Delft University of Technology

Faculty of Civil Engineering and Geosciences

AESB2120: Instrumentation and signals with Matlab (5 EC)
TOETS3

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responsible instructors P. López Dekker and S. Basu

name: [redacted]

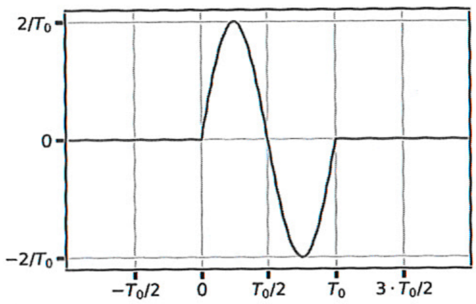
student-number: [redacted]

1. This test consists of 5 open questions.
2. The values of the questions are indicated between brackets, and the total score adds up to 16 points.
3. This test (**toets03**) counts for **30%** of the final grade for the course AESB2120.
4. The minimum grade to pass this course is 5.75, which will be rounded to 6.0.
5. The maximum grade for this course is 10.0.

6. This is a **closed-book**, written test (hence: no book, no slides, no notes).
7. A copy of the **formula-sheet** – as used during the course – will be **provided**.
8. The use of a pocket-calculator is allowed; mobile phones and other electronic devices should be switched off, and stored away.
9. Answers can be given in English or Dutch.
10. By default, time *t* is expressed in seconds [s], and frequency *f* in Hertz [Hz].

Question 1 [4pt] Certain LTI system has the following impulse response

$$h(t) = \frac{2}{T_0} \cdot \sin(2\pi f_0 t) \cdot \Pi\left(\frac{t - \frac{T_0}{2}}{T_0}\right),$$



as shown in the figure, with $f_0 = 1/T_0$. Consider now the frequency response $H(f)$.

a. Without doing any calculation nor any Fourier transform (yet), determine the frequency response at $f = 0$.

b. Using the properties of the Fourier transform, and the Fourier transform table, show that

$$H(f) = j\left(\operatorname{sinc}\left(\frac{f - f_0}{f_0}\right) - \operatorname{sinc}\left(\frac{f + f_0}{f_0}\right)\right) \cdot e^{-j\pi T_0 f}$$

Math tips:

- $e^{-j\pi T_0 (f - f_0)} = e^{-j\pi T_0 f} \cdot e^{j\pi T_0 f_0} = -e^{-j\pi T_0 f};$
- $\frac{1}{j} = -j$

c. Does this behave more like a low-pass filter, or like a band-pass filter? Please explain.

d. The system is fed with the following signal:

$$x(t) = 1 + 2 \cdot \cos(2\pi f_0 t) + 4 \cdot \cos(2\pi \cdot 2 f_0 t)$$

What will be the power of the signal at the output of the filter?

Question 2 [4pt] Consider the Gaussian pulse signal

$$x(t) = e^{-\left(\frac{t}{2}\right)^2}$$

and its convolution with itself reversed (which we call the autocorrelation):

$$r(t) = x(t) * x(-t)$$

We also know that the Fourier transform of a Gaussian signal is another Gaussian:

$$\mathcal{F}\left\{e^{-\left(\frac{t}{\tau}\right)^2}\right\} = \tau\sqrt{\pi} \cdot e^{-(\pi f \tau)^2}$$

a. Show that the Fourier transform of $r(t)$ is given by

$$R(f) = 4\pi e^{-2(2\pi f)^2}$$

b. Find $r(t)$.

You could of course write $r(t)$ as

$$r(t) = \int_{-\infty}^{\infty} R(f)\, e^{j2\pi f t} df$$

c. Show that

$$\int_{-\infty}^{\infty} |x(t)|^2 dt = r(0)$$

d. What is the Energy of $x(t)$?

Question 3 [3pt] Imagine the following situation. For a sea-level rise research project we want to deploy a network of (many) very simple tide gauges. Tides have a semi-diurnal cycle (two cycles per day), whose amplitude depends on the lunar phase, the position of the Earth with respect to the Sun. We are also interested in investigating possible sea level dependencies with respect to the Solar cycle, which has a period of 11 years. In other words, we want to be able to resolve a frequency component with frequency 1/11 cyles per year. Considering the components discussed:

a. With what frequency would you need to sample the tidal signal in order to be able to reconstruct it? You may want to use samples/day as a frequency unit.
b. For how long should the experiment run?

You may have thought of this: if we are not careful, ocean-waves would corrupt the measurements. Usually, tide gauges incorporate mechanism to filter out the waves. Imagine that our tide gauges filter out all signals with period shorter than 1 minute.

c. Would this modify the answer to the previous sections?

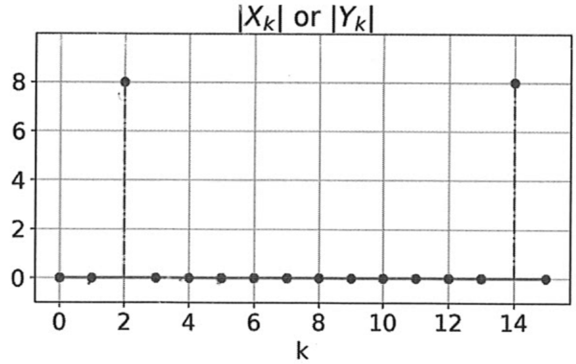
Question 4 [2pt] A continuous time signal x(t) is filtered with an ideal low pass filter with frequency response

$$H(f) = \Pi\left(\frac{f}{16}\right)$$

producing a signal y(t). Both x(t) and y(t) are ideally sampled, producing the sequences

$$x[n] = x(n \cdot T_s)$$
$$y[n] = y(n \cdot T_s),$$

with $T_s = 1/16$. Consider the DFTs of 16 samples of both signals, and the figure below:

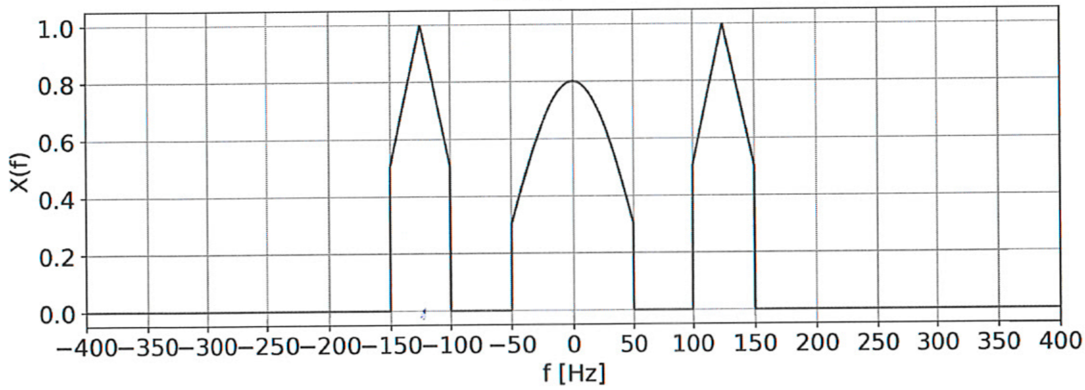


In the following table indicate with True (T) or False (F) for which of the signals given the DFTs , $X[k]$ and $Y[k]$ correspond to the figure. (Note, for each row you get 0.5 pts if both answers are correct, 0.25 if one is correct and the other left blank, and 0 pts if any of the answers is wrong).

Signal	X_k	Y_k
$x(t) = \cos(2\pi \cdot 2 \cdot t)$	Y	Y
$x(t) = \cos(2\pi \cdot 18 \cdot t)$		
$x(t) = \cos(2\pi \cdot 2 \cdot t) + \cos(2\pi \cdot 10 \cdot t)$		
$x(t) = \cos(2\pi \cdot 2 \cdot t) + \cos(2\pi \cdot 16 \cdot t)$		
$x(t) = \cos(2\pi \cdot 2 \cdot t) + \sin(2\pi \cdot 16 \cdot t)$		

Question 5 [3pt] The x(t) signal whose spectrum is sketched in the figure below, is ideally sampled:

$$x_s(t) = x(t) \cdot \sum_{n=-\infty}^{\infty} T_s \cdot \delta(t - n \cdot T_s).$$



a. Assuming a sampling frequency of $F_s = 200$ Hz, sketch the spectrum of the sampled signal, $X_s(f)$. You may do this directly on the figure.

b. Assuming that you knew what ranges of frequencies contained signal, would it be possible to reconstruct the original signal from its samples? Please explain.

c. **If you answered yes to b**: please sketch how this reconstruction could be implemented. If you propose to use any filters, please sketch their frequency response.
If you answered no to be: please indicate the minimum sampling frequency required to allow perfect reconstruction of $x(t)$.