AESB 2320 Part 1 Exam June 2018

1.a) Consider plane 1 just above entrance to tube (i.e., bottom of tank), plane 2 at end of tube.* Let v_2 = vel at end of tube.

Use eq. 7.5-12

$\frac{1}{2}(v_2^2 - v_1^2) = \frac{1}{2}v^2 \quad (v_1 = 0)$

$g(h_2 - h_1) = (9.82)(-0.5) = -4.91$

$\frac{P_2 - P_1}{\rho} = \frac{1}{1000}\left(10^5 - (10^5 + (1000)(9.82)(0.2))\right) = -1.964$

$W_m = 0$

$2v^2 \frac{L}{D} f(Re) = 2v^2\left(\frac{0.5}{0.01}\right) f = 100 f v^2$ Need trial + error to find f.

$\Sigma \frac{1}{2} v^2 e_v = \frac{1}{2}v^2 [0.45]$ [sudden contraction]

Put it all together:

$\frac{1}{2}v^2 - 4.91 - 1.964 = -[100f + 0.225]v^2$

$-6.874 = -[100f + 0.725]v^2$

trial + error. Guess $v = 1$ m/s $\quad Re = \frac{Dv\rho}{\mu} = \frac{0.01\, v\, 1000}{0.001}$

$Re = 10^4 v = 10^4 \quad \frac{k}{D} = \frac{0.04 \cdot 10^{-3}}{0.01} = 0.004$

From Fig 6.2-2, $f \sim 0.0092$

$6.874 = [0.92 + 0.725]v^2 \rightarrow v = 2.01$

$v = 2.01 \rightarrow Re = 2.01 \cdot 10^4 \rightarrow f = 0.0085 \rightarrow v = 2.09$

$2.09 \rightarrow 2.09 \cdot 10^4$ ... still $f = 0.0085$ to within my ability to read chart.

$v = 2.09$ m/s

b) Now surface 1 is just above hole, surface 2 just below it.

$g(h_2 - h_1) \approx 0$

Ignore term for drag in (nonexistent) tube

$\frac{1}{2}v^2 - 1.964 = -\frac{1}{2}v^2(0.45)$

$1.964 = 0.725 v^2 \rightarrow v = 1.64$

* If surface 1 is at top of tank, $v_1 = 0$ as before, but $(h_2 - h_1) = -0.7$, and $P_2 - P_1 = 0$. So get same final equation.

Further notes on problem 1.

This problem is based on a demonstration I learned about from Prof. Heemink from the Electrical Engineering, Mathematics and Computer Science faculty. The demonstration used to be performed by Prof. Battjes, who is now retired, in a class WI2 090, "Mecchanica van Continue Media," i.e. Continuum Mechanics, (taught in Dutch). In the demonstration, a tank with a tube drained faster than the tank without a tube, or perhaps with a shorter tube (as in our result). The extra difference in potential energy provided by the longer tube more than made up for the drag in the tube.

In the Continuum Mechanics course the solution is presented in terms of the Bernoulli equation, which excludes drag. (The Macroscopic Mechanical Energy Balance in our course includes drag through the f(Re) and e_d terms; if you're interested in the Bernoulli equation see text starting on p. 88 of BSLK, and compare the result on p. 89 to the Macroscopic Mechanical Energy Balance.) In the demonstration done in that course, if I recall correctly, the tank without the tube drains more quickly at first, but as the levels drop the tank with the tube drains more quickly. This makes sense: if the extra change in height provided by the tube is relatively insignificant (as in a tall tank with a short tube), or the drag in the tube is dominant in the overall resistance to flow, then the drag in the tube might slow down flow more than its length helps the flow potential. If most of the flow potential is provided by the tube (as in our exam problem), then the tube's length adds more to the flow than its dissipation reduces it.

Prof. Heemink provides a link to a demonstration on YouTube of a tank draining through a tube:
https://www.youtube.com/watch?v=VsenKlfPGoc
The video is entitled "Tank Draining Experiment Lab 1" so you can find it by going to YouTube and searching for that title. There is no comparison here to a tank with a hole but no tube in this video, but there's a separate video with no tube at
https://www.youtube.com/watch?v=oX7i4ZiRjPY
This video is entitled "Tank Draining Analysis." There's an explanation of flow out of a tank with a hole, using the Bernoulli equation, at
https://www.youtube.com/watch?v=QiWsX9JRb6c
The video is entitled "Inviscid Flow: Bernoulli Equation and Draining tank example problem." Note from the title "Inviscid Flow" it neglects drag. I'm a bit reluctant to share this, because if you learn to rely on the Bernoulli equation you'll forget to take drag into account, as it is included in the Macroscopic Mechanical Energy Balance.

2. a) Shell balance is same up to point of integrating eq. for τ_{xz} (Eq. 2.2-14 BSLK). Note one can't use Eq. 2.2-15 because $\rho \neq$ constant

$$\frac{d\tau_{xz}}{dx} = \rho g \cos\beta = (a + bx)\, g \cos\beta$$

$$\tau_{xz} = \left(ax + \frac{b}{2}x^2\right) g \cos\beta + C_1$$

BC: $\tau_{xz} = 0$ at $x = 0$ $\rightarrow$ $0 = (a \cdot 0 + b \cdot 0)\, g \cos\beta + C_1$

$$C_1 = 0$$

$$\tau_{xz} = \left[ax + b(x^2/2)\right] g \cos\beta = -\mu \frac{dv_z}{dx}$$

b) $$\frac{dv_z}{dx} = -\frac{g\cos\beta}{\mu}\left[ax + b\frac{x^2}{2}\right]$$ integrate

$$v_z = -\frac{g\cos\beta}{\mu}\left[a\frac{x^2}{2} + b\frac{x^3}{6}\right] + C_2$$

BC: $v_z = 0$ at $x = \delta$

$$0 = -\frac{g\cos\beta}{\mu}\left[a\frac{\delta^2}{2} + b\frac{\delta^3}{6}\right] + C_2$$

$$C_2 = \frac{g\cos\beta}{\mu}\left[a\frac{\delta^2}{2} + b\frac{\delta^3}{6}\right]$$

$$v_z = \frac{g\cos\beta}{\mu}\left[a\frac{(\delta^2 - x^2)}{2} + b\frac{(\delta^3 - x^3)}{6}\right]$$

Note if $b = 0$, $a = \rho$ and we get back results in Sect. 2.2.

3. a) To go just past the transition, $Re = 2100 = \frac{Dv\rho}{\mu}$

$$Q = \left(\frac{D^2}{4}\pi\right)v \rightarrow v = Q\left[\pi\frac{D^2}{4}\right]^{-1} = 1.67 \cdot 10^{-7}\left[\frac{\pi}{4}D^2\right]^{-1} = 2.12 \cdot 10^{-7}\, D^{-2}$$

Then $Re = 2100 = \frac{D\rho}{\mu}(2.12 \cdot 10^{-7} D^{-2}) = 0.0424\, D^{-1}$

$$2100 = 0.0424\,(D^{-1}) \rightarrow D = 2.01 \cdot 10^{-5}\ m$$

{confirm: $\frac{Dv\rho}{\mu} = \frac{(2.01 \cdot 10^{-5})\, 524\, (1000)}{(0.005)} \cong 2100$

$v = 2.12 \cdot 10^{-7}(2.01 \cdot 10^{-5})^{-2} = 524\ m/s$ } (!)

b) At $Re = 2100$, $f \cong 0.012$ whatever the roughness. at least for values of K/D in BSLK Fig 6.2-2.

Eq. 6.1-4 $\quad f = \frac{1}{4}\frac{D}{L}\left(\frac{\Delta P}{\frac{1}{2}\rho v^2}\right)$

$$0.012 = \frac{2.01 \cdot 10^{-5}}{2\,(1000)(524)^2}\frac{\Delta P}{L} \rightarrow \frac{\Delta P}{L} = 3.28 \cdot 10^{11}\ Pa/m$$

($\sim$3 million bar/m!)

Note the goal is to produce turbulence. One can't use eqs. for laminar flow.

4. A. For laminar flow of Newtonian fluid, Q is proportional to $(\Delta \mathcal{P}/L)$. ✓

B. In highly turbulent flow, $\Delta \mathcal{P}$ increases like Q^2. X

C. For a Bingham plastic, if it flows at all, Q increases more than proportional to $\Delta \mathcal{P}$. X

D. For a shear-thinning PL fluid, Q increases more than proportionately to $\Delta \mathcal{P}$. X

E. For shear thickening PL fluid, Q increases less than proportionately to $\Delta \mathcal{P}$. X

F. If the tube is vertical, with outlet higher than inlet, then $\Delta \mathcal{P} = \Delta p - \rho g H$. Doubling $\Delta p \rightarrow$ more than doubling $\Delta \mathcal{P}$. Thus, if Q doubles, it responds less than proportionately to $\Delta \mathcal{P}$. That's a shear-<u>thickening</u> PL fluid, not shear-thinning. X

G. In this case, $\Delta \mathcal{P} = \Delta p + \rho g H$. Doubling $\Delta p \rightarrow$ less than doubling $\Delta \mathcal{P}$. Q doubles, which is more than proportional to change in $\Delta \mathcal{P}$. That's a shear-thinning PL fluid. ✓ *

Ans: A, G

[C could be an answer if both Q_1 and $Q_2 = 0$. But the problem says the Bingham plastic is <u>flowing</u> $[Q \neq 0]$.]

* Of course, it would be a coincidence if it worked out just exactly so that Q doubles for a doubling in Δp. But it is possible