ta 3220 Final Exam (retake)
27 June 2013
Solution

1. a) Flow is downward, so gravit assists the flow.

$\Delta \mathcal{P} = 9080 + \rho g h = 9080 + 1000(9.8)0.6) = 15058$ Pa

b) <u>If</u> flow is in the low-Re limit, Darcy's law applies.

$K = \frac{D_p^2}{150} \frac{\varepsilon^3}{(1-\varepsilon)^2} = \frac{(0.002)^2}{150} \frac{(0.35)^3}{0.65^2} = 2.706 \cdot 10^{-9}$ m^2 (2706 d)

[ε = porosity; sometimes written φ; 150 factor sometimes replaced by 180.]

Darcy's law: $\frac{Q}{A} = \frac{K}{\mu} \frac{\Delta \mathcal{P}}{L}$

$$\frac{Q}{\pi(0.025)^2} = \frac{2.706 \cdot 10^{-9}}{0.001} \frac{15058}{0.61} = \frac{Q}{1.963 \cdot 10^{-3}} = 6.67 \cdot 10^{-2} = V_o$$

$Q = 1.31 \cdot 10^{-4}$ m^3/s

<u>BUT</u> if we assume Darcy's law, must check that Re condition is satisfied.

$$Re = \frac{D_p V_o \rho}{\mu} \frac{1}{1-\varepsilon} = \frac{(0.002)(6.67 \cdot 10^{-2}) 1000}{0.001} \frac{1}{0.65} = 205 > 10$$

Darcy's law is not accurate. Would have been better at start to use Ergun Eq., which does not assume Re < 10. This is version from BSL; similar version in FT: (p. 234)

$$\frac{\Delta \mathcal{P}}{L} = \frac{150 \mu V_o}{D_p^2} \frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75 \rho V_o^2}{D_p} \frac{(1-\varepsilon)}{\varepsilon^3}$$

$$\frac{15058}{0.61} = \frac{150(0.001) V_o}{(0.002)^2} \frac{(0.65)^2}{(0.35)^3} + \frac{1.75(1000) V_o^2}{(0.002)} \frac{0.65}{(0.35)^3}$$

$$2.469 \cdot 10^4 = 3.695 \cdot 10^5 V_o + 1.327 \cdot 10^7 V_o^2$$

$$1.327 \cdot 10^7 V_o^2 + 3.695 \cdot 10^5 V_o - 2.469 \cdot 10^4 = 0$$

This is a quadratic eq. The positive root is

$V_o = 0.0314$ m/s. $Q = (0.0314)(\pi(0.025)^2) = 6.16 \cdot 10^{-5} \frac{m^3}{s}$

Though this may look small, it's 61.6 ml/s.

2. a) The only difference w/ BSL 10.2 is the final BC., at Eq. 10.2-12 therefore $\boxed{\text{Eq. 10.2-11}}$ is the last eq. that applies.

b) The plan: plug q_r (Eq. 10.2-9) into left side of stated BC., and eq. 10.2-11 for T in right side; solve for C_2.

$$q_r|_{r=R} = \frac{S_e R}{2} = A(T^4 - T_0^4) = A\left\{\left[-\frac{S_e R^2}{4k} + C_2\right]^4 - T_0^4\right\}$$

Then solve for C_2 + plug into eq. 10.2-11.

$$\left\{\left[\frac{S_e R}{2A} + T_0^4\right]\right\}^{1/4} = -\frac{S_e R^2}{4K} + C_2 \; ; \; C_2 = \left[\frac{S_e R}{2A} + T_0^4\right]^{1/4} + \frac{S_e R^2}{4K}$$

$$\boxed{T = \frac{S_e R^2}{4K}\left(1 - \left(\frac{r}{R}\right)^2\right) + \left[\frac{S_e R}{2A} + T_0^4\right]^{1/4}}$$

3. a) What is Re? $Dv\rho/\mu = (0.1)(2)(1000)/(0.001) = 200{,}000$. This is off end of Fig 14.3-2; must use eq. 14.3-16 + Eq. III

$$Nu = (0.026)\, Re^{0.8} Pr^{1/3} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) Re\, Pr\, \frac{D}{4L}$$

$Re = 200{,}000$ $\quad Pr = C_p\mu/K = \frac{(4190)0.001}{0.68} = 6.16$; $\quad D = .1$, $\quad 4L = 80$

$$\ln\left(\frac{60-20}{60-T_{b2}}\right)\cdot(200{,}000)(6.16)\,\frac{0.1}{80} = (0.026)(200{,}000)^{0.8}(6.16)^{1/3}$$

$$\ln\left(\frac{40}{60-T_{b2}}\right)\cdot 1540 = 829$$

$$\ln\left(\frac{40}{60-T_{b2}}\right) = 0.539 \rightarrow T_{b2} = 36.7\,^\circ C$$

b) $Q = w\rho\hat{C}_p(T_{b2} - T_{b1}) = \pi R^2 v \rho \hat{C}_p (T_{b2} - T_{b1})$ (Q out of solid is represented in heating the water passing through)

$Q = \pi(0.05)^2(2)(1000)(4190)(36.7 - 20)$

$= 1.099 \cdot 10^6$ W

c) Here we need Fig 42 from Carslaw + Jaeger. $\alpha = (K/\rho C_p)_{\text{concrete}} = \frac{0.6}{(2000)(2200)}$

$\alpha = 1.36 \cdot 10^{-7}$. $\alpha t / R_w^2 = (1.36 \cdot 10^{-7})(30 \cdot 60^2 \cdot 24)/(0.05)^2 = 142$

$\log[\alpha t/(R_w)^2] = 2.15$. From chart, $\log_{10}\left[\frac{R_w q}{K(T_w - T_0)}\right] = -0.5$

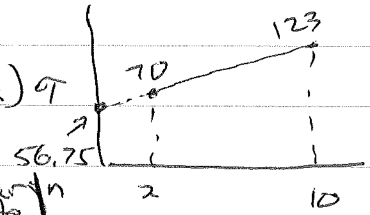
$$10^{-0.5} = \frac{(0.05)\, q_w}{(0.6)(20-60)} \rightarrow q_w = 152 \text{ W/m}^2$$

$Q = q_w 2\pi R L = 152(2\pi)(0.05)(20) = 955$ W

(Incidentally, for $\alpha t/R_w^2 = 142$ From Fig 41 of Carslaw, the well only affects $T_\wedge$ for $\log_{10}(r/R_w) \cong 1.5$, or ~1.6 m)

d) The conduction to the pipes is in series with convective heat transfer in the pipes. Therefore the slower step dominates the overall process. Here that is clearly the step in (c), unsteady conduction.

4 a) If the fluid is a Bingham plastic, τ is linear with shear stress. If τ is 123 (in whatever units) at shear rate 10 (rev/min), and 70 at 2 rev/min, τ is 56.75 Pa at τ = 0. One could estimate this using the linear graph provided. (the units (rev/min) drop out.)



b) If the fluid were a power-law fluid, then shear stress τ is proportional to (shear rate)^n
(the units of shear rate drop out when one takes the ratio.)

$$\left(\frac{10}{2}\right)^n = \left(\frac{123}{70}\right) \rightarrow n \ln(10/2) = \ln(123/70)$$

$$n = 0.35$$

c) One needs to specify another shear rate and what the result would be either way. The clearest difference is for shear rate close to zero: if τ is close to 56.75, it's a Bingham plastic; if it's a power-law fluid, τ would be close to zero.