4. steady 1D conduction in a rectangular solid with no generation

- on preceding page

a. an aside: momentum-transport analog to BSL Sect. 10.4 (BSL1e 10.8)

BSL Sect. 10.4	analog in momentum transport

geometry

rectangular

Energy balance:

convection

in = out; cancels

molecular transport

Fourier's law

generation

uniform Sv

accumulation

none (steady state)

boundary conditions

$T = T_b$ at $x = b$

$T = T_o$ at $x = 0$

momentum-transport problem:

4b. "conduction with a viscous heat source (BSL 1[st] ed. Sect. 9.4)

5. **conduction through composite walls** (BSL sect. 10.6) (BSLK 10.3)
BSL7 sect. 9.6

a. **Initial notes**

suppose there are n layers

each layer has different properties (e.g., thermal conductivity k)

- ∴ need separate shell balance on each layer
- → n second-order differential equations for T
- need 2n boundary conditions

Boundary conditions:

T continuous at contacts between layers	(n−1)
q continuous at contacts between layers	(n−1)
$q = h \, \Delta T$ at both outer surfaces	2
Total:	2n

b. **rectangular layers** (BSL² section 10.6) BSL7 9.6.
BSLK. 10.3

i. **what if T_o is specified instead of T_a?**

Replace T_a by T_o in equations 9.6-15 & 16
delete the term $\frac{1}{h_o}$ in eqs. 9.6-15, 17

ii. **analogy to Darcy flow in layered rock**

c. cylindrical layers (BSL Ex. 10.6-1) BSL1 9.6-1

i. analogy to radial Darcy flow

d. spherical layers

energy balance:

$$(\text{flux x area})|_r \quad - \quad (\text{flux x area})|_{r+\Delta r} \quad = \quad 0$$

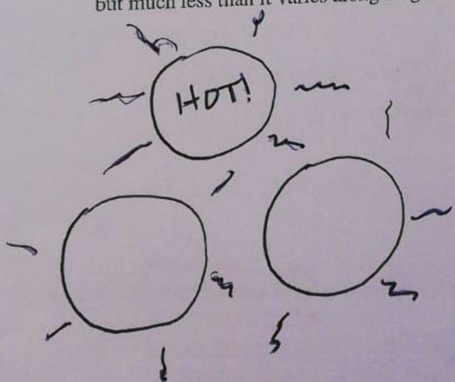
$$q_r(4\pi r^2)|_r - q_r(4\pi r^2)|_{r+\Delta r} = 0$$

for a more-complicated example in spherical geometry, see BSL Sect. 10.3

6. Conduction and convection in a porous medium (BSL Sect. 10.5)

a. simplifying the problem

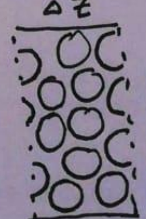
complex reality: T differs slightly between solids and nearby fluid, but much less than it varies along length of porous medium



Heat is released b
reaction at s
surface
Solid is hotter tha
surrounding flu
T varies with
around each
and with p
z along pack
Heat transfer fro
to fluid involves
tion, and may
lence, ra

simplified model: assume T is uniform between catalyst and fluid at any given value of z; T = T(z)

Δz



(BSL1 Sect 9.5)

b. setting up differential equation (BSL 2, Sect. 10.5) (BSLK 10.7)
(details of solution of differential equation not important)

7. Conduction in spherical geometry (BSL 2, Sect. 10.3) - BSL 1 Sect 9.3 Not in BSLK

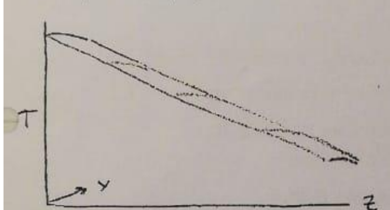
8. Conduction in a (very thin) cooling fin (BSL Sect. 10.7) BSL 1 Sect 9.7; BSLK Sect. 10.7

a. simplifying the problem

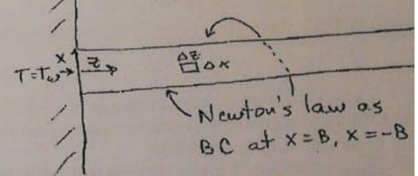
complex reality: $T = T(x,z)$. Newton's law of cooling is B.C. at $x = \pm B$.

Variation of T in x direction is small compared to variation along z direction.

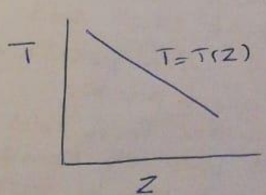
T = T(x, z)

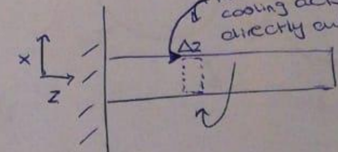


control volume thin in x and z directions



simplified model: assume uniform T across fin for any given z: $T = T(z)$. Control volume now extends across fin, from $x = B$ to $x = -B$. Newton's law of cooling now applies at surface of control volume, *enters energy balance directly.*





b. setting up differential equation (BSL Sect. 10.5) BSL1 9.5; BSLK 10.7
(details of solution of differential equation not important)

c. **an aside: how to distinguish terms in energy balance from boundary conditions.?**
Terms in energy balance are governed by definition of control volume

- fluxes across system boundary
- generation + accumulation within control volume

If a physical constraint does not apply directly to control volume, it is probably a boundary condition

(BSL1 9.7, BSLK 10.5)
In Section 10.7, control volume extends to surfaces ($x = \pm B$) where Newton's law of cooling applies. Therefore, in this problem, Newton's law of cooling enters energy balance

(BSL1 9.5 9.7)
d. **notes on examples 6 and 8** (BSL sects. 10.5 and 10.7)
(BSLK 10.7 10.5)

some surprises:

- control volume can contain solid and fluid (example 6)
- Newton's law of cooling can enter energy balance directly, not as boundary condition to dif. eq. (example 9)

key is definition of control volume

control volume "shell" is thin in direction in which T varies significantly

- if solid and fluid are at same T, can include both in control volume
- if T does not vary much with x, control volume can extend to $\pm B$
  - if control volume extends to surface where Newton's law applies, then N law enters energy balance and dif. eq.

one can't just memorize how to handle each type of term in balance; must

- define control volume properly
- consider how each process interacts with interior + boundaries of control volume

9. **forced convection in laminar flow in a tube** (BSL[2] sect. 10.8) BSL1 9.8;
(details of method of solution of partial differential equation not important; only setting up differential equation from shell balance)

Advance of a Thermal Front in a Geological Formation

Suppose a geological formation, and the fluid in it, are at temperature T_0. The formation has porosity ϕ; the rock grains have density ρ_g and heat capacity (per unit mass) C_{pg}. The fluid in the formation has density ρ_f and heat capacity C_{pf}. The layer has height H and width W and is sealed to flow above and below and on the sides.

A fluid at temperature T_1 is injected from one end. For simplicity, assume $T_1 < T_0$: cold water is injected into a warm formation. We want to know how fast the temperature front penetrates the formation. For instance, in a geothermal project, hot water is pumped out of a formation, cooled down, and reinjected: how long would it take the cold water to reach the production well (and end the process)?

We can solve for the velocity of the cold-water front using a shell balance. But our assumptions and approach are quite different from the other shell-balance heat-transfer problems we've solved so far. Up to now, the control volume has been fixed in place. There might or might not be convection involved, but heat conduction always plays a key role. So far all our cases have assumed steady state.

For this problem we make different assumptions. First,

neglect heat conduction (Fouriers law)

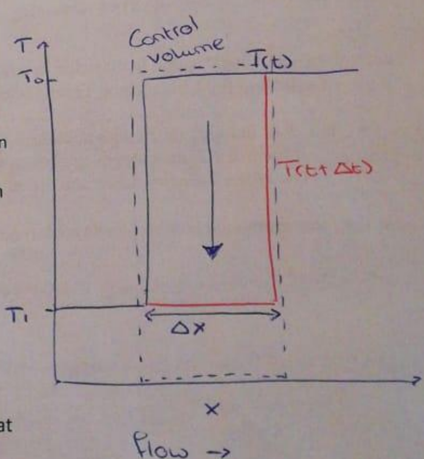
As in the case of flow through the catalyst bed (BSL1 Sect. 9.5; BSLK Sect. 10.7), we assume

At any location, rock + fluid are at the same T as each other

(As mentioned in class, we will show later in the quarter that this is an excellent assumption for porous geological formations.)

For now we assume a rectangular geometry. Fluid flows in the x direction with volumetric flow rate Q. The formation has width Y (perpendicular to the page) and height Z. There is no heat conduction to the rock layers above and below. With no heat conduction between hot and cold regions, along the flow direction, there is a sharp front between the regions of the reservoir with $T = T_1$ and T_0.

Suppose at time t the cold front is at position x. At some time later, the cold front will be at position $(x + \Delta x)$. (The point of the exercise is to solve for Δx) We consider a control volume with one boundary *just* behind x, and the other boundary *just* ahead of $(x + \Delta x)$. Note that cold fluid (T_1) flows in at x and hot fluid (T_0) flows out at $(x + \Delta x)$ throughout this period form t to $(t+\Delta t)$.



Perform now an energy balance on this control volume over the period from t to (t + Δt).

Heat in: $(Q\rho_f C_{pf} \cdot T_1)\Delta t$

Heat out: $(Q\rho_f C_{pf} T_0)\Delta t$

Accumulation: this is the change in the energy of the fluid and the rock over the distance Δx.

$$\underbrace{HW\Delta x}_{\text{volume}}\left(\varphi\rho_f C_{pf} + (1-\varphi)\rho_g C_{pg}\right)(T_1 - T_0)$$

We ignore conduction and generation. Combining terms in the energy balance,

$$Q\rho_f C_{pf}(T_1 - T_0)\Delta t =$$
$$HW\Delta x\left(\varphi C_{pf}\cdot\rho_f + (1-\varphi)\rho_g C_{pg}\right)(T_1 - T_0)$$

The velocity of the cold-fluid front is

$$\frac{\Delta x}{\Delta t} = \left(\frac{Q}{HW}\right)\cdot\frac{\rho_f C_{pf}}{\varphi\rho_f C_{pf} + (1-\varphi)\rho_g C_{pg}}$$

"superficial" or "Darcy" velocity Q/A, u

Note that this is slowed down from the velocity of the fluid.
The fluid itself is traveling with a (interstitial) velocity of

$$\frac{u}{\varphi}$$

The cold-fluid front is slowed down because

injected fluid must heat (or cool) rock as well as fill the pore space

This is good news for geothermal processes. Much of the heat that is being extracted from t formation was originally in the rock, not in the fluid itself

Consider a formation with porosity $\phi = 0.15$, grain density $\rho_g = 2500$ kg/m^3, and heat capacity $C_{pg} = 850$ J/(kg K). Assume water properties are $\rho_f = 1000$ kg/m^3 and $C_{pf} = 4150$ J/(kg K).

The water itself travels with a velocity $\frac{u}{0{,}15} = 6{,}67u$

The cold-temperature front travels with a velocity

$$u \cdot \frac{1000 \cdot 4150}{0{,}15 \cdot 1000 \cdot 4150 + 0{,}85 \cdot 2500 \cdot 850} = 1{,}70u$$

which is $\frac{1{,}7}{6{,}67} \approx 0{,}25$ times the (interstitial) velocity of the fluid itself.

This principle applies in many subsurface flow applications where fluids interact with the minerals and absorb or leave material or energy behind. Injected chemicals that interact with the rock, heat, geochemical reactions ... In all these cases, the advance of a thermal or chemical front can be very different from the velocity of the injected fluid that is causing the change in the formation.

It also explains in part why hot water can be slow to arrive at a cold shower head. First, of course, the hot water from the heater has to displace the cold water initially in the pipe. Beyond that, it loses heat to the cold pipe itself. The hot water is not at instantaneous equilibrium with the pipe at a given location, as we assume for porous rock, but still the hot water at the hot-temperature front loses its heat to the cold pipe. As a result, *hot* water arrives at the shower later than the water itself.

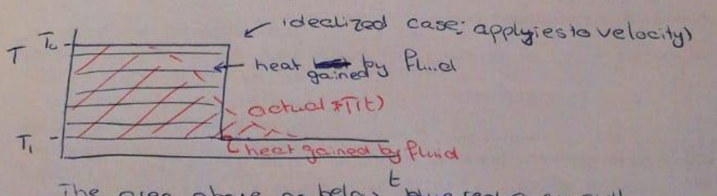
Note that in this shell-balance problem we made assumptions very different from those made in the cases in BSL1 (ch. 9) and BSLK (ch. 10). A key lesson is

check problem statement for assumptions to be made.

Don't assume that the next problem is like the previous one

In reality, the sharp step-change in T assumed here is smeared out by heat conduction and non-uniform velocity in different layers (injected fluid going faster in some layers than others), followed by heat conduction between the layers. One can reconstruct where the front should have been had it stayed sharp as follows:

In a ~~plot~~ plot of T(t), the area above (or below) the ~~pl~~ plot represents the energy absorbed (or released) from fluid



The area above or below ~~blue~~ red curve still represents energy ~~gained or~~ lost or gained by fluid

- equate area to rectangle - time for rectangle fits equation