TM 4780 ta re-sit spring 2014

25 June 2014

1. Since T stays uniform at all times we can use a macro balance.

$$-(2\pi R L + \pi R^2 \cdot 2)\, h\, (T - T_a) = \pi R^2 L \rho C_p \frac{dT}{dt}$$

$$\frac{dT}{T - T_a} = \frac{d(T - T_a)}{T - T_a} = -\frac{(2\pi R L + 2\pi R^2) h}{\pi R^2 L \rho C_p} \equiv -K$$

$\ln(T - T_a) = -Kt + C$; at $t = 0$, $T = T_0$; $C = \ln(T_0 - T_a)$

$$\left(\frac{T - T_a}{T_0 - T_a}\right) = \exp(-Kt)$$

$$K = \frac{(2\pi(0.025)(0.3) + 2\pi(0.025)^2)\, 10}{\pi(0.025)^2 (0.3)\, 900 (2500)} = \frac{(0.0471 + 0.0039)\, 10}{(0.000589)(900)(2500)}$$

$= 0.000385$

$$\frac{25 - 35}{0 - 35} = \exp(-0.000385\, t); \quad t = 5054 \text{ s}$$

(abt 84 minutes)

2. This is unsteady conduction. BSL Fig 12.1-2 applies *

$$\alpha = k/\rho C_p = \frac{0.25}{900 \cdot 2500} = 1.11 \cdot 10^{-7}$$

We want $\frac{T - T_0}{T_1 - T_0} = \frac{25 - 0}{35 - 0} = \frac{5}{7} = 0.714$ at $r = 0$.

From chart, $\frac{\alpha t}{R^2} \approx 0.3 = 1.11 \cdot 10^{-7}\, t / (0.025)^2$

(chart to interpolate)

$t = 1690$ s (about 25 min.)

3. These two processes (convective heat transfer to surface, internal conduction) are in series. The slower answer (5054 s, question 1) is the better answer.

4. This is a process of superposition. The first change sets dimensionless T: $\frac{T - 50}{100 - 50}$. The second change is (−2) units in this dimensionless T. Therefore

$$\frac{T - 50}{100 - 50} = \left(\begin{array}{c}\text{value of chart}\\ \text{for 30 hr}\end{array}\right) - 2 \left(\begin{array}{c}\text{value of chart}\\ \text{for 9 hr}\end{array}\right)$$

The chart is for unstead conduction outwards from cylinder, from Carslaw + Jaeger.

*strictly, problem 2 involves the product method, but one dimension is much larger than the other. You can confirm for yourselve that conduction in the z direction is unimportant.

$\alpha = \frac{k}{\rho C_p} = \frac{0.831}{2008 \cdot 726} = 5.70 \cdot 10^{-7}$

at 30 hr, $\frac{\alpha t}{R^2} = (5.70 \cdot 10^{-7})(30 \cdot 3600)/(0.025)^2 = 98$

at 9 hr, $\frac{\alpha t}{R^2} = 29.5$

$r/R = 25/2.5 = 10$. $\ln(r/R) = 1$.

From the chart, $\frac{T - T_0}{T - T_{well}} \cong 0.1$ for $\frac{\alpha t}{R^2} = 29.5$

≈ 0.22 $\quad \frac{\alpha t}{R^2} = 98$

$$\frac{T - 50}{100 - 50} = (0.22) - 2(0.1) = 0.02$$

$$T \cong 51^\circ C$$

5. What is Re? D_h for slit is $2 \cdot$ width $= 0.04$ m.

$Re = \frac{D_h v \rho}{\mu} = \frac{(0.04)(1)\,800}{0.000106} = 3.02 \cdot 10^5$ Highly turbulent, so hydraulic dia. approx. is OK.

For highly turbulent flow, $Nu = \frac{h D_h}{k} = 0.026\, Re^{(0.8)} Pr^{1/3}$

$Pr = \frac{c_p \mu}{k} = \frac{4857 \cdot 0.000106}{0.62} = 0.829$

$Nu = (0.026)(3.02 \cdot 10^5)^{0.8}(0.829)^{1/3} = 591 = \frac{h(0.04)}{(0.62)} \rightarrow h = 9166 \frac{W}{m^2 K}$

a) $Q = hA\Delta T$; $\frac{Q}{Area} = (9166)(150) = 1.37 \cdot 10^6$ W/m²

b) If the water cools but we assume the rock hold its same T, "Eq. III" applies:

$$Nu = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) Re\, Pr \left(\frac{D_h}{4L}\right)$$

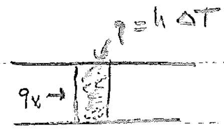
$$591 = \ln\left(\frac{100 - 350}{100 - 200}\right)(3.02 \cdot 10^5)(0.829)\left(\frac{0.04}{4L}\right)$$

$$591 = \ln\left(\frac{250}{100}\right) 2504 \frac{1}{L}; \quad L = 3.88 \text{ m}$$

The water would flow only ≈ 4 m. before cooling to 200°C. (i.e., in 4 sec.)

In reality, the rock would heat + the hot fluid would flow further before cooling.

6. Because T does not vary w/ r, the control volume can extend across the wire. Newton's law at the surface enters the shell balance, as in the "cooling fin" problem worked in class. Energy balance:



$$\pi R^2 q_x\big|_x - \pi R^2 q_x\big|_{x+\Delta x} - h 2\pi R \Delta x (T - T_a) + \pi R^2 \Delta x S = 0$$

conduction in/out — Newton's law at surface — gen. — accum (zero at s.s.)

divide by $\pi R \Delta x$

$$\frac{R q_x|_x - R q_x|_{x+\Delta x}}{\Delta x} - h 2 (T - T_a) + R S = 0$$

let $\Delta x \to 0$

$$-R \frac{dq_x}{dx} - 2h (T - T_a) + R S = 0$$

plug in Fourier's law: $q_x = -k \frac{dT}{dx}$

$$R k \frac{d^2T}{dx^2} - 2h (T - T_a) + R S = 0$$

BC: at $x = 0$, $T = T_0$

$x = L$ $\quad \frac{dT}{dx} = 0$ (reflecting $q_x = 0$ at $x = L$)