TN4780TA Part 2 Exam 15 April 2014

1. The properties of aluminum are irrelevant here; the focus is on convective heat transfer from air to the surface, in flow through a slit. We can use the hydraulic radius approx. <u>if</u> flow is turbulent.

What is Re? $D_h = 0.008\,m$ * $Re = \frac{0.008(1.26)20}{1.75\cdot 10^{-5}} = 11520$ turbulent.

One can use Fig 14.3-2 from BSL or, arguably, ** Eq. 14.3-16 (FT Eq. 3.120)

Using Fig. 14.3-2, $\frac{h_{ln}D}{k}(Re)^{-1}(Pr)^{-1/3} \cong 0.0039$

$Pr = c_p \mu / k = \frac{(1006)(1.75\cdot 10^{-5})}{0.025} = 0.7042$

$h_{ln} = (0.0039)(11520)(0.7042)^{1/3}(0.025)/(0.008) = 125\ W/m^2K$

Using Eq. 14.3-16,

$Nu = (0.026)(11520)^{0.8}(0.7042)^{1/3} = 41.06 = \frac{h_{ln}D}{k}$

$h_{ln} = (41.06)(0.025)/(0.008) = 128\ W/m^2K$

* Note width of slit in wide direction is 0.5 m. 5 cm is length in direction of flow. 0.5 m is <u>much</u> greater than 4 mm in other direction. Strictly, $D_h = 0.00794$

** BSL says eq. is valid for Re > 20,000. FT says it's ok for Re > 10,000, and uses factor (0.027) instead of (0.026)

2. This requires a shell balance in spherical geometry. BSL sect. 10.3 gives an ~~a~~ analogous problem (one could jump to Eq. 10.3-5, with S a constant); FT has a problem in spherical geometry without generation (Eq. 3.33).

Here I start at beginning. Spherical shell of thickness Δr.

conduction in $4\pi r^2 q_r|_r$

" out $4\pi r^2 q_r|_{r+\Delta r}$

generation $4\pi r^2 \Delta r S$

$4\pi r^2 q_r|_r - 4\pi r^2 q_r|_{r+\Delta r} + 4\pi r^2 \Delta r S = 0$. Divide by $4\pi \Delta r$; let $\Delta r \to 0$.

$-\frac{d}{dr}(r^2 q_r) + r^2 S = 0$ or $\frac{d}{dr}(r^2 q_r) = r^2 S$ Integrate

$\to r^2 q_r = \frac{r^3}{3} S + C_1$; $q_r = \frac{r}{3} S + \frac{C_1}{r^2}$

BC: q_r cannot be infinite at $r=0 \to C_1 = 0$*

$q_r = -k\frac{dT}{dr} = \frac{r}{3} S$ $\boxed{I} \to T = -\frac{r^2}{2}\frac{S}{3k} + C_2$ $\boxed{II}$

BC2: $q_r = A(T^4 - T_0^4)$ at $r=R$. Plug in from $\boxed{I}$ and T fr. $\boxed{II}$

$\frac{R}{3} S = A\left[\left(\frac{-R^2}{2}\frac{S}{3k} + C_2\right)^4 - T_0^4\right]$

$\left(\frac{R}{3A} S + T_0^4\right) = \left(\frac{-R^2}{2}\frac{S}{3k} + C_2\right)^4$

$\left[\left(\frac{R}{3A} S + T_0^4\right)\right]^{1/4} + \frac{R^2}{2}\frac{S}{3k} = C_2$

*We can't use BC2 yet because we don't have an expression for T(r).

$$T = -\frac{r^2}{2}\frac{S}{3k} + \left[\left(\frac{R}{3A} S + T_0^4\right)\right]^{1/4} + \frac{R^2}{2}\frac{S}{3k}$$

$$= \frac{R^2 S}{6k}\left(1 - \left(\frac{r}{R}\right)^2\right) + \left[\frac{R}{3A} S + T_0^4\right]^{1/4}$$

3. These two modes are in parallel, so the most efficient mode is the important one. When I first wrote the solution to this problem, I wrote that one would compute Q at the surface for each process, and the mode with the larger value of Q is the important mode. I gave full credit for this answer. Actually, though, Q at the surface is controlled by the generation process in the spher: Q = [(4/3) πR^3 S] *must* be the heat transfer rate at the surface for *each* process if solved for by itself (as it is in problem 2). The more efficient process would accomplish this heat transfer with a lower temperature at the sphere surface (T(r=R)). Thus one would compute the temperature at the sphere surface, for each mode separately, and the mode with the lower surface temperature is the important mode.

4. With no variation of T in the sphere, we can use a macroscopic balance. (Actually, you don't need to assume absence of T gradients in the solid. With no heat transfer at the surface, the solid would heat uniformly.)

$$\underbrace{\tfrac{4}{3}\pi R^3 S}_{\text{generation}} = \underbrace{\tfrac{4}{3}\pi R^3 \rho C_p \frac{dT}{dt}}_{\text{accum}} \qquad \text{divide by } \tfrac{4}{3}\pi R^3$$

$$\frac{S}{\rho C_p} = \frac{dT}{dt} \longrightarrow \frac{S}{\rho C_p} t + C_1 = T$$

at $t = 0$, $T = T_i$

$$T = T_i + \frac{S}{\rho C_p} t$$

time to reach T_A: $T_A = T_i + \frac{S}{\rho C_p} t$

$$t = (T_A - T_i)\frac{\rho C_p}{S}$$

5. [For this problem, both changes go to equilibrium. This stumped some students. At long times $\frac{T_1 - T_c}{T_1 - T_0} \to 0$ and $\frac{T - T_0}{T_1 - T_0} \to 1$.

This is superposition of two changes. First change defines dim'less T: $T_0 = 0°C$, $T_1 = 300°C$. Dim'less change at 4 min is $\left(-\frac{2}{3}\right)$

$$\frac{T - T_0}{T_1 - T_0} = \frac{T - 0}{300 - 0} = \left[\frac{T - T_0}{T_1 - T_0} \text{ for } 6\text{ min}\right] - \frac{2}{3}\left[\frac{T - T_0}{T_1 - T_0} \text{ for } 2 \text{ min}\right]$$

$$\alpha = \frac{k}{\rho C_p} = \frac{229}{2701 \cdot 938.3} = 9.03 \cdot 10^{-5} \qquad \frac{\alpha t}{R^2} = \frac{9.03 \cdot 10^{-5}}{(0.1)^2} \cdot 360 = 3.25 \text{ for } 6\text{ min}$$

$\frac{\alpha t}{R^2} = 1.08$ for 2 min ($F_o = 0.81$ and 0.27)

From BSL Fig 12.1-3, for 6 min $\frac{T_c - T_0}{T_1 - T_0} \simeq 1.0$; for 2 min, ~ 1.0

$$\frac{T - T_0}{T_1 - T_0} = \frac{T - 0}{300 - 0} = 1 - \frac{2}{3}(1) = 0.33; \quad T = 100\,°C$$

Using FT p. 133, at 6 min $\frac{T_1 - T_c}{T_1 - T_0} \simeq 0$ for $F_o = 0.81$; $\frac{T - T_0}{T_1 - T_0} = 1.0$

2 min: $\frac{T_1 - T_c}{T_1 - T_0} \approx < 0.001$ for $F_o = 0.27$. I get $\frac{T - T_0}{T_1 - T_0} = 1 - \frac{2}{3}(1) = 0.33$

$T_c = 100\,°C$

[For practice, try D = 60 cm. Then $T \simeq 200\,°C$.]