naam *name*	
studienummer *student number*	
vak *course*	
code *code*	datum *date*
opleiding *program*	
aantal ingeleverde vellen *total number of sheets*	opgave nummer *question number* 1

TU Delft

Technische Universiteit Delft

$\boxed{1}$ a) $g(t) = \sin t - u_\pi(t)\cdot \sin t$
$= \sin t - u_\pi(t)\cdot \sin((t-\pi)+\pi)$
$= \sin t + u_\pi(t)\cdot \sin(t-\pi)$, want $\sin(t+\pi) = -\sin t$

Via tabel: $G(s) = \frac{1}{s^2+1} + \frac{e^{-\pi s}}{s^2+1}$

b) $y''(t) + 4y(t) = \sin t,\ y(0) = a,\ y'(0) = b \xrightarrow{\mathcal{L}}$

$(s^2Y(s) - as - b) + 4Y(s) = \frac{1}{s^2+1}$

$\Leftrightarrow Y(s) = \underbrace{\frac{1}{(s^2+1)(s^2+4)}} + a\frac{s}{s^2+4} + \frac{b}{s^2+4}$

$= \frac{A}{s^2+1} + \frac{B}{s^2+4} \longleftrightarrow A = \frac{1}{3},\ B = -\frac{1}{3}$

terugtransformeren:

$y(t) = \frac{1}{3}\sin t - \frac{1}{3}\cdot\frac{1}{2}\sin(2t) + a\cos(2t) + \frac{1}{2}b\sin(2t)$

c) $\left.\begin{array}{l} y''(t) + 4y(t) = g(t) \\ y(0) = 0,\ y'(0) = 2 \end{array}\right\} \xrightarrow{\mathcal{L}} s^2Y(s) - 2 + 4Y(s) = G(s) \longleftrightarrow$

$Y(s) = \frac{2}{s^2+4} + \frac{1}{(s^2+4)(s^2+1)} + e^{-\pi s}\cdot\frac{1}{(s^2+4)(s^2+1)}$

$y(t) = \sin(2t) + \frac{1}{3}\sin t - \frac{1}{3}\cdot\frac{1}{2}\sin(2t)$
$+ u_\pi(t)\cdot\left[\frac{1}{3}\sin(t-\pi) - \frac{1}{3}\cdot\frac{1}{2}\sin(2(t-\pi))\right]$

$= \begin{cases} \sin(2t) + \frac{1}{3}\sin t - \frac{1}{6}\sin(2t), & \text{als } t \leq \pi \\ \frac{2}{3}\sin(2t), & \text{als } t \geq \pi \end{cases}$

naam *name*	
studienummer *student number*	
vak *course*	
code *code*	datum *date*
opleiding *program*	
aantal ingeleverde vellen *total number of sheets*	opgave nummer *question number*: 2

TU Delft Technische Universiteit Delft

a) $\underline{x}' = \begin{bmatrix} 1 & 2 \\ -4 & -3 \end{bmatrix} \underline{x} = A\underline{x}$. Oplossen via e.w.^{n}/e.v.^{n} van A:

$\text{Det}(A - \lambda I) = (1-\lambda)(-3-\lambda) + 8 = \lambda^2 + 2\lambda + 5$
$= (\lambda+1)^2 + 4$

e.w.^{n} van A: $\lambda_{1,2} = -1 \pm 2i$

e.v. bij $\lambda = -1+2i$: $\left[\begin{array}{cc|c} 1-(-1+2i) & 2 & 0 \\ -4 & -3-(-1+2i) & 0 \end{array}\right] =$

$= \left[\begin{array}{cc|c} 2-2i & 2 & 0 \\ -4 & -2-2i & 0 \end{array}\right] \downarrow 1+i \sim \left[\begin{array}{cc|c} 2-2i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$

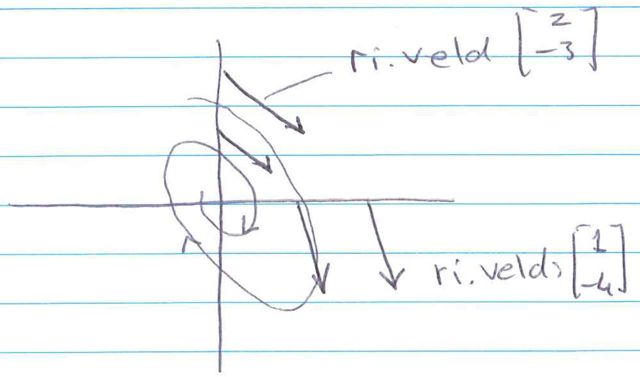
e.v. $\underline{v} = \begin{bmatrix} -1 \\ 1-i \end{bmatrix}$

opl. v. $\underline{x}' = A\underline{x}$: $\begin{bmatrix} -1 \\ 1-i \end{bmatrix} e^{(-1+2i)t} = \begin{bmatrix} -1 \\ 1-i \end{bmatrix} e^{-t}(\cos(2t) + i\sin(2t))$

$= \underbrace{\begin{bmatrix} -\cos(2t) \\ \cos(2t)+\sin(2t) \end{bmatrix} e^{-t}}_{\underline{x}_1(t)} + i \underbrace{\begin{bmatrix} -\sin(2t) \\ -\cos(2t)+\sin(2t) \end{bmatrix} e^{-t}}_{\underline{x}_2(t)}$: reële opl^n

Alg. opl.: $\underline{x} = c_1 \underline{x}_1(t) + c_2 \underline{x}_2(t)$, $c_1, c_2 \in \mathbb{R}$

b) e.w.^{n}: $-1 \pm 2i$:
stabiel
spiraalpunt



2 e $\underline{x}(t) = c_1 \underline{x}_1(t) + c_2 \underline{x}_2(t)$

$\underline{x}(0) = \begin{bmatrix} 1 \\ 4 \end{bmatrix} = c_1 \begin{bmatrix} -1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ -1 \end{bmatrix}$ geeft $c_1 = -1,\ c_2 = 3$

Antw: $\underline{x}(t) = (-1) \cdot \begin{bmatrix} -\cos(2t) \\ \cos(2t) + \sin(2t) \end{bmatrix} e^{-t} +$

$+ 3 \begin{bmatrix} -\sin(2t) \\ -\cos(2t) + \sin(2t) \end{bmatrix} e^{-t} = \begin{bmatrix} \cos(2t) - 3\sin(2t) \\ -4\cos(2t) + 4\sin(2t) \end{bmatrix} e^{-t}$

d Aanwijzing volgen: $(\underline{w} e^{-t})' = A \cdot (\underline{w} e^{-t}) + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t}$

$\Leftrightarrow -\underline{w} e^{-t} = (A\underline{w}) \cdot e^{-t} + \begin{bmatrix} 1 \\ -3 \end{bmatrix} e^{-t}$

$\Leftrightarrow A\underline{w} + \underline{w} = (A + I)\underline{w} = -\begin{bmatrix} 1 \\ -3 \end{bmatrix} = \begin{bmatrix} -1 \\ 3 \end{bmatrix}$

$\left[\begin{array}{cc|c} 2 & 2 & -1 \\ -4 & -2 & 3 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 2 & -1 \\ 0 & 2 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 2 & 0 & -2 \\ 0 & 2 & 1 \end{array}\right] \Rightarrow \underline{w} = \begin{bmatrix} -1 \\ 1/2 \end{bmatrix}$

particuliere oplossing: $\underline{x}_p(t) = \begin{bmatrix} -1 \\ +\frac{1}{2} \end{bmatrix} e^{-t}$.

naam *name*			
studienummer *student number*			
vak *course*			
code *code*		datum *date*	
opleiding *program*			
aantal ingeleverde vellen *total number of sheets*		opgave nummer *question number*	3

Technische Universiteit Delft

a $\begin{cases} dx/dt = 0 \\ dy/dt = 0 \end{cases} \longleftrightarrow \begin{cases} x^2 - y = 0 \\ (x-1)(y-4) = 0 \longleftrightarrow x = 1 \text{ of } y = 4 \end{cases}$

Rustpunten: $(1, 1)$, $(2, 4)$ en $(-2, 4)$

b Lineariseringen: $J(x,y) = \begin{bmatrix} 2x & -1 \\ y-4 & x-1 \end{bmatrix}$

in $(1,1)$: $\begin{bmatrix} 2 & -1 \\ -3 & 0 \end{bmatrix}$ e.w^n: $(\lambda - 2)(\lambda - 0) - 3 = 0$

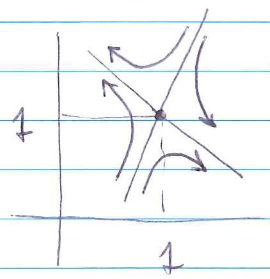
$\lambda^2 - 2\lambda - 3 = 0$

$(\lambda - 3)(\lambda + 1) = 0$

$\lambda_1 = 3 > 0$, $\lambda_2 = -1 < 0 \Longrightarrow$ zadelpunt

e.v.^{n}: bij λ_1: $\left[\begin{array}{cc|c} -1 & -1 & 0 \\ -3 & -3 & 0 \end{array}\right] \rightarrow \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

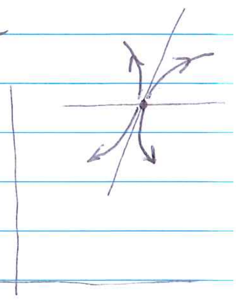
bij λ_2: $\left[\begin{array}{cc|c} 3 & -1 & 0 \\ -3 & 1 & 0 \end{array}\right] \rightarrow \begin{bmatrix} 1 \\ 3 \end{bmatrix}$



in $(2,4)$: $J = \begin{bmatrix} 4 & -1 \\ 0 & 1 \end{bmatrix}$ e.w^n: $4 \rightarrow$ e.v. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$1 \rightarrow$ e.v. $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

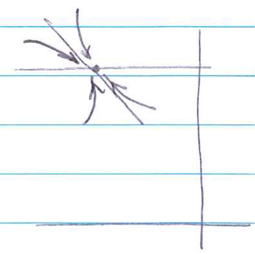
instabiele knoop



in $(-2,4)$: $J = \begin{bmatrix} -4 & -1 \\ 0 & -3 \end{bmatrix}$ e.w^n: $-4 \rightarrow$ e.v. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

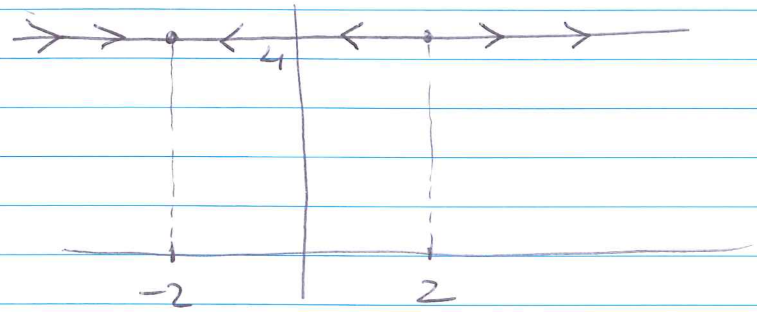
$-3 \rightarrow$ e.v. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

stabiele knoop



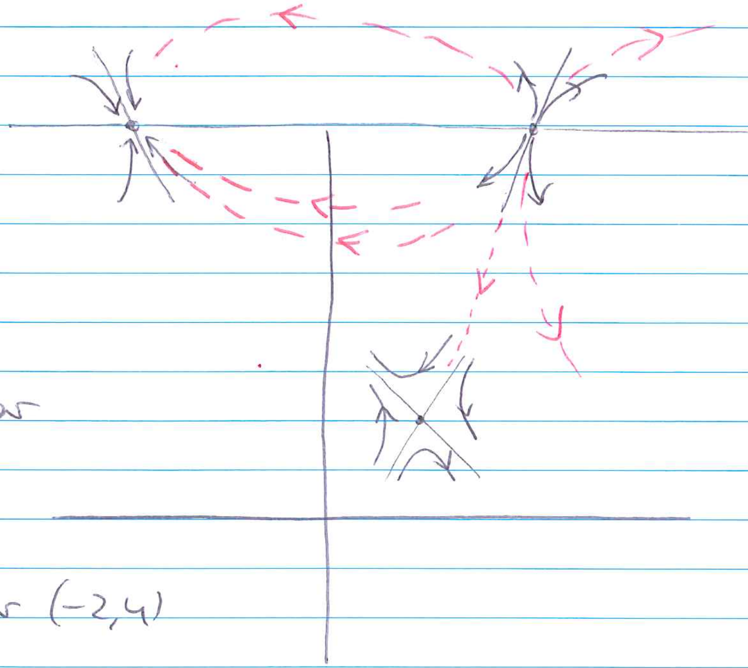
3 c $y' = 0$ als $y = 4$, dus oplossingen die starten vanuit een punt $(a, 4)$ blijven op de lijn $y=4$

Verder, op deze lijn $x' = x^2 - 4$ is negatief tussen $x = -2$ en $x = 2$ en daarbuiten positief



merk op: dit is consistent met de twee knopen in $(\pm 2, 4)$

d overall picture (sort of)



er zullen ∞ veel opl^n van $(2,4)$ naar $(-2,4)$ gaan

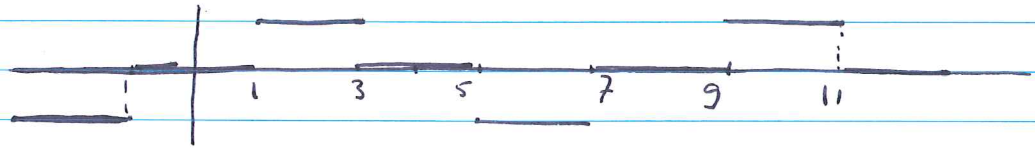
er zullen ∞ veel opl^n van $(1,1)$ naar $(-2,4)$ gaan

tussen de oplossingen die vanuit $(2,4)$ naar beneden gaan en naar links dan wel naar rechts afbuigen zal er 1 zijn die uitkomt in $(1,1)$

naam *name*			
studienummer *student number*			
vak *course*			
code *code*		datum *date*	4
opleiding *program*			
aantal ingeleverde vellen *total number of sheets*		opgave nummer *question number*	

Technische Universiteit Delft

s is de oneven periodieke voortzetting van f met periode 8:



$$s(x) = \sum_{k=1}^{\infty} b_k \sin\left(k \cdot \frac{\pi}{4} x\right)$$

$$b_k = \frac{2}{4} \int_0^4 f(x) \sin\left(\frac{k\pi}{4} x\right) dx = \frac{2}{4} \int_1^3 1 \cdot \sin\left(\frac{k\pi}{4} x\right) dx$$

"L"

$$= \frac{2}{4} \left[-\frac{4}{k\pi} \cos\left(\frac{k\pi}{4} x\right) \right]_1^3 = \frac{2}{k\pi} \left(\cos\left(\frac{k\pi}{4}\right) - \cos\left(\frac{3k\pi}{4}\right) \right).$$

k	1	2	3	4	5	6	7	8	9
$\cos(k\pi/4)$	$\frac{1}{2}\sqrt{2}$	0	$-\frac{1}{2}\sqrt{2}$	-1	$-\frac{1}{2}\sqrt{2}$	0	$\frac{1}{2}\sqrt{2}$	1	
$\cos(3k\pi/4)$	$-\frac{1}{2}\sqrt{2}$	0	$\frac{1}{2}\sqrt{2}$	-1	$\frac{1}{2}\sqrt{2}$	0	$-\frac{1}{2}\sqrt{2}$	1	
b_k	$\frac{2\sqrt{2}}{\pi}$	0	$-\frac{2\sqrt{2}}{3\pi}$	0	$-\frac{2\sqrt{2}}{5\pi}$	0	$\frac{2\sqrt{2}}{7\pi}$	0	

merk op: $b_{k+8} = b_k$, dus $(b_k)_{k=1}$ geeft een repeterend patroon.

Eerste vijf termen $\neq 0$ v.d. sinusreeks:

$$\frac{2\sqrt{2}}{\pi} \cdot \left(\sin \frac{\pi x}{4} - \frac{1}{3} \sin\left(\frac{3\pi x}{4}\right) - \frac{1}{5} \sin\left(\frac{5\pi x}{4}\right) + \frac{1}{7} \sin\left(\frac{7\pi x}{4}\right) + \frac{1}{9} \sin\left(\frac{9\pi x}{4}\right) \right)$$

naam *name*	
studienummer *student number*	
vak *course*	
code *code*	datum *date*
opleiding *program*	
aantal ingeleverde vellen *total number of sheets*	opgave nummer *question number* 5

TU Delft
Technische Universiteit Delft

standaardprobleem!

<u>korte</u> uitwerking (want na te lezen in B&dP)

$$u(x,y) = X(x)Y(y) \longrightarrow \dots \longrightarrow \frac{X''(x)}{X(x)} = -\frac{Y''(y)}{Y(y)} = c$$

$$\left.\begin{array}{l} u(x,0)=0 \Rightarrow Y(0)=0 \\ u(x,4)=0 \Rightarrow Y(4)=0 \\ \frac{Y''(y)}{Y(y)} = c \end{array}\right\} \Rightarrow Y_n(y) = \sin\left(\frac{n\pi}{4}y\right), \quad n=1,2,\dots$$

voor $c = -\left(\frac{n\pi}{4}\right)^2$

$$\left.\begin{array}{l} \frac{X''(x)}{X(x)} = -c = +\left(\frac{n\pi}{4}\right)^2 \\ u(0,y)=0 \Rightarrow X(0)=0 \end{array}\right\} X_n(x) = (A)\cdot \sinh\left(\frac{n\pi}{4}x\right)$$

$$u(x,y) = \sum_{n=1}^{\infty} a_n \sinh\left(\frac{n\pi}{4}x\right)\cdot \sin\left(\frac{n\pi}{4}y\right)$$

$$u(2,y) = \sum_{n=1}^{\infty} \underbrace{a_n \sinh\left(\frac{n\pi}{2}\right)}_{=\begin{cases}0 & \text{als } n\neq 4\\ 1 & \text{als } n=4\end{cases}}\cdot \sin\left(\frac{n\pi}{4}y\right) = \sin(\pi y) \quad \text{(III)}$$

$$\Rightarrow a_4 = \frac{1}{\sinh\left(\frac{4\pi}{2}\right)}$$

$a_n = 0, \; n \neq 4$

<u>Antwoord</u>: $u(x,y) = \dfrac{\sinh\left(\frac{4\pi}{4}x\right)\cdot \sin(\pi y)}{\sinh(2\pi)}$

<u>b</u> om aan (III') te voldoen: vervang x door $(2-x)$:

$$\tilde{u}(x,y) = \frac{\sinh(\pi\cdot(2-x))\cdot \sin(\pi y)}{\sinh(2\pi)}$$