ta 3220 Final Exam Spring 2013 Solution

1. a) This is unsteady cond. in a flat plate w/ insulated side.

Using FT, p. 130, we want $\frac{T_1 - T_c}{T_1 - T_0} = \frac{20 - 19.5}{20 - 15} = \frac{.5}{5} = 0.1$ *

$\rightarrow F_o \cong 0.26$, $F_o = \frac{\alpha t}{D^2} = \left(\frac{0.68}{1000 \cdot 4190}\right)\frac{t}{(0.002)^2} = (1.62 \cdot 10^{-7})\frac{t}{4 \cdot 10^{-6}} = 4.05 \cdot 10^{-2} t$

↖ 2×0.001 to account for insulated surface

6.40 s

b) We need overall U (heat-transfer coef.) for water layer. Only 1 layer, no convection, $U = \left(\frac{0.001}{0.68}\right)^{-1} = 680$ W/m²K

Macro balance on Al layer: $AU(T_1 - T) = A\Delta x \rho C_p \frac{dT}{dt}$
(note sign is right. $T_1 = 20°C > T \rightarrow dT/dt > 0$).

$(680)(T_1 - T) = (0.005)(2700)(938)\frac{dT}{dt} = -1.266 \cdot 10^4 \frac{d(T_1 - T)}{dt}$

$\frac{d(T_1 - T)}{(T_1 - T)} = -\frac{680}{1.266 \cdot 10^4} dt = \frac{U}{\Delta x \rho C_p} dt$ Note balance is on Al plate. ρ, C_p refer to Aluminum

$\ln(T_1 - T) = -0.0537\, t + C$

at $t = 0$, $(T_1 - T) = 5°C$; thus $C = \ln 5$

$\ln\left(\frac{T_1 - T}{5}\right) = -0.0537\, t = \ln\left(\frac{0.5}{5}\right) = -2.302$

$t = 42.8$ s

c) This is same as pt a, but with different α and D.
We want $\frac{T_1 - T_c}{T_1 - T_c} = 0.1$ $F_o = 0.26 = \frac{\alpha t}{D^2}$ **

$\alpha = \frac{230}{2700 \cdot 938} = 9.08 \cdot 10^{-5} \rightarrow 0.26 = \frac{(9.08 \cdot 10^{-5})t}{(0.010)^2} = 0.908 t$

0.28 s.

d) Slowest step is conduction across the water layer; t ≈ 43 s.
Actual time is longer, because all these steps are in series.
(But probably not much longer, because magnitudes are so different).

* Using BSL Fig 12.1-1 for part (a), we want $\frac{\alpha t}{b^2} \cong 1.0$, $b = 0.001$ m
** " " " (c) " $b = 0.005$

Same final answers.

This problem was motivated by an actual lab apparatus. The conclusion is that if we wait about a minute, it's OK.

2. Initial change sets dimensionless T's: $T_1 = 0°C$, $T_0 = 100$.

At 30 min., it has been 30 min. since first change.

$\alpha = 9.08 \cdot 10^{-5}$ as in problem 1 (c).

After 30 min, $\frac{\alpha t}{D^2} = \frac{(9.08 \cdot 10^{-5})\, 30 \cdot 60}{(1.5)^2} = 0.0726 = F_o$

From FT p. 130, $\frac{T_1 - T_c}{T_1 - T_c} \approx 0.1$; $\frac{T_c - T_0}{T_1 - T_0} = 0.9$

(From BSL Fig 12.1-3, $\frac{\alpha t}{R^2} = 0.29$; $\frac{T_c - T_0}{T_1 - T_0}$ at center $\doteq 0.87$)

It has been 20 min since 2nd change.

$\frac{\alpha t}{D^2} = \frac{(9.08 \cdot 10^{-5})\, 20 \cdot 60}{(1.5)^2} = 0.0484$

From FT p. 130, $\frac{T_1 - T_c}{T_1 - T_e} \doteq .30$; $\frac{T - T_0}{T_1 - T_0} = 0.70$

(From BSL Fig 12.1-3, $(\alpha t / R^2) = 0.193$; $\frac{T_1 - T_0}{T_1 - T_0} \doteq 0.69$)

$\frac{T_c - T_0}{T_1 - T_0} = (0.9) - (0.7) = 0.20$ $\quad T_c = \underline{80°C}$

3. Need a macro balance on sphere, since sphere is always at uniform T.

b) $\underset{\text{heat out (note the sign)}}{-A \sigma e (T^4 - T_0^4)} + \underset{\text{gen.}}{V S} = \underset{\text{accum}}{V \rho C_p \, dT/dt}$ $\qquad A = 4\pi R^2$; $V = \frac{4}{3}\pi R^3$

$-4\pi R^2 \sigma e (T^4 - T_0^4) + (\frac{4}{3}\pi R^3) S = (4/3)\pi R^3 \rho C_p \frac{dT}{dt}$, Divide by $4\pi R^2$

$-\sigma e (T^4 - T_0^4) + \frac{R}{3} S = \frac{R}{3} \rho C_p \frac{dT}{dt}$

a) The final s.s. is when generation = heat lost, and $\frac{dT}{dt} = 0$.

$A \sigma e (T^4 - T_0^4) = V S \rightarrow 4\pi R^2 \sigma e (T^4 - T_0^4) = \frac{4}{3}\pi R^3 S$.

$(T^4 - T_0^4) = \frac{RS}{3} \frac{1}{\sigma e}$; $\quad T = \left[\frac{RS}{3} \frac{1}{\sigma e} + T_0^4\right]^{1/4}$

4. We don't know V or D_h, so this is trial + error.

$D_h = 4 \cdot [1/(1+1+1)] = 4/3$

$Re = D_h V \rho / \mu = (4/3) V\, 1000 / 0.001 = 1.333 \cdot 10^6\, V$

$V = \left(\frac{D_h}{L} \frac{2\, \Delta P}{\rho} \frac{1}{4f}\right)^{1/2}$ $\qquad \frac{\Delta P}{L} = \rho g \cos(87°) = 1000\, (9.8)(0.0523) = 512.9$

$V = \left(\frac{(4/3)}{1000}\, 2\, (512.9) \frac{1}{4f}\right)^{1/2} = (1.368 / 4f)^{1/2}$; $\quad \varepsilon / D = \frac{0.05}{(4/3)} = 0.0375$

Guess $V = 1$ m/s,	$Re = 1.33 \cdot 10^6$	$4f \doteq 0.06$	$V = 4.77$ m/s
4.77	$6.36 \cdot 10^6$	0.06	4.77 m/s

$Q = VA = 4.77\ m^3/s$

5. This problem is analogous to heat transfer in tube flow.

What is Re? $Re = \frac{DU\rho}{\mu} = \frac{(1\cdot10^{-4})(5\cdot10^{-5})1000}{0.001} = 0.005$. Clearly laminar.

Are we relatively close to equil.?

No. $\frac{0 - 0.5C_A^0}{0 - C_A^0} = 0.5 > 0.25$

∴ Can use analog to Eq. 3.124

$Nu = 0.332\, Re^{1/2} Pr^{0.33} \rightarrow Sh = 0.332\, Re^{1/2} Sc^{0.33}$

$Sc = \frac{\mu}{\rho \mathcal{D}} = \frac{0.001}{1000\cdot 10^{-9}} = 1000$; $Sh = 0.332(0.005)^{1/2}(1000)^{1/3} = 0.235$

Using "Eq. IIIm" in notes,

"Nu_{AB}" $= Sh = \ln\left(\frac{0 - C_A^0}{0 - 0.5C_A^0}\right) Re\, Sc\, \frac{D}{4L}$

$0.235 = \ln(2)(0.005)(1000)\frac{10^{-4}}{4L}$

$L = 3.69\cdot 10^{-4}$ m (370 μm)

6. a) In highly turbulent flow, f becomes independent of Re.

$f = \text{const} \rightarrow V = \left(\frac{D}{L}\frac{2\Delta P}{\rho}\frac{1}{4f}\right)^{1/2} = \text{Constant}\left(\frac{\Delta P}{L}\right)^{1/2}$

or $\frac{\Delta P}{L} = \text{Const.} \times V^2$. Doubling $Q \rightarrow \frac{\Delta P}{L}$ rises 4x.

b) For a power-law fluid, Q is proportional to $\left(\frac{\Delta P}{L}\right)^{1/n}$.

Or $\left(\frac{\Delta P}{L}\right)$ is proportional to Q^n. If Q doubles and $\left(\frac{\Delta P}{L}\right)$ rises by 4x, then $n=2$. He thinks he's got a shear-thickening fluid.

7. a) Assume flow is up.

P inlet $= 6\cdot10^5$

outlet $= 2\cdot10^5$

pipe rises, so must subtract effect of gravity.

$\frac{\Delta P}{L} = \frac{(6\cdot10^5 - 2\cdot10^5) - 1000(9.8)100}{1000} = -\frac{5.80\cdot10^5}{1000} = -580$ Pa/m (if flow is up)

b) assumption was wrong. Flow is down, or from right to left.