TN4780ta Part 1 Exam
Jan 24 2014

1.a) This problem, once simplified, is a repeat of the thought experiment behind Newton's law of viscosity. * $\frac{dv_x}{dy} = \frac{13.4}{0.0005}$ $= 2.68 \cdot 10^4$ m/s² (in magnitude). The shear force is $-\mu \frac{dv_x}{dy} =$ $0.001(2.68 \cdot 10^4) = 26.8$ N/m². The ~~force~~ area per tire for each of four tires is 0.3 m × 0.15 m = 0.045 m². Thus the force for deceleration is $(26.8)(0.045)(4) = 4.824$ N

From Newton's law of motion $F = m\frac{dv_x}{dt}$; $-4.84 = 900\frac{dv_x}{dt}$;
$\frac{dv_x}{dt} = 0.00536$. The car hardly slows down at all.

b.) For shearing flow between plates, the relevant definition of Re is $\frac{YV\rho}{\mu}$; Y = gap width, V = vel. of moving plate and the upperlimit for laminar flow is 3000.
Here $Re = \frac{(0.0005)(13.4)(1000)}{0.001} = 6700 > 3000$. The analysis in part (a) is not valid, or at least not quantitatively correct.

My solution above is a bit loose about +/- signs. Physically, it is clear the automobile must exert force through the film of water to maintain its velocity, and that force represents a deceleration of the auto.

* IF one started with momentum balance (no gravity, no Δp, no accum)

$$LW\tau_{xz}|_x - LW\tau_{xz}|_{x+\Delta x} + W\Delta x\,\cancel{\rho v_z^2}|_{z=0} - W\Delta x\,\cancel{\rho v_z^2}|_{z=L} = 0$$

cancel because v_z = const along streamline

$$\rightarrow LW\frac{\tau_{xz}|_x - \tau_{xz}|_{x+\Delta x}}{\Delta x} = 0$$

$$\rightarrow -\frac{d\tau_{xz}}{dx} = 0 \rightarrow \tau_{xz} = C_1 \quad \boxed{I}$$ No BC on τ.

$$-\mu\frac{dv_z}{dx} = C_1 \rightarrow v_z = -\frac{C_1}{\mu}x + C_2.$$ BC: $v_z = 0$ at $x = 0 \rightarrow C_2 = 0$

BC $v_z = V = 13.4$ at $x = \delta = 0.005 \rightarrow 13.4 = -\frac{C_1}{\mu}\delta \rightarrow C_1 = -\frac{13.4 \cdot 0.001}{0.0005}$

$\rightarrow C_1 = -26.8 = \tau_{xz}$ fr. eq. $\boxed{I}$ above

2. Velocity is unknown. $Re = \frac{DV\rho}{\mu} = \frac{(0.005)V(1.26)}{1.75\cdot10^{-5}} = 360\,V$

Eq. 6.1-7: $f = \frac{4}{3}\frac{gD}{V^2}\left(\frac{\rho_s-\rho}{\rho}\right) = \frac{4}{3}\frac{(9.8)(0.005)}{V^2}\left(\frac{125-1.26}{1.26}\right) = 6.42/V^2$

There are various ways to solve this, but all involve making a guess + checking it when done. One could guess the range of Re and use an analytical approx. formula, then check Re, and, if necessary, try a different range. Here I use "successive substitution" and Fig 6.3-1.

Guess $V = 0.01$ m/s.	$Re = 3.60$	$f \approx 10$ → $10 = 6.42/V^2$ →	$V = 0.80$
0.80	288	0.7	3.03
3.03	1090	0.48	3.66
3.66	1320	0.45	3.77
3.77	1360	[I can't read chart closely enough to see a difference. Iteration is over]	

$V = 3.77$ m/s

Intuitively, this seems 'way too fast. I suspect I over-estimated the density of the bug.

One could also use the other trial + error method, e.g.,

Guess $Re > 500$. $f = 0.44 = 6.42/V^2 \rightarrow V = 3.81$ $Re = 1375$ ✓

or, guess $f = \left(\sqrt{\frac{24}{Re}} + .5407\right)^2$ (BSL eq. 6.3-16). You need trial + error, but this converges on $3.76 = V \rightarrow Re = 1350 < 6000$ ✓

But if you guess $Re < 0.1$ (BSL 6.3-15)

or $1 < Re < 1000$ (eq. given in class)

...you find Re too large + must guess again

Note that because of the role of kinetic energy + fittings in problem 3, one can't simply use Eq. 6.1-4 to solve it.

3. a) There are two equivalent ways to proceed: a) take the entrance surface "1" at the top of the tank. H then enters the problem through the change in height from "1" to "2". b) take the entrance just upstream of the pipe at the bottom of the tank, H enters through the pressure difference between "1" and "2".

Here I use the first approach:

Eq. 7.5-10 applies. Take the terms one at a time:

$\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}[(5)^2 - 0^2] = 12.5$ (velocity $\cong 0$ in tank)

$g(h_2 - h_1) = (9.8)(2 - 0.5 - H) = 9.8(1.5 - H)$

$\int \frac{1}{\rho} dP = 0$. No Δp; both surfaces at 1 atm.

$\hat{W}_m = 0$. No work in or out.

$\Sigma\left(\frac{1}{2}v^2 \frac{L}{R_h} f\right) = \frac{1}{2}(5)^2 \frac{(2+1+0.5)}{0.005} f$

Note since pipe dia is uniform, $v = 5$ m/s everywhere.

Recall for circular pipe $R_h = D/4 = \frac{0.02}{4} = 0.005$

For f, we need Re. $Re = \frac{Dv\rho}{\mu} = \frac{(0.02)5(1000)}{0.001} = 10^5$

$\frac{K}{D} = \frac{0.005}{2} = 0.0025$, From Fig 6.2-2, for $\frac{K}{D} = 0.0025$ and $Re = 10^5$,

$f \cong 0.0062$

$\therefore$ term is $\frac{1}{2}(5)^2 \frac{3.5}{0.005}(0.0062) = 54.25$

$\Sigma\left(\frac{1}{2}v^2 e_v\right)_L = \left[\underset{\text{contraction}}{0.45} + \underset{\text{elbows (middle of range)}}{1.6 \times 2}\right]\frac{1}{2}(5)^2 = 45.65$

Putting it together:

$12.5 + 9.8(1.5 - H) = -54.25 - 45.65$ ①

$1.5 - H = (-112.4)/9.8 = -11.46$

$H = 12.97$ m

b) ① Note that very roughly 50% of dissipation is in pipes, 40% in fittings, 10% → kinetic energy. If the drag in the pipe were 2 or 3x greater, it would greatly change value of H. Note from signs of terms in this eq., large H gives energy for dissipation in pipe + fittings, + kinetic energy of water leaving pipe.

4. a) Everything is the same up to the statement of Newton's law of viscosity (with μ a constant). Thus the last eq to use unchanged is Eq. 2.3-12(b)*. Combining that eq. with this expression for $\mu(r)$ gives

$$\left(\frac{\Delta \mathcal{P}}{2L}\right) r = -\left(\frac{1}{A+B(r/R)}\right)\frac{dv_z}{dr}$$

$$-\left(\frac{\Delta \mathcal{P}}{2L}\right) r \left(A + B\left(\frac{r}{R}\right)\right) = \frac{dv_z}{dr} = -\left(\frac{\Delta \mathcal{P}}{2L}\right)\left(Ar + B\left(\frac{r^2}{R}\right)\right)$$, Integrate:

$$v_z = -\left(\frac{\Delta \mathcal{P}}{2L}\right)\left[A\left(\frac{r^2}{2}\right) + B\left(\frac{r^3}{3R}\right)\right] + C_1$$

BC: $v_z = 0$ at $r = R \rightarrow 0 = -\left(\frac{\Delta \mathcal{P}}{2L}\right)\left[A\left(\frac{R^2}{2}\right) + B\left(\frac{R^3}{3R}\right)\right] + C_1$

$$C_1 = \frac{\Delta \mathcal{P}}{2L}\left[A\left(\frac{R^2}{2}\right) + B\left(\frac{R^3}{3R}\right)\right]$$

$$v_z = \frac{\Delta \mathcal{P}}{2L}\left[A\left(\frac{R^2-r^2}{2}\right) + B\left(\frac{R^3-r^3}{3R}\right)\right] = \frac{\Delta \mathcal{P} R^2}{4L}\left[A\left(1-\left(\frac{r}{R}\right)^2\right) + B\,\frac{2}{3}\left(1-\left(\frac{r}{R}\right)^3\right)\right]$$

Notes: there are probably more elegant ways to group the terms in this equation.

This problem is only very loosely related to polymer flow in porous media. The polymer avoids the pore wall, and viscosity is less there. The actual expressions for this are too complicated to put on an exam.

* strictly, one could say 2.3-13 is still valid, if one doesn't treat μ as a constant. I accepted either answer. I also accepted 2.3-14 if the subsequent derivation made clear μ was not a const.