DELFT UNIVERSITY OF TECHNOLOGY
Faculty of Electrical Engineering, Mathematics and Computer Science

TEST NUMERICAL METHODS FOR DIFFERENTIAL EQUATIONS (WI3097 TU)
Thursday April 19 2012, 18:30-21:30

1. We consider the following method for the integration of the initial value problem $y' = f(t, y)$, $y(t_0) = y_0$

$$\begin{cases} w_{n+1}^* = w_n + hf(t_n, w_n) \\ \\ w_{n+1} = w_n + h\left(a_1 f(t_n, w_n) + a_2 f(t_{n+1}, w_{n+1}^*)\right) \end{cases} \tag{1}$$

a Show that the local truncation error of the above method has order $O(h)$ if $a_1 + a_2 = 1$. Which value for a_1 and a_2 will give a local truncation error of order $O(h^2)$? (3 pt.)

b Demonstrate that for general values of a_1 and a_2 the amplification factor is given by

$$Q(h\lambda) = 1 + (a_1 + a_2)h\lambda + a_2(h\lambda)^2. \tag{2}$$

(2 pt.)

c Consider $\lambda < 0$ and $(a_1 + a_2)^2 - 8a_2 < 0$. Derive the condition for stability, to be fullfilled by h. (2 pt.)

d Consider the following system

$$\begin{cases} y_1' = -y_1 y_2, \\ y_2' = y_1 y_2 - y_2, \end{cases} \tag{3}$$

Show that the Jacobian of the right hand side of the above system (which is used for the linearization of the above system) for the initial condition $y_1(0) = 1$ and $y_2(0) = 2$ is given by

$$\begin{pmatrix} -2 & -1 \\ 2 & 0 \end{pmatrix}.$$

(1.5 pt.)

e We apply the numerical method in equation (1) for the case that $a_1 = a_2 = 1/2$ to system (3). Is the method stable near the initial condition $y_1(0) = 1$ and $y_2(0) = 2$, and step size $h = 1$ (+ motivation)? (1.5 pt.)

[0]**please turn over, For the answers of this test we refer to: http://ta.twi.tudelft.nl/nw/users/vuik/wi3097/tentamen.html**

2. We consider the following boundary value problem:

$$(P_1)\begin{cases} -v'' + v = 2 + x(2-x), & x \in (0,1), \\ v(0) = 0, & v'(1) = 0. \end{cases}$$

a Show that $v(x) = x(2-x)$ is the solution to the boundary value problem (P_1). (1 pt.)

b Give a finite difference discretization with error of $\mathrm{O}(h^2)$ (+ proof), where h represents the distance between adjacent gridnodes (*Hint: use a virtual gridnode near* $x=1$). The discretization must be symmetric. (3 pt.)

c Give the system of equations that results from the finite difference discretization with three (after processing the virtual node) unknowns ($h = 1/3$). (2 pt.)

d Compute the error of the numerical solution, and explain your answer. (1 pt.)

e Next, we consider the following system of nonlinear equations

$$\begin{cases} 18v_1 - 9v_2 + v_1^2 = \frac{20}{9}, \\ -9v_1 + 18v_2 + v_2^2 = \frac{20}{9}. \end{cases}$$

Carry out one step of Newton's method on the above system where you use $v_1 = v_2 = 0$ as the initial guess. (3 pt.)