ta 322D Extra exam 5 Nov. 2013 Solution

1. a) The momentum balance is the same ($\beta = 0$), as is the B.C. at $x = 0$: $\tau = 0$ @ $x = 0$. Thus Eq. 2.2-11 is still valid, but Eq. 2.2-11 does not apply. $\tau_{xz} = \rho g x \cos\beta$

b) In this case, $\cos\beta = 1$. The film start to flow when $\tau_{xz}\big|_{x=\delta} \geq \tau_0$. Thus it start to flow if

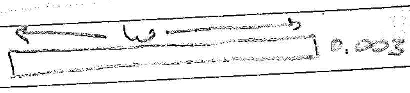
$$\rho g \delta \geq \tau_0 \qquad \text{max } \delta \text{ for no flow: } \delta = \frac{\tau_0}{\rho g}$$

c) In this case, $(1200)(9.8)\,\delta \geq 1.3$

max Thickness of film $= \dfrac{1.3}{1200 \cdot 9.8} = 1.10 \cdot 10^{-4}\,m$ ($\sim 1/10$ mm)

Question dropped from exam

2. What is Re? $Re = \dfrac{D_h V \rho}{\mu}$, $D_h = \dfrac{4(0.003)W}{2W + \cancel{2(0.003)}}$ ~ 0



$D_h = 2(0.003) = 0.006$

$Re = \dfrac{(0.006)(1)\,1000}{0.001} = 6000.$

$Pr = \dfrac{C_p \mu}{k} = \dfrac{4190(0.001)}{0.680} = 6.16.$

What is $\dfrac{T_{02} - T_{b2}}{T_{01} - T_{b1}}$? $\dfrac{40-45}{40-90} = \dfrac{5}{50} = 0.1$. Only valid correlation is BSL Fig 14.3-2, (Eq. 3.120 is for $Re > 10{,}000$; 3.116 for $Re < 2100$)

From BSL Fig 14.3-2, $\ln\left(\dfrac{T_{01} - T_{b1}}{T_{02} - T_{b2}}\right)\left(\dfrac{D}{4L}\right)(Pr)^{2/3} = 0.004$

$\ln(10)\left(\dfrac{0.006}{4L}\right)(6.16)^{2/3} = 0.004 \rightarrow L = 2.90\,m$

or, from chart $Nu\,Re^{-1}\,Pr^{-1/3} = 0.004 \rightarrow Nu = (0.004)(6000)(6.16)^{1/3} = 44.0$

$Nu = \ln\left(\dfrac{T_{01} - T_{b1}}{T_{02} - T_{b2}}\right) Re\,Pr\,\dfrac{D}{4L} \rightarrow 44.0 = \ln(10)(6000)(6.16)\,\dfrac{.006}{4L}$

$L = 2.90\,m.$

3. $\Delta \mathcal{P}$ across the sample: $\mathcal{P}$ at inlet reflects 1 m of water above. sample itself is 10 cm lower at outlet than inlet.

$$\Delta \mathcal{P} = (1.1)(1000)\,9.8 = 10780$$

If we were sure flow is in Darcy regime, we could use Darcy's law. But we don't know that. Thus must use eq. that applies at all Re: Eq. 5.64 in FT, or Eq 6.4-12 in BSL:

$$\frac{\Delta \mathcal{P}}{L} = 150\left(\frac{\mu v_0}{D_p^2}\right)\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{7}{4}\left(\frac{\rho v_0^2}{D_p}\right)\frac{1-\varepsilon}{\varepsilon^3}$$

[The 150 is 170 in the Eq. in FT]

$$v_0 = \frac{Q}{A} = \frac{2\cdot 10^{-5}}{\pi (0.025)^2} = 0.0102$$

$$10780 = 150\left(\frac{0.001(0.0102)}{D_p^2}\right)\frac{(0.65)^2}{(0.35)^3} + \frac{7}{4}\left(\frac{(1000)(0.0102)^2}{D_p}\right)\frac{0.65}{(0.35)^3}$$

$$10780 = 0.01508/D_p + 2.7602/D_p^2 \qquad \text{Mult. by } D_p^2$$

$$0 = 0.01508 + 2.76\,D_p - 10780\,D_p^2 \qquad \text{Quadratic eq.}$$

$$D_p = \frac{-2.76 \pm \sqrt{(2.76)^2 + 4(10780)(0.01508)}}{-2\,(10780)} = \frac{2.76 \pm 25.65}{21560}$$

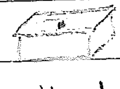
(1.32 mm dia.)

positive root applies: $D_p = 0.00132$

Note $Re = \frac{D_p \rho v_0}{\mu}\frac{1}{1-\varepsilon} = \frac{(0.00132)(1000)0.102}{0.001}\frac{1}{0.65} = 207 \gg 10$

Darcy's law does not apply

4. Because of insulated surface, this is like a cube, 0.5 m on a side, cooled on all sides.

a) Slowest place to cool off is in center on top 

b) Could use chart in BSL (12.1-1) and product method, or FT Fig 3.17. Using BSL, $\alpha = k/\rho C_p = 17/(7820 \cdot 461) = 4.71\cdot 10^{-6}$

$\frac{\alpha t}{b^2} = \frac{(4.71\cdot 10^{-6})(3600)}{(0.25)^2} = 0.271$. Interpolating, from Fig 12.1-1, $\frac{T_1 - T}{T_1 - T_0} \doteq 0.61$ at center. That is for slab. For cube, $\frac{T_1 - T}{T_1 - T_0} = (0.61)^3 \approx 0.23 = \frac{0 - T}{0 - 100}$; $T = 23\,^\circ C$

Using FT chart 3.16, $F_0 = \frac{(4.71\cdot 10^{-6})3600}{(0.5)^2} = 0.068$. $\frac{T_1 - T_c}{T_1 - T_0} = 0.21$. About same.

5. From BSL Sect 9.6 (1st ed.) or FT eq. 3.6,

a)
$$q = \frac{K}{0.002}(T_1 - T); \qquad T_1 = \text{surface } T$$

Energy balance on solid:

$$(0.5)^2(0.25)\rho C_p \frac{dT}{dt} = A \cdot \frac{K}{0.002}(T_1 - T) \qquad A = (0.5)^2 + 4(0.25)(0.5) = 0.75$$

(accum) (heat in through plastic)

$$(0.5)^2(0.25)(7820)(471)\frac{dT}{dt} = 0.75\left(\frac{0.16}{0.002}\right)(T_1 - T)$$

$$2{,}302 \cdot 10^5 \frac{dT}{dt} = 60\,(T_1 - T)$$

$$-\frac{dT}{(T_1 - T)} = 2.606 \cdot 10^{-4}\,dt$$

$$\ln(T_1 - T) = -2.606 \cdot 10^{-4} t + C; \text{ at } t = 0, T_1 - T = 100 \rightarrow C = \ln(100)$$

$$\ln\left(\frac{T_1 - T}{100}\right) = -2.606 \cdot 10^{-4} t$$

for $t = 3600$ s, $(T_1 - T) = 39°$.

b) This process is in series with internal heat conduction. Conduction through the plastic is slower, and therefore controlling. Answer (b) is the better answer (but both processes are probably important).