**Delft University of Technology**

**Faculty of Civil Engineering and Geosciences**

TU Delft

# AESB2120: Instrumentation and signals with Matlab (5 EC)
# TOETS02

4th October, 2018

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| name: |
|---|
| student-number: |

1. This test consists of 5 open questions, adding up to 15 points.
2. All sub-questions are worth 1 point, unless otherwise indicated.
3. This test (**toets02**) counts for **30%** of the final grade for the course AESB2120.
4. The minimum grade to pass this course is 5.75, which will be rounded to 6.0.
5. The maximum grade for this course is 10.0.
6. The final grade will be calculated as $\text{grade} = 1 + 9 \cdot \frac{\text{points obtained}}{15}$
7. This is a **closed-book**, written test (hence: no book, no slides, no notes).
8. A copy of the **formula-sheet** – as used during the course – will be **provided**.
9. The use of a pocket-calculator is allowed; mobile phones and other electronic devices should be switched off, and stored away.
10. For question 2 you may fill the answers in the table provided.
11. Answers can be given in English or Dutch.
12. By default, time $t$ is expressed in seconds [s], and frequency $f$ in Hertz [Hz].

**Question 1 [2pt]**

a) Sketch the following signal

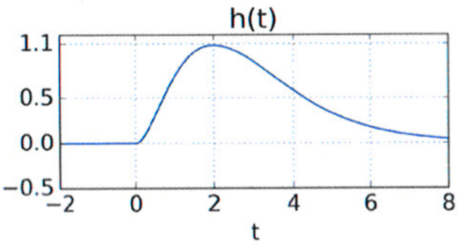
$$x(t) = e^{-t} \cdot u(t-1).$$

b) This signal is now given as an input to an LTI system with impulse response

$$h(t) = -\delta(t-2).$$

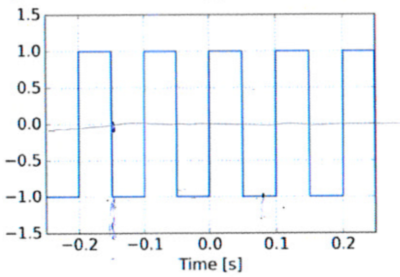
Sketch the output signal (i.e. $y(t) = x(t) * h(t)$).

**Question 2 [2pt]** For the systems given in the following table indicate (as in the example in the first row) which properties hold. For each correctly filled cell you will receive 1/6 pts. Wrong answers subtract 1/6 point each, provided that the total score for each system remains larger or equal to zero.

| **System** | **Linear** | **Time-Invariant** | **Causal** |
|---|---|---|---|
| A system with impulse response $h(t) = \Pi\left(\frac{t-1}{2}\right)$ | Yes | Yes | Yes |
| A solar-powered thermometer such that: $y(t) = \begin{cases} x(t) & \text{at day time} \\ 0 & \text{at night time} \end{cases}$ | | | |
| A system where the output $y(t)$ to a signal $x(t)$ is given by the convolution $y(t) = x(t) * h(t)$, with h(t) as shown below.   | | | |
| A system for which the output y(t) to an input x(t) is given by $y(t) = \cos \omega_1 t \cdot x(t+1)^2$ | | | |
| A physical system described by the following ODE: $\frac{d^2}{dt^2}y(t) - \frac{d}{dt}y(t) + y(t) = a \cdot x(t)$ | | | |

**Question 3 [4pt]**

Consider the periodic signal, $x(t)$, represented in the figure below.

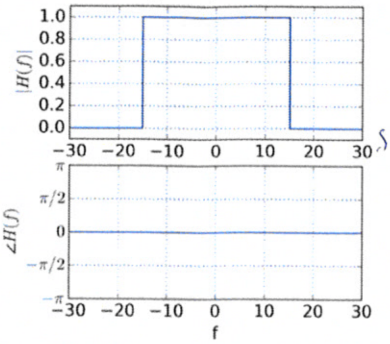



Consider also its trigonometric Fourier expansion,

$$x(t) = \sum_{n=0}^{\infty} a_n \cdot \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \cdot \sin(n\omega_0 t)$$

a) Calculate the power of the signal (*Hint: remember to look for the easiest approach*).
b) Without doing any calculations, determine the values of $a_n$. Justify your answer.

The signal is filtered with a system with the frequency response represented below




c) Write down an expression of the output signal, $y(t)$, showing only the relevant frequency components.
d) Calculate the power of $y(t)$.

**Question 4 [4pt]**
A certain system is described by the following relation:

$$y(t) = \int_{-\infty}^{t} x(\alpha) \cdot e^{-(t-\alpha)} d\alpha.$$

a) Show that the impulse response of the system is $h(t) = u(t) \cdot e^{-t}$.
b) Determine its frequency response, $H(f)$.
c) The system is fed with a constant signal, $x(t) = a_0$. What will be the output signal?
d) Sketch the output system if the system is fed with a signal $x(t) = \Pi\left(\frac{t-4}{4}\right)$. If there are changes of trends in the output signal, please indicate in the graph at which times these changes take place.

**Question 5 [3pt]**
Consider a train of Dirac-delta functions

$$x(t) = \sum_{n=-\infty}^{\infty} \delta(t - n \cdot 0.1).$$

a) Show that the complex Fourier series coefficients are $X_n = 10$ for all values of n.[0.5pt]
b) Show that this signal has infinite power. [0.5pt]

The signal is now given as an input to a system with the following frequency response

$$H(f) = \frac{1}{2} \cdot \Pi\left(\frac{f+15}{20}\right) + \frac{1}{2} \cdot \Pi\left(\frac{f-15}{20}\right)$$

obtaining a signal $y(t)$.
c) What is the power of $y(t)$?
d) Determine the time-domain expression of $y(t)$