C. Examples in BSL. 1. Heating in electrical wire

dependence of either the thermal or electrical conductivity need be considered. The surface of the wire is maintained at temperature T_0. We now show how one can determine the radial temperature distribution within the heated wire.

For the energy balance we select as the system a cylindrical shell of thickness Δr and length L. (See Fig. 9.2–1.) The various contributions to the energy balance are

rate of thermal energy in across cylindrical surface at r	$(2\pi rL)(q_r\|_r)$	(9.2–2)
rate of thermal energy out across cylindrical surface at $r + \Delta r$	$(2\pi(r + \Delta r)L)(q_r\|_{r+\Delta r})$	(9.2–3)
rate of production of thermal energy by electrical dissipation	$(2\pi r\,\Delta rL)S_e$	(9.2–4)

The notation q_r means "flux of energy in the r-direction," and $|_r$ means "evaluated at r." Note that we take "in" and "out" to be in the positive r-direction.

We now substitute these three expressions into Eq. 9.1–1. Division by $2\pi L\,\Delta r$ and taking the limit as Δr goes to zero gives

$$\left\{\lim_{\Delta r \to 0} \frac{(rq_r)|_{r+\Delta r} - (rq_r)|_r}{\Delta r}\right\} = S_e r \qquad (9.2\text{–}5)$$

The expression within braces is just the first derivative of rq_r with respect to r, so that Eq. 9.2–5 becomes

$$\frac{d}{dr}(rq_r) = S_e r \qquad (9.2\text{–}6)$$

This is a first-order ordinary differential equation for the energy flux, which may be integrated to give

$$q_r = \frac{S_e r}{2} + \frac{C_1}{r} \qquad (9.2\text{–}7)$$

The integration constant C_1 must be zero because of the boundary condition

B.C. 1: at $r = 0$ q_r is not infinite (9.2–8)

Hence the final expression for the energy flux distribution is

$$\boxed{q_r = \frac{S_e r}{2}} \qquad (9.2\text{–}9)$$

This states that the heat flux increases linearly with r.

We now substitute Fourier's law (see Eq. 8.1–2) in the form $q_r = -k(dT/dr)$ into Eq. 9.2–9 to obtain

$$-k\frac{dT}{dr} = \frac{S_e r}{2} \tag{9.2–10}$$

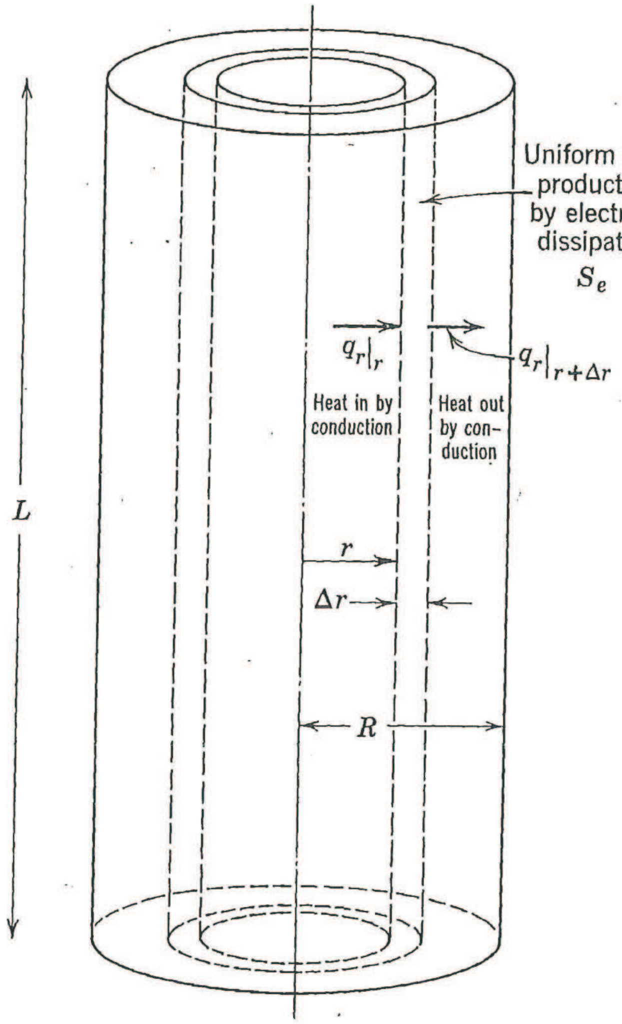

Fig. 9.2–1. Cylindrical shell over which energy balance is made in order to get temperature distribution in an electrically heated wire.

When k is assumed to be constant, this first-order differential equation may be integrated to give

$$T = -\frac{S_e r^2}{4k} + C_2 \tag{9.2–11}$$

The integration constant C_2 is determined from

B.C. 2: at $r = R$ $T = T_0$ (9.2–12)

Hence C_2 is found to be $T_0 + (S_e R^2/4k)$ and Eq. 9.2–11 becomes

$$\boxed{T - T_0 = \frac{S_e R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]} \tag{9.2–13}$$