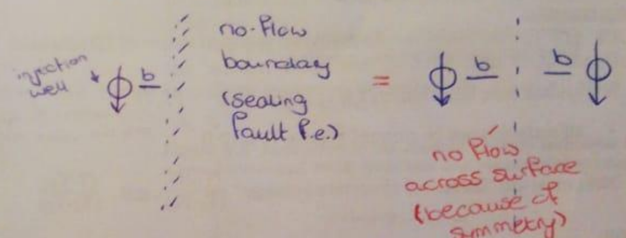
## C. Extending the 1D solutions

### 1. zero-flux boundary conditions (such as perfectly insulated boundaries)

**a. principle:**

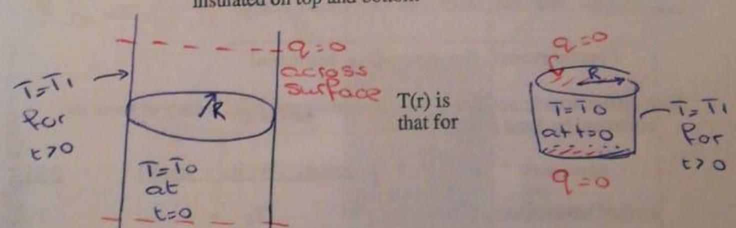
If tabulated solution (chart) has any surface for q=0 at ~~all~~ t, then one can replace that surface with a ~~pose~~ perfectly insulated boundary

**b. analogy to "method of images" in reservoir engineering**

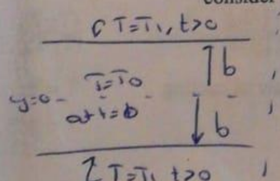



**c. Specific examples**

**i. opposide flat sides perfectly insulated (q=0 across surface)**
for example, finite cylinder
insulated on top and bottom

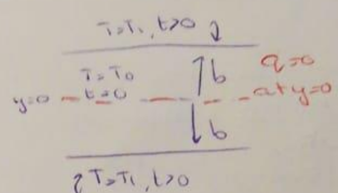



consider how one could extend solution for finite-width slab . . .

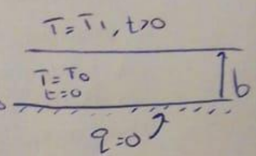



any slice perpendicular to slab has q=0

ii. one flat side perfectly insulated ($q=0$ across surface)




T(r) is that for y=0




2. **"Orthogonal conduction"** - see attached handout from F. M. White, *Heat Transfer*, 1984.

Requirements:

- system shape must be orthogonal intersection of shapes of 1D tabulated solutions (see handout for examples)
- system must be a uniform $T = T_0$ at time $t = 0$
- all surfaces must be changed to same $T_1$ at $t \geq 0$

except for perfectly insulated surfaces as handled above

Note must use "unrealized temperature change" $\frac{(T_1-T)}{(T_1-T_0)}$, *not* $\frac{(T-T_0)}{(T_1-T_0)}$

Then:

$\frac{(T_1-T)}{(T_1-T_0)}$ for any position in 3D solid is the product of $\left(\frac{(T_1-T)}{(T_1-T_0)}\right)_i$ values for the corresponding position in each of the 1D solutions.

! ! Note: must use "unrealized temperature change" $\frac{(T_1-T)}{(T_1-T_0)}$, *not* $\frac{(T-T_0)}{(T_1-T_0)}$ ! !

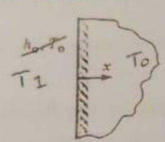
**Please reread the preceding sentence!**

Note differences in notation between handout and BSL. Many of these are corrected by hand in your handout:

| | handout (White's book) | BSL |
|---|---|---|
| surface temperature | $T_0$ | $T_1$ |
| initial temperature | $T_i$ | $T_0$ |
| dimensionless temperature | $\theta \equiv \frac{(T-T_0)}{(T_i-T_0)}$ | $\frac{(T_1-T)}{(T_1-T_o)}$ |
| B.C. at surfaces for $t > 0$ | Newton's law of cooling | $T = T_1$ |

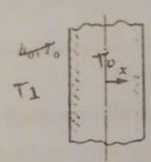
**Table 4.2** Multidimensional sudden immersion solutions as a product of one-dimensional results

*0. Basic Solutions*




$S(x, t) = 1 - \Theta$
Eq. (4.30)

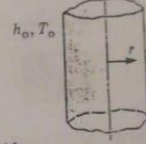
Semi-infinite slab




$P(x, t)$
Eq. (4.36)

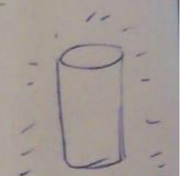
Finite-width slab

etc. ...




$C(r, t)$
Eq. (4.42)

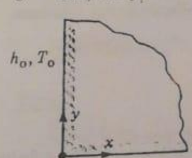
Conduction within cylinder



Conduction outside cylinder

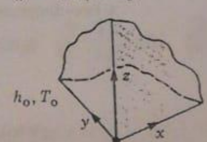
*1. Two-Dimensional Corner Region*

$\Theta = S(x, t)S(y, t)$



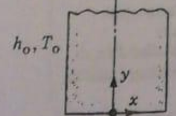

*2. Three-Dimensional Corner Region*

$\Theta = S(x, t)S(y, t)S(z, t)$



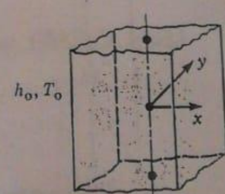

*3. Semi-Infinite Plate*

$\Theta = P(x, t)S(y, t)$




*4. Infinite Rectangular Bar (origin at centerline)*

$\Theta = P(x, t)P(y, t)$



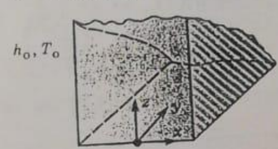
: (Cont.)

*nfinite Rectangular rigin in the center of ttom face)*

$(x, t)P(y, t)S(z, t)$

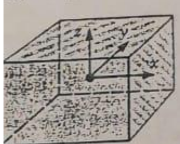
*6. Finite-Width Corner Region (origin in the center of the corner)*

$\Theta = P(x, t)S(y, t)S(z, t)$



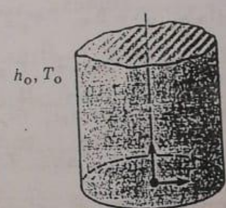

*ape (origin at exact of box)*

$(x, t)P(y, t)P(z, t)$



*8. Semi-Infinite Cylinder (origin at center of bottom face)*

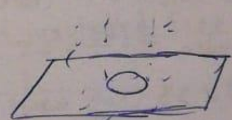
$\Theta = S(x, t)C(r, t)$



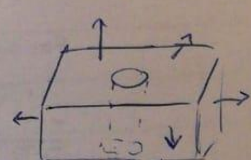

*Length Cylinder (ori- exact center)*

$(x, t)C(r, t)$

10. Cylindrical hole in semi-infinite slab



11. Finite-width slab with cylindrical hole in it



XII. C. 3 example problems in
Unsteady Heat conduction in Solids & Product Solutions

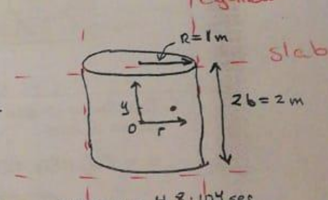
Example 1

0.1 m dia steel ball, w/ $k = 16.3\ W/(m\cdot K)$, $\rho = 7700\ kg/m^3$, $\hat{C}_p = 500\ \frac{J}{kg K}$.
initial $T \equiv T_0 = 300\ K$. At time $t \geq 0$, surface temperature is $T_1 = 600\ K$.
How long until center rises to 500 K?

solution: compute $\alpha \equiv \frac{k}{\rho \hat{C}_p} = \frac{16.3}{(7700)(500)} = 4.23 \cdot 10^{-6}\ m^2/s$

want $\frac{T_1 - T}{T_1 - T_0} = \frac{600-500}{600-300} = 0.33$

From Fig 12.1-3, for $r/R = 0$ (center of sphere), $\frac{T_1-T}{T_1-T_0} = 0.33$ for
$\frac{\alpha t}{R^2} \cong 0.18 \rightarrow t = (0.18)\frac{(0.05)^2}{4.2\cdot 10^{-6}} = 106$ (Note $r = \frac{0.1}{2}$)




Example 2

Cylinder of dimensions shown. Note height of cylinder corresponds to 2 × (half-width of slab, b) = 2b. $T_0 = 300\ K$, $T_1 = 600\ K$.
$\alpha = 4.2 \cdot 10^{-6}$ Want T at point shown, at $t = 800\ min = 4.8 \cdot 10^4$ sec
0.2m up and 0.5m out from center of cylinder. ($y = .2\ m$, $r = .5\ m$. Note origin is in center of solid.)

From handout, Table 4.2, $\frac{T_1 - T}{T_1 - T_0}$ (*) is given by item 9 (p. 185); it is product of P, solution for finite-width slab (or "plate"), and C, solution for cylinder.

For plate, (finite-width slab) $\frac{y}{b} = \frac{0.2}{1} = 0.2$; $\alpha t / b^2 = \frac{(4.2\cdot 10^{-6})(4.8\cdot 10^4)}{(1)^2} \cong 0.2$, From Fig 12.1-1,
$P \equiv \frac{T_1 - T}{T_1 - T_0} \cong 0.74$ (note P is given by scale on <u>right</u> of Figure)

For cylinder, $\frac{r}{R} = \frac{0.5}{1} = 0.5$; $\alpha t / R^2 = \frac{(4.2\cdot 10^{-6})(4.8\cdot 10^4)}{(1)^2} \cong 0.2$. From Figure 12.1-
$C \cong 0.33$ (note again we use scale on right.)

$\frac{T_1 - T}{T_1 - T_0} = (0.74)(0.33) \cong 0.25 \rightarrow T \doteq 525\ K$

* this is equivalent to the defs given on the handout on p. 126; they use d