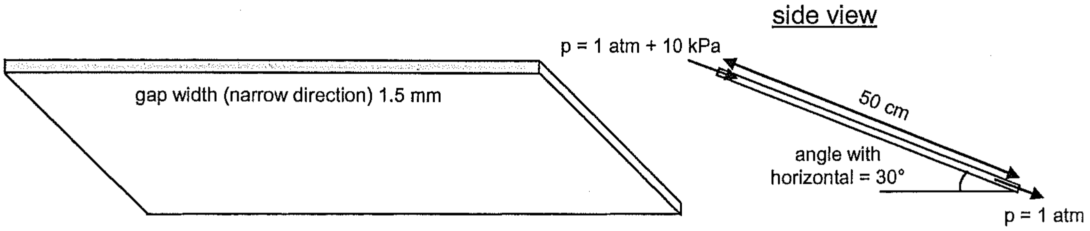
ta3220 Final Examination
Spring 2009
3 April 2009

Write your solutions *on your answer sheet*, not here. In all cases ***show your work.***
Use SI units in your work.

1. A Newtonian fluid flows through a narrow slit with gap width 1.5 mm. The width in the other direction is much greater. The length of the slit in the direction of flow is 50 cm. The slit is held at an angle of 30° to the horizontal. Water fills the slit. The pressure at the inlet is 10 kPa greater than at the outlet. The walls are very smooth.
   a) What is $\Delta\mathcal{P}/L$, the total potential gradient driving the flow?
   b) What is the average velocity of the water in the slit?
   Show your work.
   (20 pts)

<u>properties of water</u>

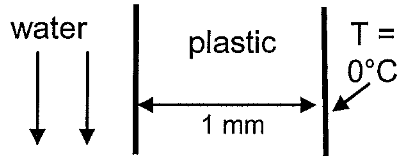
$\rho = 1000$ kg/m^3 $\quad$ $C_p = 4190$ J/(kg K) $\quad$ k = .680 W/(m K) $\quad$ $\mu = 0.001$ Pa s



2. a) Suppose the same slit were filled with a Bingham plastic with the same density as that of water. What minimum yield stress would be required to prevent the Bingham plastic from flowing at all in the slit?
   b) Suppose the slit were filled with a shear-thickening power-law fluid with n = 3. With the slit held at the angle 30° from horizontal, the power-law fluid flows with total flow rate (in m^3/s) of Q_1. Then the potential gradient driving the flow, $\Delta\mathcal{P}/L$, is increased to 1.5 times it original value, and the fluid flows at a rate Q_2. What is this new flow rate Q_2 in terms of the old flow rate Q_1? Assume laminar flow at both flow rates
   (15 pts)

3. Consider again water in the slit as described in problem 1. Now suppose that water flows through the slit at a velocity of 1 m/s. Water flows in with temperature 25°C, and the walls are maintained at temperature 0°C.
   a) What is the heat-transfer coefficient between the flowing water and the wall?
   b) What would be the temperature of water as it leaves the slit after a distance of 50 cm?
   (20 pts)

4. Consider again the situation in problem 3. Actually, the wall does not have temperature fixed at 0°C. Rather, the wall itself is a layer of plastic, 1 mm thick, and on the *other* side of the plastic the temperature is fixed at 0°C. This plastic has a thermal conductivity of 0.25 W/(m K).



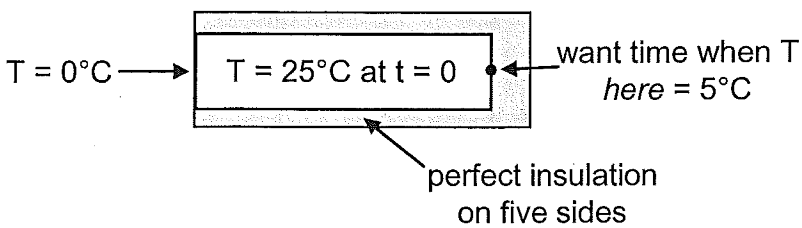
   a) What is the effective heat-transfer coefficient between the flowing water and the other side of the plastic layer?
   b) Which is more important to the heat-transfer process: the plastic layer or the heat-transfer process in the water? Briefly justify your answer.

   If you were unable to complete problem 3, do as much of this problem as you can, and explain how you would use the answer to problem 3 to finish this problem.
   (15 pts)

5. A solid cylinder is in the shape of a long annulus. Its inner surface, at $r = R_o$, is maintained at T_o. The cylinder's outer surface, at $r = R_1 > R_o$, is perfectly insulated. Within the solid, energy is released at a rate S (in units W/m^3). Solve for steady-state temperature within the solid, T(r), in terms of these parameters and other properties of the solid.

   Attached is the derivation of steady-state temperature for a heated cylindrical wire, from BSL. Feel free to use any parts of that derivation that apply to this problem. If you do use any parts of this derivation, state exactly what you are using (e.g., by giving the equation number).
   (20 pts)

6. One way to test for the purity of silver is to test its thermal properties, because it has one of the largest values of thermal conductivity (α, or a in the FT text) among metals. For silver, $\alpha = 1.66\ 10^{-4}\ m^2/s$. Suppose one has a rectangular bar of silver, 20 cm long and 5 cm wide in the other two directions. The bar is initially at 25°C. All the surfaces of the bar are perfectly insulated except end; starting at time t = 0, this surface is suddenly cooled to 0°C. How long would it take the temperature at the center of opposite surface of the bar, 20 cm away from the cooled end, to cool to 5°C?
   (10 pts)



dependence of either the thermal or electrical conductivity need be considered. The surface of the wire is maintained at temperature T_0. We now show how one can determine the radial temperature distribution within the heated wire.

For the energy balance we select as the system a cylindrical shell of thickness Δr and length L. (See Fig. 9.2–1.) The various contributions to the energy balance are

rate of thermal energy in across cylindrical surface at r	$(2\pi rL)(q_r\|_r)$	(9.2–2)
rate of thermal energy out across cylindrical surface at $r + \Delta r$	$(2\pi(r+\Delta r)L)(q_r\|_{r+\Delta r})$	(9.2–3)
rate of production of thermal energy by electrical dissipation	$(2\pi r\,\Delta rL)S_e$	(9.2–4)

The notation q_r means "flux of energy in the r-direction," and $|_r$ means "evaluated at r." Note that we take "in" and "out" to be in the positive r-direction.

We now substitute these three expressions into Eq. 9.1–1. Division by $2\pi L\,\Delta r$ and taking the limit as Δr goes to zero gives

$$\left\{\lim_{\Delta r\to 0}\frac{(rq_r)|_{r+\Delta r}-(rq_r)|_r}{\Delta r}\right\} = S_e r \qquad (9.2\text{–}5)$$

The expression within braces is just the first derivative of rq_r with respect to r, so that Eq. 9.2–5 becomes

$$\frac{d}{dr}(rq_r) = S_e r \qquad (9.2\text{–}6)$$

This is a first-order ordinary differential equation for the energy flux, which may be integrated to give

$$q_r = \frac{S_e r}{2} + \frac{C_1}{r} \qquad (9.2\text{–}7)$$

The integration constant C_1 must be zero because of the boundary condition

B.C. 1: at $r = 0$ q_r is not infinite (9.2–8)

Hence the final expression for the energy flux distribution is

$$\boxed{q_r = \frac{S_e r}{2}} \qquad (9.2\text{–}9)$$

This states that the heat flux increases linearly with r.

We now substitute Fourier's law (see Eq. 8.1–2) in the form $q_r = -k(dT/dr)$ into Eq. 9.2–9 to obtain

$$-k\,\frac{dT}{dr} = \frac{S_e r}{2} \tag{9.2–10}$$

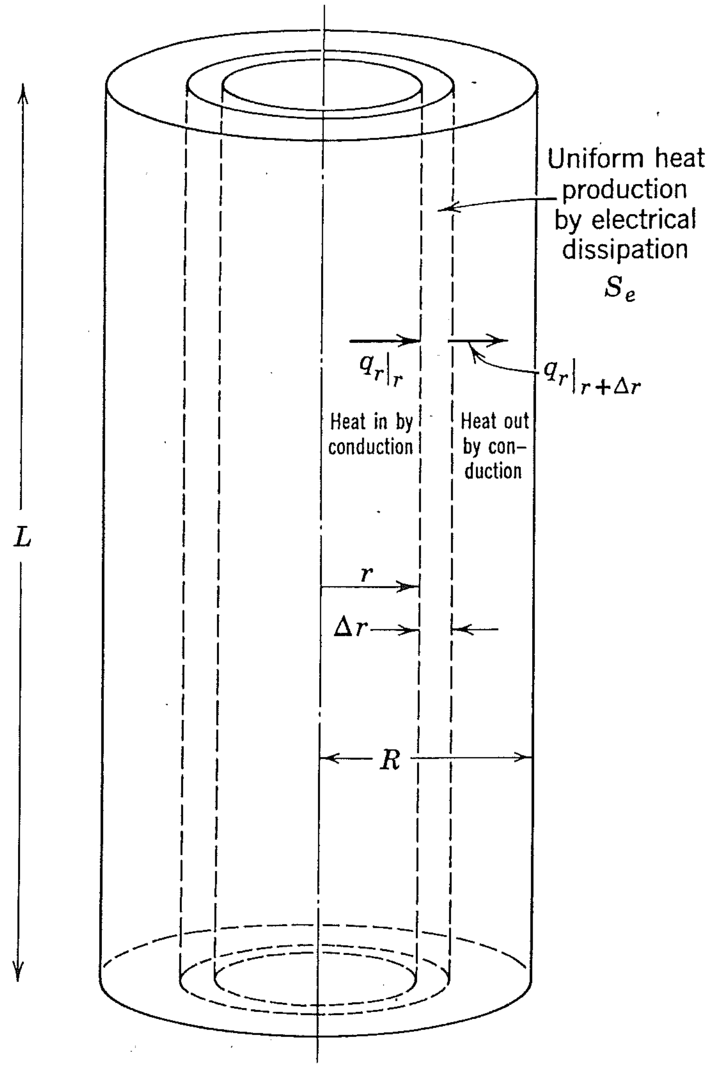


Fig. 9.2–1. Cylindrical shell over which energy balance is made in order to get temperature distribution in an electrically heated wire.

When k is assumed to be constant, this first-order differential equation may be integrated to give

$$T = -\frac{S_e r^2}{4k} + C_2 \tag{9.2–11}$$

The integration constant C_2 is determined from

B.C. 2: at $r = R$ $T = T_0$ (9.2–12)

Hence C_2 is found to be $T_0 + (S_e R^2/4k)$ and Eq. 9.2–11 becomes

$$\boxed{T - T_0 = \frac{S_e R^2}{4k}\left[1 - \left(\frac{r}{R}\right)^2\right]} \tag{9.2–13}$$