AESB 2320 pt 2 Exam 13 April, 2016

1. a) As mentioned in class, orthogonal conduction is an example of "parallel" modes of heat transfer: each additional mode accelerates the process. In this case, that means T would be lower (closer to equilibrium) than the calculation excluding horizontal conduction.

b) ... but not a lot lower. The length scale in the horizontal direction is of order 0.5-1 m, whereas it is 0.1 m vertically. The short dimension drives unsteady conduction to equilibrium before longer dimensions.

[See note at end.]

2. Shell balance has generation and conduction, no convection or accumulation:

conduction in: $4\pi r^2 q_r|_r$

" out: $4\pi r^2 q_r|_{r+\Delta r}$

generation: $4\pi r^2 \Delta r S = 4\pi r^2 \Delta r \left(\frac{A}{r}\right)$

$$4\pi r^2 q_r|_r - 4\pi r^2 q_r|_{r+\Delta r} + 4\pi r^2 \Delta r\left(\frac{A}{r}\right) = 0$$

divide by $4\pi \Delta r$; let $\Delta r \to 0$

$$-\frac{d}{dr}(r^2 q_r) + Ar = 0$$

$$\frac{d}{dr}(r^2 q_r) = Ar$$

$$r^2 q_r = A(r^2/2) + C_1$$

$$q_r = \frac{A}{2} + \frac{C_1}{r^2}$$

B.C.: $q_r = q^*$ at $r = \alpha R$

$$q^* = \frac{A}{2} + \frac{C_1}{(\alpha R)^2}$$

$$C_1 = \left(q^* - \frac{A}{2}\right)(\alpha R)^2$$

$$q_r = \frac{A}{2} + \left(q^* - \frac{A}{2}\right)\left(\frac{\alpha R}{r}\right)^2 = -k\frac{dT}{dr}$$

b)

$$\frac{dT}{dr} = -\frac{1}{k}\frac{A}{2} + \left(\frac{A}{2} - q^*\right)\frac{1}{k}(\alpha R)^2\frac{1}{r^2}$$

$$T = -\frac{A}{2k}r + \left(\frac{A}{2} - q^*\right)\frac{1}{k}(\alpha R)^2\left(-\frac{1}{r}\right) + C_2$$

BC: $T = T_0$ at $r = R$

$$T_0 = -\frac{A}{2k}R + \left(\frac{A}{2} - q^*\right)\frac{(\alpha R)^2}{k}\left(-\frac{1}{R}\right) + C_2$$

$$C_2 = \frac{A}{2k}R + \left(\frac{A}{2} - q^*\right)\left(\frac{\alpha R^2}{k}\right)\frac{1}{R}$$

$$T = \frac{A}{2k}(R-r) + \left(\frac{A}{2} - q^*\right)\frac{1}{k}(\alpha R)^2\left(\frac{1}{R} - \frac{1}{r}\right)$$

c) $Q = 4\pi R^2 q_r|_{r=R} = 4\pi R^2\left(\frac{A}{2} + \left(q^* - \frac{A}{2}\right)\alpha^2\right)$

[see note at end]

3. What is Re? $\quad Re = \frac{Dv\rho}{\mu} = \frac{(0.1)(0.2)1000}{0.001} = 20{,}000$

Using Eq. 14.3-16:

$$Nu = 0.026\, Re^{0.8}\, Pr^{1/3} (\mu_0/\mu_0)$$

$$Pr = \frac{c_p \mu}{k} = \frac{(4190)(0.001)}{0.68} = 6.17 \quad \leftarrow \text{we'll neglect this}$$

$$Nu = 0.026(20000)^{0.8}(6.17)^{1/3} = 131.6$$

"Eq III": $\quad Nu = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) Re\, Pr \frac{D}{4L} = (20000)(6.17)\frac{0.1}{80} \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right)$

$$= 154 \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right)$$

$$\ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) = \frac{131.6}{154} = 0.853$$

$$\frac{T_0 - T_{b1}}{T_0 - T_{b2}} = 2.35$$

One could solve by trial + error. But consider that T_0 is 2.35 times further from 100° than it is from 50°. In other words $(T_0 - T_{b2}) = \frac{50}{1.35} = 37$

$\boxed{T_0 = 13\,°C.}$ $\quad T_0 - T_{b1} = 100 - 13 = 87$

$T_0 - T_{b2} = 50 - 13 = 37$

$$\frac{T_0 - T_{b1}}{T_0 - T_{b2}} = \frac{87}{37} = 2.35 \checkmark$$

Alternative solution: Using Fig 14.3-2,

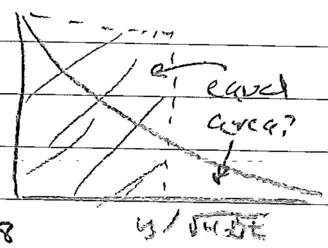
$$\ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right)\frac{D}{4L}(Pr)^{2/3} \cong 0.0036 = \ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right)\frac{0.1}{80}(6.17)^{2/3}$$

$$\ln\left(\frac{T_0 - T_{b1}}{T_0 - T_{b2}}\right) = 0.0036\,\frac{80}{0.1}(6.17)^{-2/3} = 0.856$$ Nearly same as above

4. a) This is unsteady diffusion into a semi-infinite solid. From Fig 3.8-2 (BSLK) one can see $\frac{C_A - C_{A0}}{C_{A1} - C_{A0}} = 0.1$ for $\frac{y}{\sqrt{4Dt}} \doteq 1.18$

$$y = 1.18(4\,10^{-8}\,3600)^{1/2} = 0.0142\,m$$

b) From Fig 3.8-2, the area under the curve is the total amount of A in the solid. The area under the curve is, by my eyeball, roughly equal to being fully saturated to a distance of about 0.5 - 0.6; say 0.55. If $\frac{y}{\sqrt{4Dt}} = 1.18$ is 0.0142 m, then this distance is about $\frac{0.55}{1.18} \times 0.0142\,m = 0.0066\,m$. In other words, the amount of A in the solid is roughly equivalent to a conc. of 10 kg/m³ to a distance of 6.6 mm. That is $10 \times 0.0066 \times 1\,m^3 = 0.066\,Kg$ (66 g) of A in the plastic.



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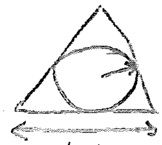
Note on problem 1b: Conduction within the triangle couldn't possibly be faster than in a circle inscribed in the triangle. If the side is 1 m, then the radius of the circle is 0.29 m. If



1 m

$\frac{T_1 - T}{T_1 - T_0} = 0.5$ for the slab, then $\frac{\alpha t}{b^2} = \frac{\alpha t}{(0.05)^2} \approx 0.4$ (Fig 11.5-1 from BSLK; Fig 12.1-1 from BSL2). For the circle, at this same time, $\frac{\alpha t}{(0.29)^2} = (0.4)\frac{(0.05)^2}{(0.29)^2} = 0.03$. As shown on Fig 11.5-2 for the circle, the center of the circle is unaffected at this time.

[This merely gives numbers to the claim that only the thinnest dimension matters if it is much thinner than the others. This analysis was not necessary to answer the question.]

Note on problem 2: I should have worded the problem statement to make clear that the entire problem was at steady state. Some students attempted part (a) allowing for accumulation in the energy balance, and then dropped the term in part (b). In that case I used the student's derivation for (b) in grading (a). Strictly, part (a) already implies steady-state. If it were not steady-state, q depends on t as well as r; $q = q(r,t) \neq q(r)$.

Many students relied too much on the example of the spherical nuclear fuel element.* In this case, there is only one layer, and it has generation; there is no B.C. at $r = 0$.

In the final part, I meant heat transfer from the outer surface, at $r = R$. If students added an expression for transfer at $r = \alpha R$, that was OK. At $r = R$, it is not convective heat transfer: $T(R)$ is given, not the T of a surrounding fluid.

* in BSL 1 and BSL2, but not BSLK.