VII. Heat transfer coefficients

A. Review General definition of h

$$Q = h \cdot A \cdot \Delta T$$

total rate of heat transfer; heat transfer coefficient; characteristic area; characteristic temperature difference

B. Flow in tubes

1. Heat transfer between wall and fluid

BSL(K) section 14.1, 14.2

$$Q = h_{ln}(\pi D L)(T_0 - T_b)_{ln} \qquad \text{Eq. I}$$

$$= h_{ln}(\pi D L)\frac{(T_{01} - T_{b1}) - (T_{02} - T_{b2})}{\ln((T_{01} - T_{b1})/(T_{02} - T_{b2}))} \qquad \text{BSLK eq. 14.1-4}$$

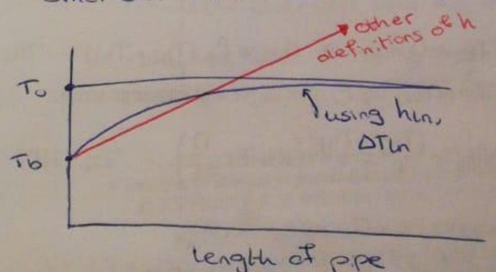
T_{b1} = fluid T entering ; T_{b2} = fluid T leaving

T_{01} = wall T entrance; T_{02} = wall T exit

units: Q = watts

h_{ln} = $W/(m^2 K)$

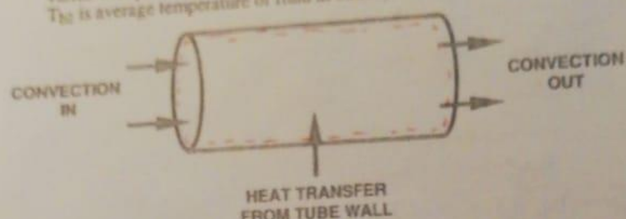
- Don't pay attention to other definitions of h in BSLK sect. 14.1
  other definitions lead to unphysical situations



2. Energy balance for flow in tube

a. macroscopic balance

(Can use macroscopic energy balance because don't care about details of how T varies with position within the tube. T_{b1} is average temperature of fluid at inlet; T_{b2} is average temperature of fluid at outlet.)



Define system as fluid within pipe. Assume fluid is incompressible. Therefore average velocity v is same at inlet and outlet.

Terms in energy balance:

- convection of thermal energy in: $\underset{\text{area}}{(\pi R^2)}\, v \rho \hat{C}_p T_{b1}$
  Define mass rate of flow $w \equiv (\pi R^2) \rho v$;
  then convection in $= w \hat{C}_p T_{b1}$
- convection of thermal energy out: $w \hat{C}_p T_{b2}$
- heat transfer from walls: Q $Q>0 \rightarrow$ heat flows in
- No accumulation at steady state - T does not change with *time* at any fixed location in the pipe (though T does vary with *position* along the pipe).
- No heat generation within fluid.

<u>Energy balance</u>:

$$w \hat{C}_p T_{b1} - w \hat{C}_p T_{b2} + Q = 0; \quad \text{or} \quad Q = w \hat{C}_p (T_{b2} - T_{b1}) \quad - \quad \text{"Eq. II"}$$

Combine with definition of h_{ln} ("Eq. I") from above and rearrange terms:

$$\frac{h_{ln} D}{k} \equiv Nu_{ln} = \frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{ln}} \left(Re\ Pr \frac{D}{4L} \right) \quad - \quad \text{"Eq. "III"}$$

Note: *IF* T_0 is uniform along tube ($T_{01} = T_{02} \equiv T_0$), then

$$\frac{(T_{b2} - T_{b1})}{(T_0 - T_b)_{ln}} = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) = \frac{\Delta T \text{ at inlet}}{\Delta T \text{ at outlet}}$$

- In ChE 383, for simplicity, we usually assume $T_{01} = T_{02} \equiv T_0$.
- Eq. "III" is derived from energy balance, and therefore represents conservation equation in solution of heat-transfer problems in tubes.

3. Correlations for h_{ln}

As with friction factors, correlations for h_{ln} relate dimensionless groups:

$(Nu)_{ln} \equiv \frac{h_{ln} D}{k}$ ← dia. of pipe ← thermal conductivity of liquid "Nusselt no."

$Re = \frac{DV\rho}{\mu} = \frac{DG}{\mu}$

$Pr = \left(\frac{\hat{C}_p \mu}{k}\right)$ ↑ fluid "Prandtl no." $= \left[\frac{(\mu/\rho)}{\alpha}\right]$

Correlations for Nu_{ln} are transport "laws" - one combines these correlations with the conservation eq., "Eq. III," to complete the solution.

The primary correlation is BSLK 14.3-2

Note:

- similarities to f(Re) chart

  $(Nu)_{ln}$ correlation with Re

  general slope

- differences with f(Re) chart

  - multiple curves at low Re, depend on L/D
  - only one curve in highly turbulent regime (strictly, this is a chart for 'smooth tubes')

  ... *because correlation is only for smooth pipes*

- solution in laminar region is from analytical solution to pde derived in BSL Sect. 10.8; in turbulent region, based on experimental results

  BSLK Sect. 10.7

- third label for the vertical axis of Fig. 14.3-2 *combines transport law and conservation eq:* thus can solve problem directly

- although $[(h_{ln} D/k) Re^{-1}]$ declines with increasing Re across much of chart, h_{ln} increases monotonically with increasing Re. *Increasing convection and turbulence increases h_{ln}.*

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4. using the correlations for Nu_{ln}

<u>Re < 2100</u>

a) ~~Is wall temperature roughly uniform?~~

- ~~no - must use differential equation for T_b with h_{loc}~~
  - ~~any correlation below (Eq. 14.3-17, Fig. 14.3-2 or Fig 14.2-1 "const. wall T (tube)" curve) is OK for h_{loc}~~
  - ~~NOT COVERED IN ChE 3~~
- ~~yes - continue to b~~

b) Re Pr D/L >> 10? (Or $[(T_{02}-T_{b2})/(T_{01}-T_{b1})] > 0.2$?)

(in other words, is T relatively <u>far</u> from equilibrium at outlet?)

- <u>yes</u>: then can solve in any of 3 ways:
  - use Fig 14.3-2 from BSL → ~~Eq~~ BSL2 or BSLK 14.3-17
  - Eq. ~~14.3-17~~ and Eq. "III;" or Eq. 3.150
  - Fig. 14.2-1 "const. wall T (tube)" curve and Eq. "III"
- <u>no</u>: then can use only Fig. 14.2-1 "const. wall T (tube)" & Eq "~~III~~" <u>III</u>

<u>Re > 2100</u>

Use Fig. 14.3-2 (or 10,000) BSL2 or BSLK 14.3-16

If Re > 20,000 can also use Eq. ~~14.3-16~~ and Eq. "III"

<u>*Non-cylindrical Tubes*</u>: Use "hydraulic radius" approximation

Same approach as in friction-factor problems

Substitute $4R_h = 4\,(S/Z)$ for D

Calculate <v> for given tube shape

Calculate h or T as for circular tube

- *VALID ONLY FOR TURBULENT FLOW*

<u>*Note:*</u> for *any* Re, if $T_{01}=T_{02}\equiv T_0$, then

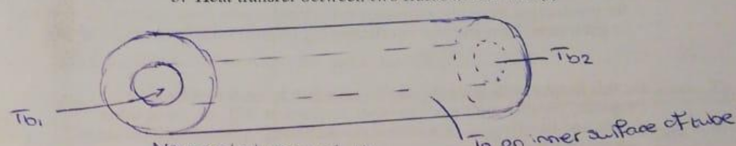
$(T_{b2}-T_{b1})/(T_0-T_b)_{ln} = \ln [(T_0-T_{b1})/(T_0-T_{b2})]$

in this course, we'll always assume $T_{01} = T_{02} = T_0$ [uniform wall T_0]

5. Additional notes

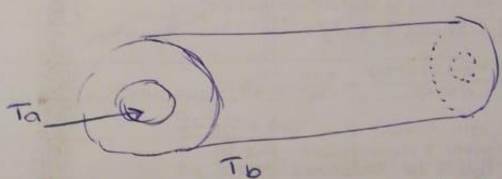
a. Don't sweat the other correlations for h_{ln} in BSL Section 14.2, 14.3

5. Heat transfer between two fluids across wall(s)



Nomenclature in ch.14

Call T of fluid outside T_f



Nomenclature in ch. 9 BSL1

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b. h_{ln} corresponds to h_o of BSL2 Sect. 10.6 (BSL 1 Sect 9.6; BSLK Sect 10.3)
- i.e., heat-transfer coefficient for Newton's law on inner surface of tube

- Can combine $h_{ln} = h_o$ derived with methods of BSL ch. 14 with formula for conduction through composite cylindrical layers from Section 10.6 to get overall heat-transfer coefficient U_o for heat transfer through tubes (BSL 1 9.6)

- (in this course, still have no way to calculate h_3 on the outer tube surface; for this problem, see other sections of BSL ch. 14 or other texts and handbooks. Principles are still same: greater convection and increasing turbulence increase h_3 as they do $h_{ln} \equiv h_o$)

c. for h for packed beds, see BSL Sect. 14.5 (BS L2 or K) - not covered in AESB 23

d. Eq. III describes heat transfer between the inner of surface of a tube (assumed to be at fixed T_0) and fluid within the tube.

What if instead have uniform T_f *of fluid outside* the tube?

Use eq. 9.6-31 BSL1 to get U_o with $h_{ln} = h_o$

BSLK 10.3-30

BSL2 Eqs. 10.6-29 through 31 apply, with

"T_b" replacing "T_a" in eq. 9.6-30 (fluid temp. inside tube)

"T_f" replacing "T_b" (fluid temp. outside tube)

calculate h_{ln} ($\equiv h_o$) from methods in Ch. 14

h_3 must be given in problem statement

$\rightarrow U_o$

e. Eqs. 10.6-29 to 31 give U_o; then how to calculate fluid T along tube?

Conservation equation is modified form of Eq. III:

From eq. 9.6-31 (BSL 1)

$$\left[\frac{U_o D_o}{k}\right] = \ln\left(\frac{T_f - T_{b1}}{T_f - T_{b2}}\right) Re\, Pr \frac{D_o}{4L}$$

k thermal conductivity inside tube

internal diam. tube

T fluid entrance

T fluid exit

T fluid outside tube

6. Example problems

a. BSL ex. 14.3-1

[illegible]. Final notes on heat transfer

The mass and energy balances in AESB2320 ~~PGE 383~~ and in thermodynamics are fundamentally the same, though they may look different.

In AESB2320 ~~PGE 383~~, we focus attention on the "Q" term, and for simplicity, leave out the following complications

- useful work output
- phase changes. assume ~~eno~~ energy/volume is $\rho \hat{C}_p T$ with constant $\rho \hat{C}_p$; excludes heat of vaporization, accumulation

- thermodynamics addresses these two complications, without devoting so much attention to the "Q" term.

In the real world, you'll need handle situations with all factors simultaneously