ta3220 Final Examination
Spring 2010
29 June 2010

Write your solutions *on your answer sheet*, not here. In all cases ***show your work***. To avoid any confusion, state the equation numbers and figure numbers of equations and figures you use. Beware of unnecessary information.

1. a) A given sandpack has permeability of 1 x 10^{-12} m^2. A typical porosity of a sandpack is 0.32. Estimate the size of the grains in this packing.
   b) Suppose water were injected into the packing (note properties below). At what superficial velocity (flow rate divided by cross-sectional area, Q/A) would flow begin to show non-Darcy effects?

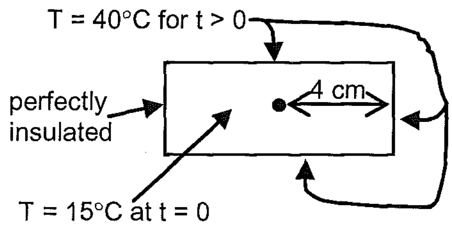
   properties of water

   ρ = 1000 kg/m^3 μ = 0.001 Pa s

   (15 points)

2. A Bingham plastic is injected into a sandstone core of permeability 100 md (10^{-13} m^2) and porosity 0.15. The Bingham plastic has a yield stress τ_o of 40 Pa. What pressure gradient could the Bingham plastic resist without flow in this rock? (For the purpose of this problem only, it is useful to assume at "bundle of tubes" description of the rock. The porous rock is represented by a bundle of tiny cylindrical tubes of radius R = $(8\ k/\phi)^{1/2}$, where k is permeability in m^2 and ϕ is porosity.)
   (10 points)

3. A given rock sample is 4 cm in diameter and 8 cm long. Initially, it is at temperature 15°C. Suddenly, at time t = 0, the cylindrical surface and one flat end are fixed at 40°C. The other flat surface is perfectly insulated. What is the temperature the position indicated by the dot, on the central axis in the middle of the block, after 2 minutes?



   properties of given rock

   ρ = 2270 kg/m^3 C_p = 999 J/(kg K) k (thermal conductivity) = 2.61 W/(m K)

   (15 points)

4. A sphere of diameter 50 cm, with properties below, is initially at 0°C. Starting at time zero the surface is raised to and maintained at 100°C. After 10 minutes the surface is returned to, and maintained at, 0°C. What is the temperature of the center of the sphere 10 minutes after the temperature is returned to 0°C?

   properties of sphere

   k = 34.6 W/(m K) C_p=125.7 J/(kg K) ρ = 11,340 kg/m^3

   (15 points)

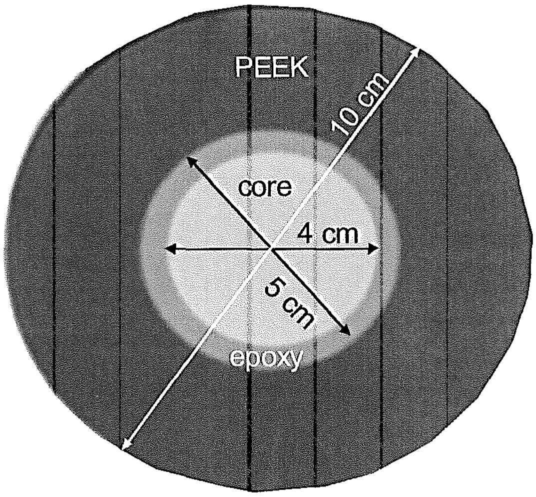
5. I am concerned about heat transfer through an experimental apparatus in the lab. In this apparatus, a 4-cm-diameter core lies inside a layer of epoxy of outer diameter 5 cm. Surrounding the epoxy is a layer of polyetheretherketone (PEEK) with outer diameter 10 cm. The properties of epoxy and PEEK are given below. The outer surface of the PEEK is maintained at T_1 and the outer surface of the core is maintained at T_2. What is the overall heat-transfer coefficient U through the two layers (epoxy and PEEK)?

<u>properties of PEEK</u>

$\rho = 2160\ kg/m^3$ $C_p = 999\ J/(kg\ K)$ $k = 0.249\ W/(m\ K)$

<u>properties of epoxy</u>

$\rho = 880\ kg/m^3$ $C_p = 1000\ J/(kg\ K)$ $k = 0.50\ W/(m\ K)$

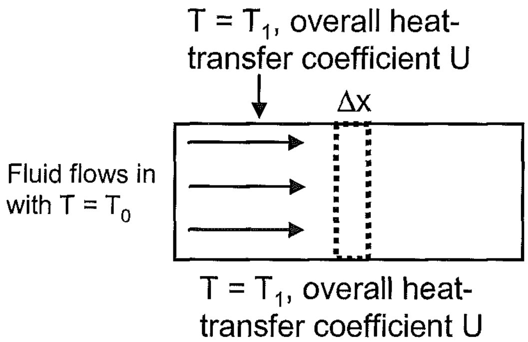


(10 points)

6. a) Fluid flows into a cylindrical porous medium of radius R with superficial velocity (Q/A) and temperature T_0. The fluid has heat capacity C_p, thermal conductivity k and density ρ. The porous medium has the following properties (when dry): porosity ϕ, heat capacity C_{ps}, density ρ_s and thermal conductivity k_s. Surrounding the porous medium is a fluid maintained at a temperature T_1, and the overall heat-transfer coefficient between this outer fluid and the porous medium is U. For simplicity, we make the following assumptions: 1) fluid and solid in the porous medium are at the same temperature at each location; 2) there are no temperature gradients in the porous medium in the radial direction; therefore we can do a shell balance across the medium as illustrated below. 3) Heat conduction in the axial direction can be neglected.

   Derive a differential equation for temperature of the fluid T as a function of position x along the core at steady state. State the boundary condition(s) on this equation.

   b) Solve this equation for T(x).



Note: If you're *very* clever, you may be able to figure out a way to solve this problem using an equation we already derived in class. For this problem, however, I want you to derive the equation yourself from a shell balance.

(25 points)

7. A certain "deviated" well (well drilled at an angle) is 1000 m long, but only 800 m deep. The pipe is of inner diameter 5 cm. The pressure at the bottom of the pipe is 8.1×10^6 Pa; the pressure at the top is 10^5 Pa. The pipe is full of oil of density 800 kg/m^3 and viscosity 10 cp. What is the total potential gradient $\Delta\mathscr{P}/L$ driving the flow? Is flow up or down?

(10 points)