AES 2320 Pt 1 Exam 10 March 2020

1 The first two viscosities should fall in the laminar regime ($Re < 2100$), and the final value in the turbulent regime. We want Re_2 as close to 2000 as possible

So, for instance, $Re_1 = 1000$, $Re_2 = 2000$, $Re_3 = 4000$

[The exact values of Re could vary, but should be close to these.]

That suggests $Re_1 = 1000 = \frac{DV\rho}{\mu_1} = \frac{(0.02)(0.1)1000}{\mu_1} \rightarrow \mu_1 = 0.002$ Pa s

(~ 2 cp)

2. a) The derivation of shear stress is the same as for a tube up until the BC (for the tube) at $r=0$ is introduced:

BSLK eq. 2.3-10: $\tau_{rz} = \frac{\mathcal{P}_0 - \mathcal{P}_L}{2L} r + \frac{C_1}{r} \leftarrow$ (BSL1 eq. 2.3-11)

Now the B.C. is $\tau_{rz} = 0$ at $r = \kappa R$ $\left[\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{L} = \rho g \text{ in this case}\right]$

$$0 = \frac{\mathcal{P}_0 - \mathcal{P}_L}{2L} \kappa R + \frac{C_1}{\kappa R} \rightarrow C_1 = -\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) \kappa R (\kappa R)$$

$$\tau_{rz} = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right)\left(r - \frac{(\kappa R)^2}{r}\right) = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) r \left(1 - \left(\frac{\kappa R}{r}\right)^2\right)$$

Note that $\tau_{rz}(r)$ increases as $r \uparrow$, from 0 at $r = \kappa R$ to max value at wall (R).

b) $\tau_{rz} = \tau_0$ at wall if

$$\tau_0 = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) R \left(1 - \left(\frac{\kappa R}{R}\right)^2\right) = \left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) R (1 - \kappa^2)$$

$$(1 - \kappa^2) = \tau_0 \left[\left(\frac{\mathcal{P}_0 - \mathcal{P}_L}{2L}\right) R\right]^{-1}$$

$$\kappa = \left[1 - \tau_0 \left[\frac{(\mathcal{P}_0 - \mathcal{P}_L)}{2L} R\right]^{-1}\right]^{1/2} = \left[1 - \tau_0 \frac{2L}{(\mathcal{P}_0 - \mathcal{P}_L) R}\right]^{1/2}$$

(Therefore shearing starts at wall)

For any κ smaller than this (i.e., thicker film), film would shear at the wall.

3. a) For small Re, creeping flow applies. Eq. 2.7-17 (BSLK)

$\mu = \frac{2}{9} \frac{R^2 (\rho_s - \rho) g}{V_t} \rightarrow V \sim R^2$. If D doubles, V increases 4x

b) For large Re, Eq. 6.1-7 applies, with $f \rightarrow$ const. (Fig BLLK 6.3-1)

$f = \text{const} = \frac{4}{3} g \frac{D}{V_\infty^2}\left(\frac{\rho_s - \rho}{\rho}\right) \rightarrow V_\infty \sim \sqrt{D}$. If D doubles, V is 1.4x greater

[see note at end]

4. a) In the macroscopic ME balance, there is drag on the pipe, pressure, gravity on abrupt constriction and kinetic energy. Using Eq. 7.5-11 or 12 ½ Draw surface 1 at top of lake and surface 2 just above outlet of tube

$\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}(v_2^2 - 0)$ $\quad v_2 = Q/(\pi D^2/4) = \frac{7.67(4)}{\pi D^2} = \frac{9.77}{D^2}$

$g(h_2 - h_1) = (9.8)(-400) = -3920$

$(p_2 - p_1)/\rho = (2\cdot10^6 - 10^5)/1000 = 1900$

$\hat{W}_m = 0$

$\frac{1}{2} v^2 \frac{L(4)}{D} f = \frac{1}{2}\left(\frac{9.77}{D^2}\right)^2 \frac{1600}{D} f = \frac{7.64\cdot10^4}{D^5} f$

$\frac{1}{2} v^2 e_v = \frac{1}{2}\left(\frac{9.77}{D^2}\right)^2 (0.45) = \frac{21.5}{D^4}$ for contraction

Putting it all together,

$$\frac{1}{2}\left(\frac{9.77}{D^2}\right)^2 - 3920 + 1900 = -\frac{7.65\cdot10^4}{D^5} f - \frac{21.5}{D^4} \quad \boxed{\text{I}}$$

Trial + error would be messy, since D enters in such a complex way.

b) If we neglect kinetic energy + the contraction, then

$$-3920 + 1900 = -\frac{7.65\cdot10^4}{D^5} f \Rightarrow -2020 = -\frac{7.65\cdot10^4}{D^5} f \quad \boxed{\text{I}}$$

Try trial + error. Guess $D = 0.1$ m.

$Re = \frac{Dv\rho}{\mu} = D\left(\frac{9.77}{D^2}\right)\frac{1000}{0.001} = 9.77\cdot10^6/D = 9.77\cdot10^7$

$f = 0.007$. Eq. I: $D = \left[\frac{7.65\cdot10^4}{2020} f\right]^{1/5} = \{37.87 f\}^{1/5} = 0.77$

If $D = 0.77$, $Re = 1.26\cdot10^7$, $f = 0.007$ still. Done. $D = 0.77$ m.

Not required for exam, but I am curious:
Was it reasonable to ignore the other terms?

$$\frac{1}{2}\left(\frac{9.77}{0.77}\right)^2 - 2020 = -\frac{7.65\cdot10^4}{(0.77)^5}(0.007) - \frac{21.5}{(0.77)^4}$$

$$\underbrace{86.5}_{\text{neglected terms}} - 2020 \cong -1978 - \underbrace{61.2}_{\text{neglected terms}}$$

Yeah, that was a pretty reasonable approximation, leaving out those terms.

Note on problem 3

One can derive the result in (a) from BSLK Eq. 6.1-7 and f(Re) for small Re:

$$f = \frac{4}{3} g \frac{D}{v^2}\left(\frac{\rho_s - \rho}{\rho}\right) = \frac{24}{Re} = \frac{24\mu}{Dv\rho}$$

eliminating constants g, ρ_s, ρ, μ:

$$\frac{D}{v^2} = \frac{1}{Dv} \rightarrow \frac{v^2}{v} = v = D^2$$

If D↑ 2x, v↑ 4x.

Note on problem 4

It surprises me that D is so large. Surely the well was not drilled with so wide a diameter (?).* A student pointed out that perhaps the hole was washed out and widened during the flood. We assume a constant + uniform D and v through the whole process.

* using a lower p at bottom would help, but not enough. Moreover, I imagine hole was rougher than we assumed.