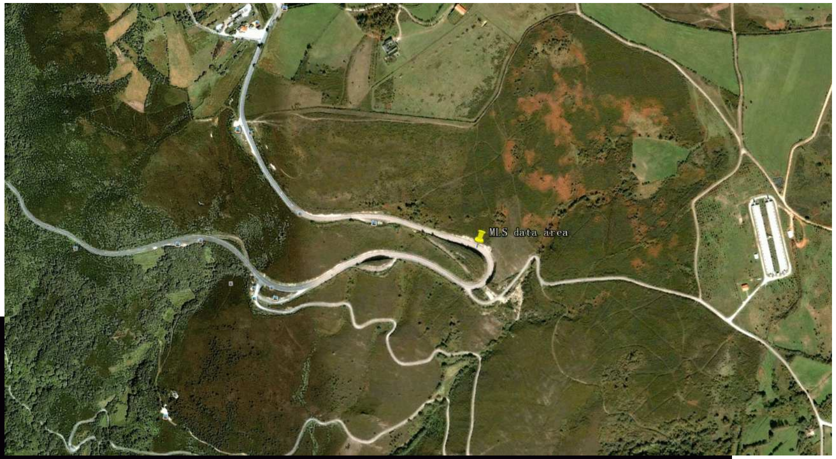
Where A_{ijk} denotes the area of the triangle Δ_{ijk}.

So, the closer u to vertex i the more weight h_i gets.

Question: (Dis)advantages?

Road TIN





Conclusions (for $1\frac{1}{2}$ lecture)

Two sensor principles

- GNSS: relatively sparse observations
- LIDAR: acquires large point clouds

Four interpolation methods

- Arithmetic mean
- Inverse distance interpolation (good for sparse data sets)
- Nearest neighbor interpolation
- Triangular interpolation (good for large, detailed data sets)

Until now: all observations are treated equal...

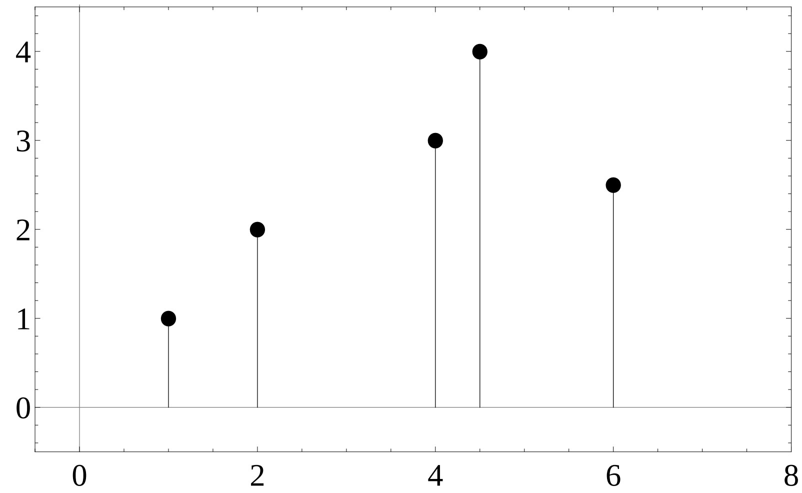
Still missing.

- Incorporating observation quality
- Incorporating correlation between observations

Exercises

Exercise

Exercise 4.1



In the figure, the points $p_1 = (1, 1)$, $p_2 = (2, 2)$, $p_3 = (4, 3)$, $p_4 = (4.5, 4)$, and $p_5 = (6, 2.5)$ are shown. Think of these points as the result of measuring some signal as function of time.

a). Interpolate the signal on the interval $(0, 8)$ using nearest neighbor interpolation

b). Interpolate the signal on the interval $(0, 8)$ using linear interpolation.

c). How is linear interpolation the equivalent of triangular interpolation for 1D?

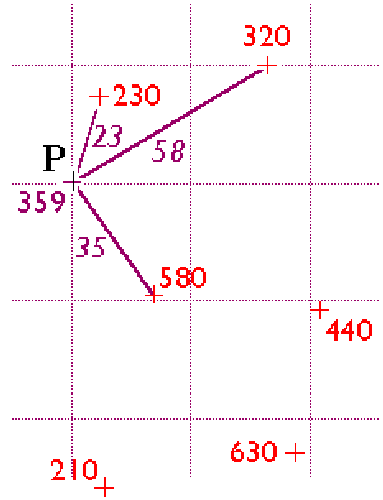
d). Interpolate the signal on the interval $(0, 8)$ using inverse distance interpolation with a power of 2. I you don't use a computer, just give a sketch.

e). Interpolate the signal in Matlab on the interval $(0, 8)$ using inverse distance interpolation with powers of $p = 0, 1, 2, 3, 1000$.

Exercise

Source: https://www.e-education.psu.edu/natureofgeoinfo/c7_p9.html

Exercise 4.2



$$z_P = \frac{\sum_{i=1}^{n} \left(\frac{z_i}{d_i}\right)}{\sum_{i=1}^{n} \left(\frac{1}{d_i}\right)}$$

$$= \frac{\frac{230}{23} + \frac{320}{58} + \frac{580}{35}}{\frac{1}{23} + \frac{1}{58} + \frac{1}{35}}$$

Consider the example of Inverse Distance Interpolation in the figure. Note that in this example only close by points are used for interpolation.

a). Give a criterion that would exactly result in the use of the measurements as indicated in the figure.

b). What is the power of the method used? That is, what value of p is used?

c). Estimate the height z_p using power $p = 0, 1, 2, 10, \infty$ from the three observations connected to point P.

d). So, what is the height using nearest neighbor interpolation?

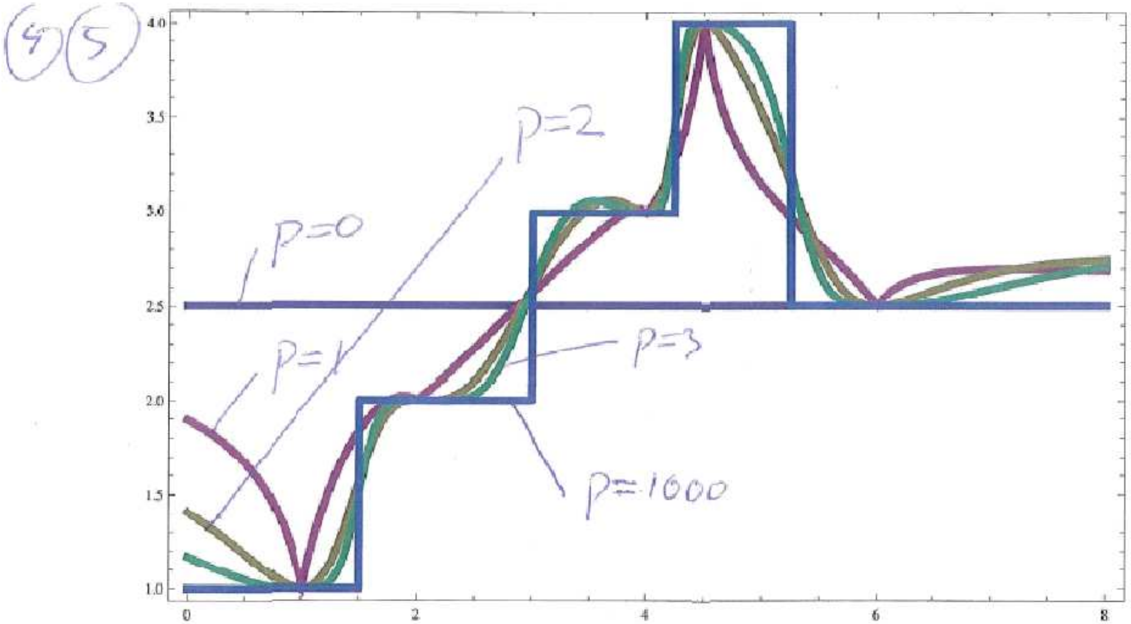
e). Can you estimate a height using Triangle Interpolation? Why not?

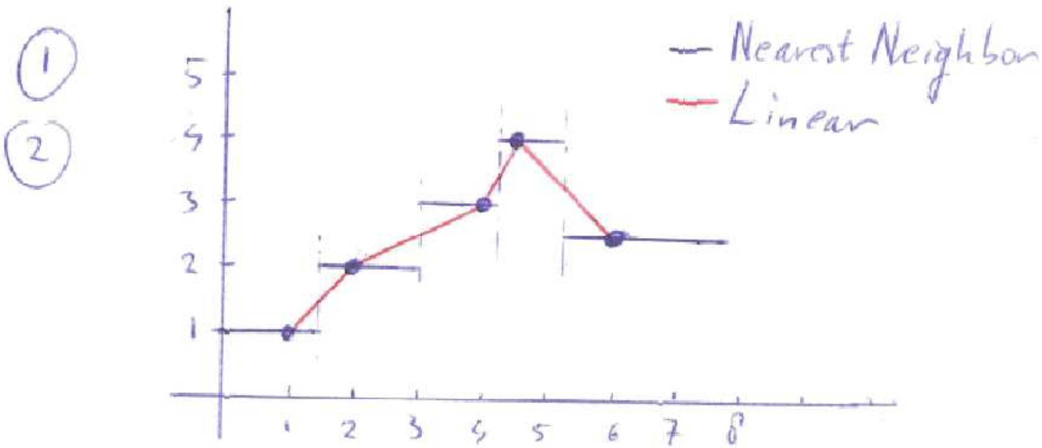
Exercise

Exercise 4.3 Draw a triangulation that is not Delaunay. Why is it not Delaunay?

Answers, Exercise 4.1

c) Following the edges of the triangles in a linear interpolation gives the same result as linear interpolation along the followed line segment





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Answers, Exercise 4.2

1. Criterion: e.g. distance: within radius $R = ..$
2. $p = 0$: $z_p = \frac{230+320+580}{3} = 377$
   $p = 1$: $z_p = 359$;
   $p = 2$: $z_p = 334$;
   $p = 10$: $z_p = 235$;
   $p = \infty$: $z_p = 1/0$, so, doesn't exist;
3. $p = \infty$: nearest neighbour; $z_p = 230$.
4. No, this is not possible. P is outside the convex hull of the observations, so it is notably not in any triangle.

Answers, Exercise 4.3

Start with a Delaunay triangulation and flip one edge.

As long as there are no four points from a configuration of points on a circle, the Delauany triangulation of these points is unique. If you change one edge, there result is therefore not Delaunay anymore.