Delft University of Technology

Faculty of Civil Engineering and Geosciences

TU Delft

AESB2120: Instrumentation and signals with Matlab (5 EC)
TOETS02

5th October, 2017

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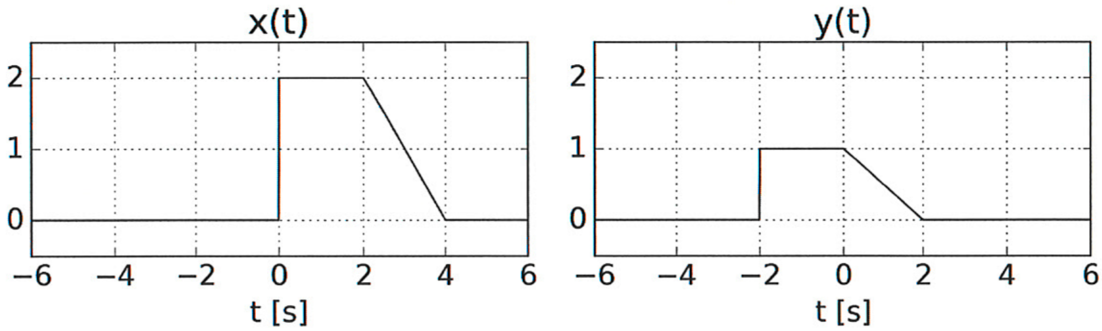
name: [redacted]	
student-number: [redacted]	

1. This test consists of 6 open questions, adding up to 14 points.
2. This test (**toets02**) counts for **30%** of the final grade for the course AESB2120.
3. The minimum grade to pass this course is 5.75, which will be rounded to 6.0.
4. The maximum grade for this course is 10.0.

5. This is a **closed-book**, written test (hence: no book, no slides, no notes).
6. A copy of the **formula-sheet** – as used during the course – will be **provided**.
7. The use of a pocket-calculator is allowed; mobile phones and other electronic devices should be switched off, and stored away.
8. For question 2 you may fill the answers in the table provided.
9. Answers can be given in English or Dutch.
10. By default, time *t* is expressed in seconds [s], and frequency *f* in Hertz [Hz].

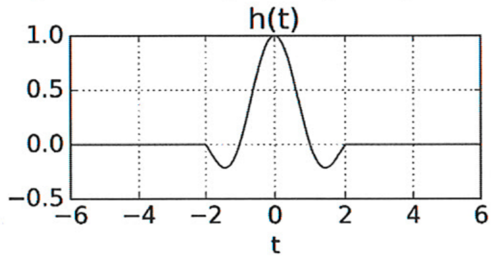
Question 1 [2pt] Consider the pairs of signals x(t), and y(t) represented in the figure below, where y(t) is the response to a certain LTI system to the input x(t).



a) Describe in one sentence what the system [redacted] a mathe[redacted]atical description for it.
b) What is the impulse response of the s[redacted] causal[redacted]

[redacted]

Question 2 [2pt] For the systems given in the following table indicate (as in the example in the first row) which properties hold.

System	Linear	Time-Invariant	Causal
A system with impulse response $h(t) = \Pi\left(\frac{t-1}{2}\right)$	Yes	Yes	Yes
A thermometer that saturates at 50 degree: $y(t) = \begin{cases} x(t) & \text{for } x(t) < 50 \\ 50 & \text{for } x(t) \geq 50 \end{cases}$			
A system with the impulse response given in the figure 			
A system for which the output y(t) to an input x(t) is given by $y(t) = 1 + 0.5 \cdot x(t)$			
A system for which the output y(t) to an input x(t) is given by $y(t) = u(-t) \cdot x(t)$			

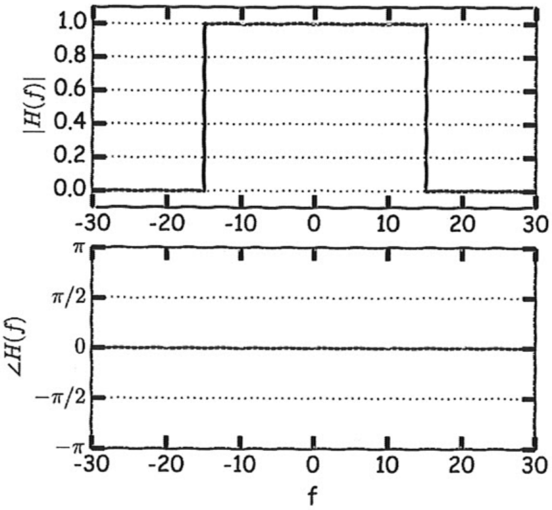
Question 3 [2pt]

a) Show that $\int_{-\infty}^{t} \delta(\alpha) d\alpha = u(t)$.

b) Find the **impulse** response, **h(t)**, of a system defined by the following relation:

$$\mathcal{H}\{x(t)\} = \int_{-\infty}^{t} x(\alpha) \cdot e^{-2(t-\alpha)} d\alpha$$

Question 4 [2pt] Consider a periodic signal ***y(t)***, with period T_0=0.1 s (i.e. fundamental frequency 10 Hz), that has been generated by filtering another periodic signal (x(t)) with a system with the frequency response represented below.



We also are now that:

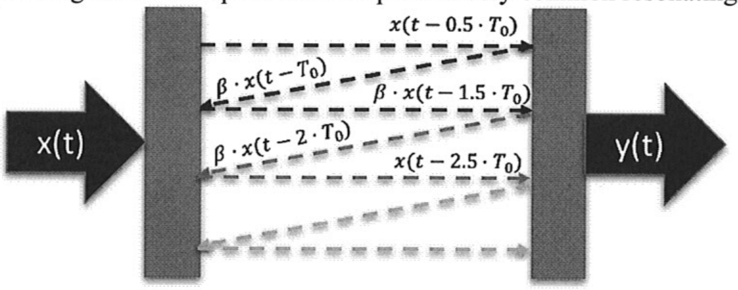
- The signal has even symmetry (y(t) = y(-t))
- The mean value of the signal is 1
- The power of the signal is 3

Because it as periodic signal, we can write is as

$$y(t) = \sum_{n=0}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

a) Explain why all $b_n$coefficients must be zero.
b) Find the values of all non-zero $a_n$coefficients.

Question 5 [5pt] The figure below represents a simple and very common resonating structure:



The signal enters the structure through the left boundary, and travels until the second boundary where it arrives after $0.5 \cdot T_0$ seconds. There, a fraction $\beta$of the signal is reflected and goes back to the first boundary, where it is reflected again, etc. The signal at the second boundary can be written as

$$y_1(t) = x(t - 0.5 \cdot T_0) + \beta \cdot x(t - 1.5 \cdot T_0) + \beta^2 \cdot x(1 - 2.5 \cdot T_0) + \cdots$$

$$= \sum_{n=0}^{\infty} \beta^n \cdot x(t - (n + 0.5)T_0)$$

The output of the system can be written as $y(t) = \alpha y_1(t)$, so that we have:

$$y(t) = \alpha \cdot \sum_{n=0}^{\infty} \beta^n \cdot x(t - (n + 0.5) \cdot T_0)$$

a) Show that the impulse response of this system is given by

$$h(t) = \alpha \cdot \delta(t - 0.5 \cdot T_0) * \sum_{n=0}^{\infty} \beta^n \cdot \delta(t - n \cdot T_0)$$

For the rest of the exercise we will simplify this slightly and assume

$$h(t) = \alpha \cdot \sum_{n=0}^{\infty} \beta^n \cdot \delta(t - n \cdot T_0)$$

b) To go step by step: consider the impulse response $h_n(t) = \beta^n \cdot \delta(t - n \cdot T_0)$; what is the corresponding frequency response, $H_n(f)$?
c) Derive the expression of the frequency response of this system, $H(f)$.
d) Show that H(f) can be written as

$$H(f) = \frac{\alpha}{1 - \beta \cdot e^{-j2\pi f T_0}}$$

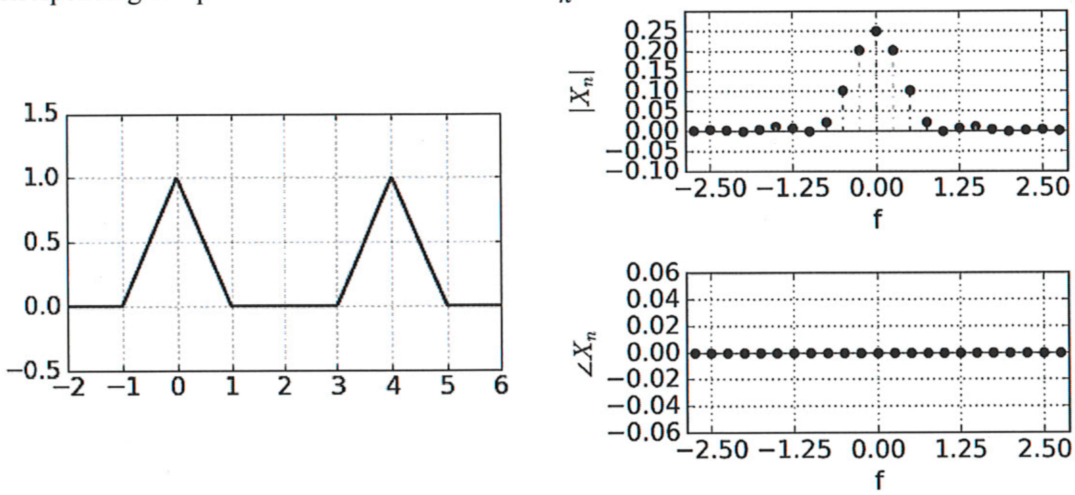
for this you may want to use the following result (valid for $|a| < 1$):

$$1 + a + a^2 + \cdots = \sum_{k=0}^{\infty} a^k = \frac{1}{1 - a}$$

e) Taking $\beta = 0.99$ and $\alpha = 0.01$, what would be output of the system if the input were

$$x(t) = \cos\frac{\pi t}{T_0} + \cos\frac{2\pi t}{T_0}$$

Question 6 [1pt] Consider the train of triangular functions shown in the Figure, and the corresponding complex Fourier series coefficients X_n.



a) Calculate

$$\sum_{n=-\infty}^{\infty} |X_n|^2$$