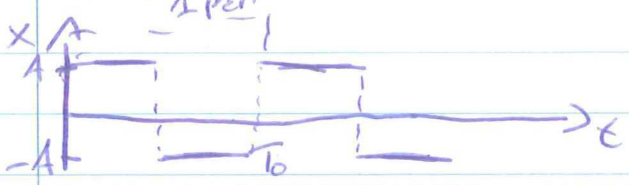
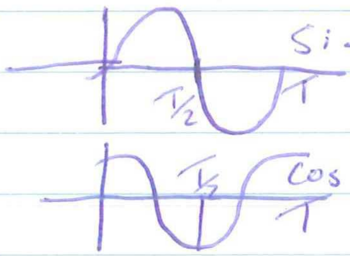
Signale H3

$x(t) = a_0 + a_n \cos(n\omega_0 t) + b_n \sin(n\omega_0 t)$

Periodiek signaal te schrijven als cos & sin.





signaal lijkt op sin. dus geen cos in vergl.

$$x(t) = \begin{cases} A : 0 < t < \frac{T_0}{2} \\ -A : \frac{T_0}{2} < t < T_0 \end{cases}$$

$$a_0 = \frac{1}{T_0} \int_{T_0} x(t)\,dt = 0$$, eerst A daarna -A

$$a_m = \frac{2}{T_0} \int_0^{T_0} x(t) \cos(m\omega_0 t)\,dt$$

$$b_m = \frac{2}{T_0} \int x(t) \sin(m\omega_0 t)\,dt$$

$$a_m = \frac{2}{T_0}\left(\int_0^{\frac{T_0}{2}} A \cos(m\omega_0 t) + \int_{T_0/2}^{T_0} (-A)\cos(m\omega_0 t) \right)$$

$$= \frac{2A}{T_0}\left(\frac{\sin(m\omega_0 t)}{m\omega_0 t}\Bigg|_0^{\frac{T_0}{2}} + \frac{-\sin(m\omega_0 t)}{m\omega_0 t}\Bigg|_{\frac{T_0}{2}}^{T_0} \right)$$

$* \; \omega_0 = 2\pi f = 2\pi \cdot \frac{1}{T_0}$

$$= \left[\frac{1}{m\omega_0} \sin(m\omega_0 t) \right]_0^{\frac{T_0}{2}} - \left[\frac{1}{m\omega_0} \sin(m\omega_0 t) \right]_{\frac{T_0}{2}}^{T_0}$$

$$\sin\left(m \cdot 2\pi \cdot \frac{1}{\cancel{T_0}} \cdot \frac{\cancel{T_0}}{2}\right) = \sin\left(m \frac{2\pi T_0}{2 T_0}\right) = \sin(\pi) = 0$$

dus $a_m = 0$

er zit dus geen cos in z'n plaats

$$b_m = \frac{2}{T_0}\int x \sin(m\omega_0 t)\, dt$$

$$= \left(\frac{2A}{T_0}\left(-\frac{\cos(m\omega_0 t)}{m\omega_0}\right]_0^{T_0/2} + \left(\frac{\cos(m\omega_0 t)}{m\omega_0}\right]_{T_0/2}^{T_0}$$

$$= -\cos\left(m\frac{2\pi}{T_0}\cdot\frac{T_0}{2}\right) + \underbrace{\cos\left(m\left(\frac{2\pi}{T_0}\right)\cdot 0\right)}_{=1}$$

$$+ \underbrace{\cos\left(m\frac{2\pi}{T_0}\cdot T_0\right)}_{=1} - \cos\left(m\frac{2\pi}{T_0}\cdot\frac{T_0}{2}\right)$$

$$= 2 - 2\cos(m\cdot\pi)$$

$m<0 \rightarrow -1$ aanneme $m>0$

$m>0 \rightarrow 1$

dus: $= 2-2=0$

aanneme $m<0$

dus: $= 2 - -2 = 4$

eindresultaat:

$a_0 = 0$

$a_n = 0$

$b_n = \rightarrow 0 \quad m>0$

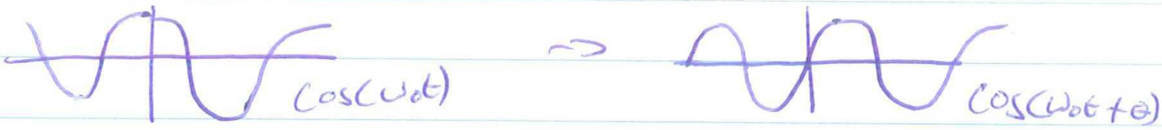
$\frac{4A}{m\pi} \quad m<0$

eind: $x = \frac{4A}{m\pi}\sin(m\omega_0 t)$

↳ half-wave oneven symetrisch

Voor thuis:

$$\cos(n\omega_0 t + \theta)$$



$$\cos(n\omega_0 t)\cdot\cos(\theta) - \sin(\omega_0 t)\cdot\sin(\theta)$$

$$\theta = 45^\circ$$

$$= \underbrace{\cos(\theta)}_{a_1}\,\underbrace{\cos(\omega_0 t)}_{m=1} - \underbrace{\sin(\theta)}_{a_1}\,\underbrace{\sin(\omega_0 t)}_{m=1}$$

$$a_0 = 0$$

$$a_m = \quad m \geq 2$$

Complexe fourier seric

$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$e^{-j\theta} = \cos\theta - j\sin(\theta)$

$$\boxed{\begin{array}{l} \frac{1}{2}e^{j\theta} + \frac{1}{2}e^{-j\theta} = \cos(\theta) \\ \frac{1}{2}e^{j\theta} - \frac{1}{2}e^{-j\theta} = j\sin(\theta) \xrightarrow{:j} \frac{1}{2j}e^{j\theta} - \frac{1}{2j}e^{-j\theta} = \sin\theta \end{array}}$$

$\cos(n\omega_0 t) = \frac{1}{2}e^{-j(\omega_0 t)} + \frac{1}{2}e^{j(\omega_0 t)}$

$\sin(\omega_0 t) = \frac{1}{2}e^{-j(\omega_0 t)} - \frac{1}{2}e^{j(\omega_0 t)}$

complexe :

$x(t) = a_0 + a_n\left(\frac{1}{2}e^{j(\omega_0 t)} + e^{-j(\omega_0 t)}\right) + \frac{1}{j}b_n\left(\frac{1}{2}e^{j(\omega_0 t)} - \frac{1}{2}e^{-j(\omega_0 t)}\right)$

$x(t) = a_0 + \underbrace{\frac{1}{2}\left(a_n + \frac{b_n}{j}\right)}_{\to X_n} e^{j(\omega_0 t)} + \underbrace{\frac{1}{2}\left(a_n - \frac{b_n}{j}\right)}_{\to X_{-n}}\left(e^{-j(\omega_0 t)}\right)$

$x(t) = a_0 + \frac{1}{2}(a_n - jb_n)e^{j(\omega_0 t)} + \frac{1}{2}(a_n + jb_n)(e^{-j(\omega_0 t)}$

$$\boxed{\frac{b_n}{j} \cdot \frac{j}{j} = \frac{jb}{\underset{=-1}{j^2}} = -jb_n}$$

$\to a_0 = \underbrace{a_0 \cdot e^{j0\omega_0 t}}_{\to X_0}$

$x(t) = X_0 e^{j0\omega_0 t} + \sum_{n=1}^{\infty} X_n e^{j\omega_0 t} + \sum_{n=1}^{\infty} X_{-n} e^{-j\omega_0 t}$

$$\boxed{= \sum_{n=-\infty}^{\infty} X_n e^{j(n\omega_0 t)}}$$

Complexe fourier

Oefeninge

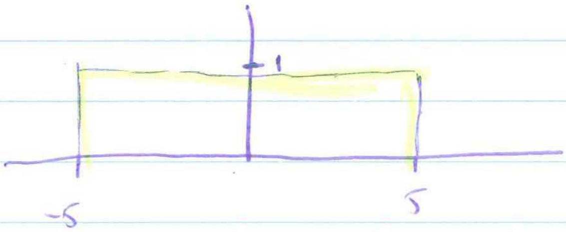
1.8 a.) $\Pi(0,1t)$

$$\omega_0 = 0,1 = 2\pi \cdot f$$

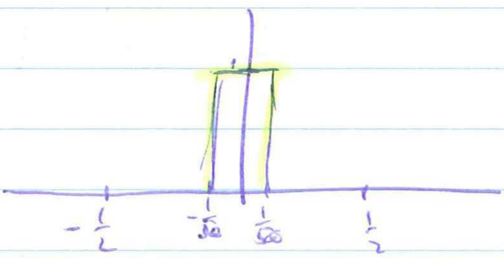
$$f = 0,016 \text{ Hz}$$

$$\Pi(0,1t) = \Pi(0,1(t))$$

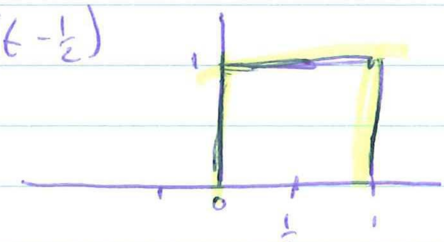
$$= \Pi = \left(\frac{t}{0,1}\right) = \Pi(10t)$$



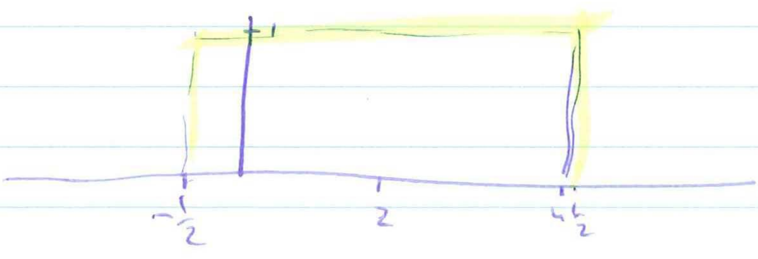
b.) $\Pi(10t) = \Pi(10(t)) = \Pi\left(\frac{t}{10}\right) = \Pi(0,1t)$



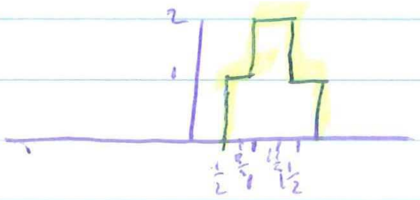
c.) $\Pi(t-\frac{1}{2})$



d.) $\Pi\left(\frac{1}{5}\cdot(t-2)\right)$



e.) $\pi\left(\frac{1}{2}(t-1)\right) + \pi(t-1)$



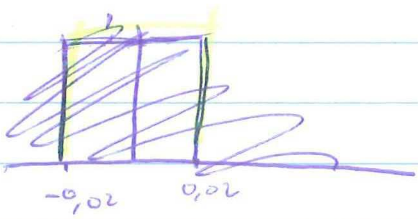
1-g a.) $\sin(50\pi t)$

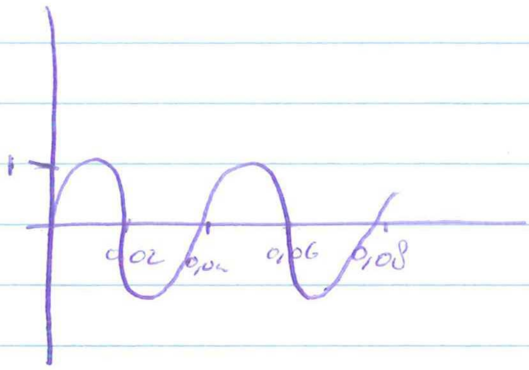
$\omega_0 = 50\pi = 2\pi f$

$f = 25\ Hz$

~~Pe~~ periode = 0,04 s

$\theta = 0$





b.) $\cos(60\pi t)$

$\omega_0 = 2\pi f = 60\pi$

$f = 30\ Hz$

periode $= \frac{1}{30}$

d.) $\sin(50\pi t) + \cos(60\pi t)$

$= \sin((25 \cdot 2\pi) t) + \cos((30 \cdot 2\pi) \cdot t)$

$= \sin(((5 \cdot 5) \cdot 2\pi) t) + \cos(((6 \cdot 5) \cdot 2\pi) t)$

gelijkovereekomst = 5 → 5 Hz

Period = 0,2 s

e.) $\sin(50\pi t) + \cos(70\pi t)$

$= \sin((25 \cdot 2\pi) t) + \cos((35 \cdot 2\pi) t)$

$= \sin((5 \cdot 5) \cdot 2\pi) t) + \cos((7 \cdot 5) 2\pi) t)$

Overee : 5 Hz

$t = 0,02$ s.

1-11 a.) $2\cos(10\pi t + \frac{\pi}{6})$

$f = 5\,Hz \qquad T = 0{,}2\,s$

b.) $17\pi t \qquad \omega_0 = 17\pi = 2\pi f$

$f = 8\frac{1}{2}\,Hz$

$T = 0{,}12\,s$

(c.) $\omega_0 = 19\pi = 2\pi f \rightarrow f = 9\frac{1}{2}$

$T = 0{,}11$ sec.

A)

1-12 a.) $2\cos(10\pi + \frac{\pi}{6})$

phasor rot $= 2 \cdot e^{\frac{\pi}{6}j} \cdot e^{10\pi t j}$

b.) $5\cos(17\pi t - \frac{\pi}{4})$

Phasor rot $= 5e^{-\frac{\pi}{4}j} e^{17\pi t j}$

(c.) $3\sin(19\pi t - \frac{\pi}{3}) \rightarrow 3\cos(19\pi t - \frac{5}{6}\pi)$

Phasor rot $= 3e^{-\frac{5}{6}\pi} e^{19\pi t}$

B)

a.) $2\cos(10\pi + \frac{\pi}{6})$

Sum phasor $= e^{(10\pi + \frac{\pi}{6})j} + e^{-(10\pi + \frac{\pi}{6})j}$

b.) $5\cos(17\pi - \frac{\pi}{4})$

sum phasor $= 2\frac{1}{2}e^{(17\pi - \frac{\pi}{4})j} + 2\frac{1}{2}e^{-(17\pi - \frac{\pi}{4})j}$

(c.) $3\sin(19\pi t - \frac{\pi}{3}) \rightarrow 3\cos(19\pi t - \frac{5}{6}\pi)$

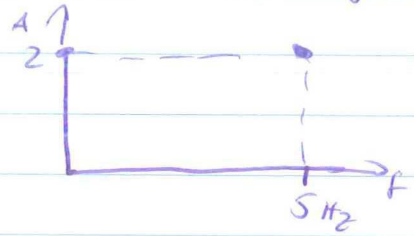
Sum phasor $= 1\frac{1}{2}e^{(19\pi - \frac{5}{6}\pi)} + 1\frac{1}{2}e^{-(19\pi t - \frac{5}{6}\pi)}$

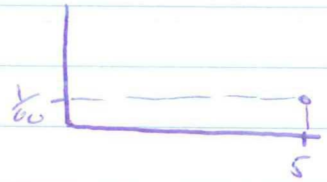
C.)

a.) $2\cos(10\pi + \frac{\pi}{6})$

Ampl. = 2

$T = 0{,}2\ s \Rightarrow f = 5\ Hz$ $\theta = \frac{\pi}{6}$



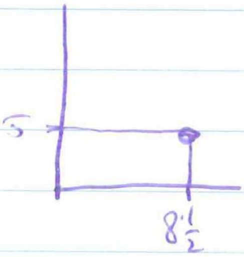


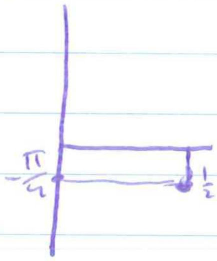
b.) $5\cos(12\pi - \frac{\pi}{4})$

Ampl. = 5

$f = 8\frac{1}{2}\ Hz$

$\theta = -\frac{\pi}{4}$



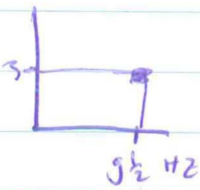


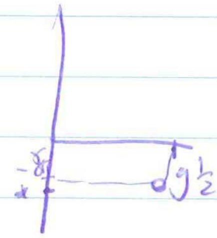
c.) $3\cos(19\pi t - \frac{5}{6}\pi)$

Ampl. = 3

$f = 9\frac{1}{2}\ Hz$

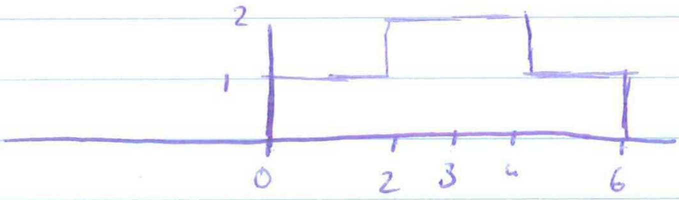
$\theta = -\frac{5}{6}\pi$





1-14 a.) $x_a = \Pi\left(\frac{1}{6}(t-3)\right) + \Pi\left(\cancel{t}\,\frac{1}{2}(t-4)\right)$

$\tau = 6 \qquad\qquad \tau = 2$



1-26. a.)

Instrumental and signals

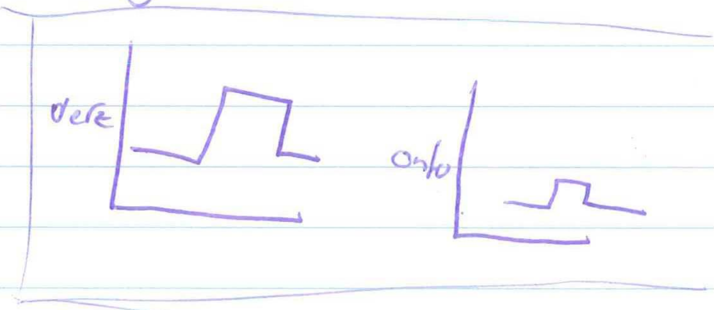
algemeen
↳ verstoringe zijn belangrijk, geven info

$\boxed{Tx}$ -> medium -> $\boxed{Rx}$

Signaal   Ontvanger

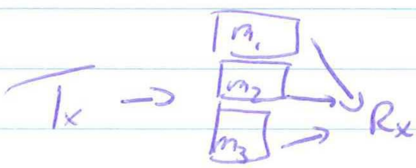
normaal: - afname door hoek van uitzending
- afname door tijdsduur

Vervorming:



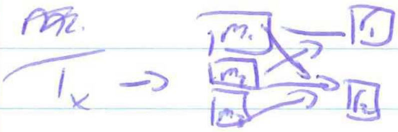
Simpel -> complex

1 signaal verzonden, meerdere signale terug, bijv. Seismic



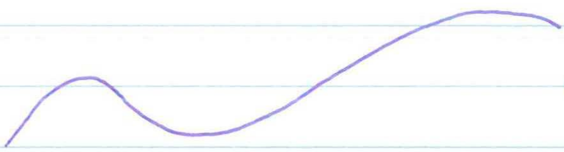
-> meerdere mediums

Bv.



-> meerdere ontvangers

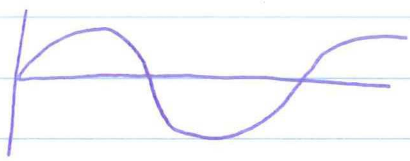
Diep gaan → acoustic signalen



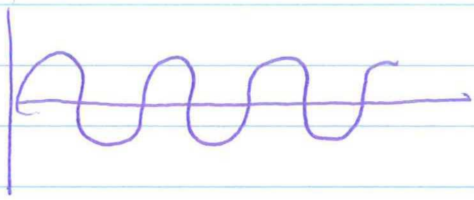
← laag reconstrueren

$$z(t) = \sum \underbrace{a_n}_{\text{const}} \cos(\underbrace{n}_{\text{variabele}} \underbrace{\omega_0}_{\text{rad/s}} \underbrace{t}_{\text{s}})$$

n=1



n=2

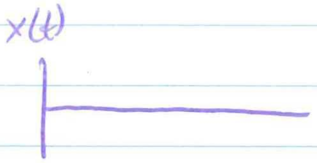


Signals   college 1
H1

1.1

Systeem t_x $\overset{medium}{\sim\sim\sim}$ R_x
$x(t)$ $y(t)$
begin   eind

$y(t)$ is omgebouwen $x(t)$
bijv. $y(t) = 3 \cdot x(t)$



Deterministic signaal
↳ functie van tijd
↳ zeker sygnaal

Vb. $x(t) = \frac{At^2}{B+t^2}$

A en B zijn constant
↳ bestaat niet in echt, lvm ruis

Stochastic signaal
↳ random signaal, altijd aanwezig

Continous signaal
↳ het is er altijd

Discreet signaal
↳ signaal op bepaalde punten
↳ omdat we maar op bepaalde punten meten dus gebruiken.

Periodick / Aperodic signaal

Sinusoidal

$x(t) = A \sin(2\pi f_0 t + \theta)$

alle constant behalve t

Signalen en ~~Signale~~ Systeme

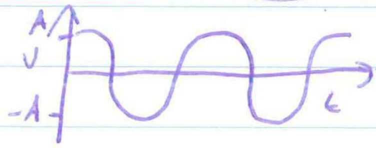
Phasors signals and Spectra

$x(t) = A\cos(\omega_0 t + \theta)$ (periodiek) ①

② $\tilde{x}(t) = Ae^{j(\omega_0 t+\theta)} = |A\cos(\omega_0 t+\theta) + jA\sin(\omega_0 t+\theta)|$

③ $= Ae^{j\theta}\cdot e^{j\omega_0 t}$

Complex
↳ hiervan reeel neme geeft van $\tilde{x}$ weer x

① 

Periodiek signaal

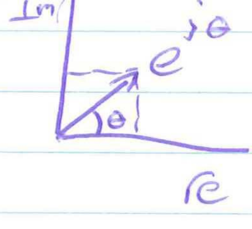
$\omega = 2\pi f$, hoeksnelheid rad/s

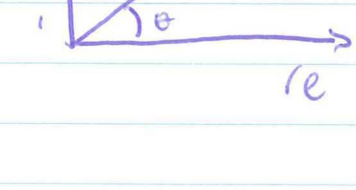
② is complexe $\tilde{x}$ deel, makkelijk met rekene
↳ later weer terug van $R(\tilde{x}) = x$

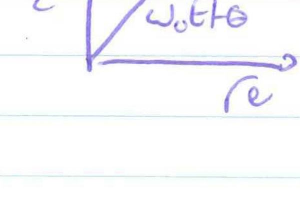
③ $\boxed{\underset{\text{Ampl}}{A}\,\underset{\text{fase}}{e^{j\theta}}\cdot e^{j\omega_0 t}}$ → rotating phasor ④
↳ phasor signaal

↓

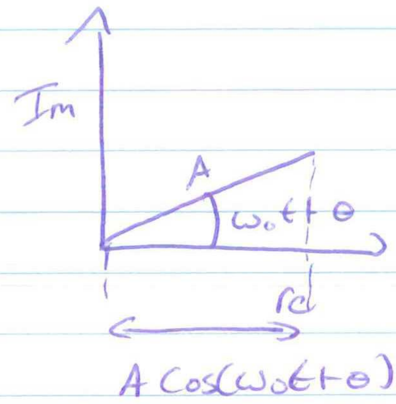
Phasor = vaste

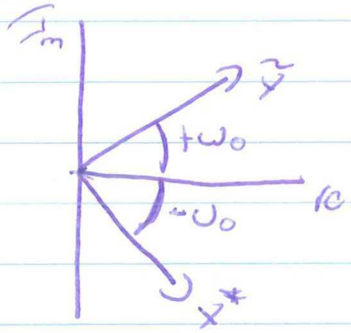
$e^{j\theta} = \underbrace{\cos\theta}_{\text{reeel}} + \underbrace{j\sin\theta}_{\text{Imaginair}}$ ⇒ 

④ 



$X = Re(\tilde{x} e^{j\omega_0 t}) = A\cos(\omega_0 t + \theta)$

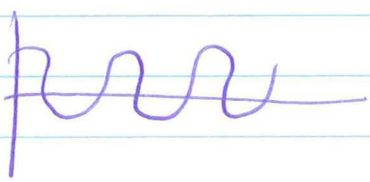




$x^* = \text{complex} = A\cos(\omega_0 t+\theta) - jA\sin(\omega_0 t+\theta)$

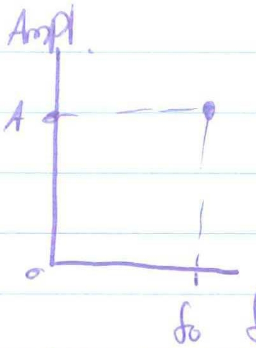
terug naar reëel : $x = \frac{1}{2}A e^{j(\omega_0 t+\theta)} + \frac{1}{2}A e^{-j(\omega_0 t+\theta)}$

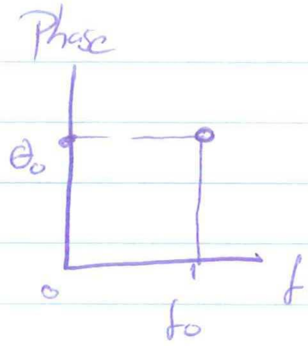
Als wij aan signale denke. zie wij



we moete in frequentie domeine gaa werken.

frequentie domein

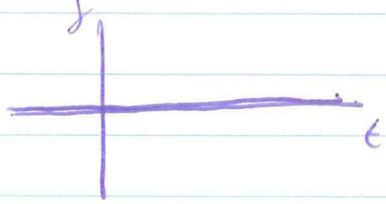




Amplitude spectrum phase spectrum

blz. 22.

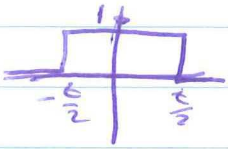
$\delta = 0 \quad , \quad t \neq 0$



$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

maar $\delta = 0$ dus bij $t=0$ komt impuls, 1 klap ervoor was niks en daarna was niks.

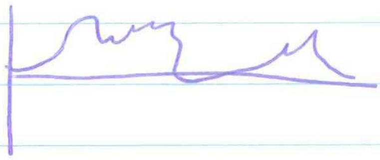
$$\Pi\left(\frac{t}{T}\right) = \begin{cases} 1 & -\frac{t}{2} < t < \frac{t}{2} \\ 0 & \dots \end{cases} \rightarrow$$

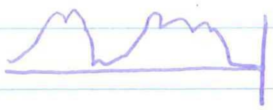


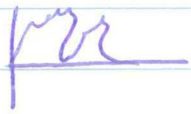
deze $-\frac{t}{2}$ e $\frac{t}{2}$ steeds kleiner make

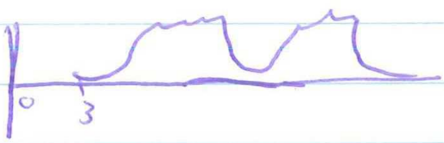
→ hierbij wordt Ampl. groter. zie blz 23 fig 1-11.

even tussendoor

$x(t)$ $\rightarrow$ 

$x(-t)$ 

$x(6t)$ (6x zo snel) 

$x(t-3)$ 

3.11

$$\sin(x)\cdot\sin(y) = -\tfrac{1}{2}\cos(x+y) + \tfrac{1}{2}\cos(x-y)$$

$$= -\tfrac{1}{2}\cos(x+y) + \tfrac{1}{2}\cos(0)$$

$$= -\tfrac{1}{2}\cos(x+y) + \tfrac{1}{2}$$