DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TEST NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS (WI3097 TU)
Thursday April 18 2013, 18:30-21:30

1. We consider the numerical integration of the following initial value problem $y' = f(t, y)$, $y(t_0) = y_0$ with the Euler forward method:

$$w_{n+1} = w_n + hf(t_n, w_n). \tag{1}$$

   a Determine the order of the local truncation error. (2.5pt.)

   b We consider the following second order initial value problem

$$\begin{cases} y'' + \varepsilon y' + y = sin(t), \\ y(0) = 1, y'(0) = 0. \end{cases} \tag{2}$$

   Rewrite this initial value problem into the form of a system of first order differential equations. Take the initial conditions into account. (1pt.)

   We continue with the following system of initial value problems

$$\begin{cases} y_1' = -y_2, \\ y_2' = y_1 + \varepsilon y_2, \end{cases} \tag{3}$$

   with initial conditions $y_1(0) = 1$ and $y_2(0) = 2$, further ε is a given real-valued constant.

   c What is the maximum allowable value of h for numerical stability if $\varepsilon = 0$? Motivate your answer. (2.5pt.)

   d For which values of ε is the given system (analytically) stable? (2pt.)

   e What is the maximum allowable value of h for numerical stability if $-2 \leq \varepsilon < 0$? Motivate this answer. (2pt.)

[0]**please turn over, For the answers of this test we refer to:**
http://ta.twi.tudelft.nl/nw/users/vuik/wi3097/tentamen.html

2. We consider the following boundary value problem:

$$\begin{cases} -\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} + y = x^2 + 2x - 2, & x \in (0,1), \\ y(0) = 0, & y'(1) = 2. \end{cases} \tag{4}$$

a Prove that $y(x) = x^2$ satisfies this boundary value problem. (1pt.)

We apply the finite difference method to approximate the solution to the above boundary value problem. Let the gridnodes be given by $x_j = jh$, with h as stepsize. Let $x_n = nh = 1$.

b Give a finite differences scheme (+ motivation) for which the local truncation error is of order $O(h^2)$. *Hint:* Use a virtual gridnode for the boundary condition at $x = 1$. (3pt.)

c Motivate, using the estimate of the local truncation error in the previous question and the exact solution to this boundary value problem, why the difference between the numerical approximation and the exact solution is equal to zero. (2pt.)

Given the following tabular values for the approximation of the function $y(x) = x^2$.

Tabel 1: Tabular values for $\tilde{y}(x)$ (rounded to three digits).

x	$\tilde{y}(x)$
0	0
0.25	0.063
0.5	0.25

d Estimate $y'(0)$ using forward differences on the values in Table 1 with $h = 0.25$ en $h = 0.5$. (1pt.)

e We consider the accuracy of the computation.

i Suppose that the tabular values contain a (rounding) error of maximum size $\varepsilon = 0.0005$, i.e. $|\tilde{y}(x_j) - y(x_j)| \leq \varepsilon$ ($y(0) = 0$ is exact), what is the influence of this rounding error on the error of the forward differences? (1pt.)

ii Demonstrate that the truncation error is of order $O(h)$. (1pt.)

iii Use Richardson's Method to estimate the error. (1pt.)