Analyse 4 Wi1300TA, 22 augustus 2011.

1 a). $\underline{r}(t) = \begin{pmatrix} \cos t \\ \sin t \\ 2t \end{pmatrix}$, $\underline{r}'(t) = \begin{pmatrix} -\sin t \\ \cos t \\ 2 \end{pmatrix}$, $|\underline{r}'(t)| = \sqrt{5}$

$$\int_C y\,ds = \int_0^{\pi} \sin t\,\sqrt{5}\,dt = \sqrt{5}\left[-\cos t\right]_0^{\pi} = 2\sqrt{5}$$

b). $$m = \int_C \rho\,ds = \rho\sqrt{5}\int_0^{\pi} dt = \rho\pi\sqrt{5}$$

$$\bar{x} = \frac{1}{m}\int_C x\rho\,ds = \frac{\rho}{m}\int_0^{\pi} \cos t\,\sqrt{5}\,dt = 0$$

$$\bar{y} = \frac{1}{m}\int_C y\rho\,ds = \frac{\rho}{m}\int_C y\,ds \overset{\text{zie (a)}}{=} \frac{\rho}{\rho\pi\sqrt{5}}\cdot 2\sqrt{5} = \frac{2}{\pi}$$

$$\bar{z} = \frac{1}{m}\int_C z\rho\,ds = \frac{\rho}{m}\int_0^{\pi} 2t\sqrt{5}\,dt = \pi$$

$$(\bar{x}, \bar{y}, \bar{z}) = \left(0, \frac{2}{\pi}, \pi\right)$$

2. a) i. $\operatorname{curl} \underline{F} = \nabla \times \underline{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} -y \\ x \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}$

ii. $\operatorname{div} \underline{F} = \nabla \cdot \underline{F} = 0$

iii. Nee, want $\operatorname{curl} \underline{F} \neq \underline{0}$

iv. Een potentiaalfunctie bestaat niet want $\underline{F}$ is niet conservatief.

b) parametrisering van C:

$$\underline{r}(t) = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} + t\left\{\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} - \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}\right\} = \begin{pmatrix} 1 \\ 2-4t \\ 3-2t \end{pmatrix}, \quad 0 \leq t \leq 1$$

$$\int_C \underline{F}\cdot d\underline{r} = \int_0^1 \underline{F}\cdot\underline{r}'(t)\,dt = \int_0^1 \begin{pmatrix} -2+4t \\ 1 \\ 1 \end{pmatrix}\cdot\begin{pmatrix} 0 \\ -4 \\ -2 \end{pmatrix} dt =$$

$$\int_0^1 (-6)\,dt = -6$$

2. c) De flux van $\underline{F}$ door B is 0 want B is een gesloten oppervlak en $\operatorname{div} \underline{F} = 0$

Divergentiestelling: $\iint_B \underline{F} \cdot d\underline{S} = \iiint_V \operatorname{div} \underline{F} \, dV = 0.$

3. a). $\underline{r}_s \times \underline{r}_t = \begin{pmatrix} 1 \\ 0 \\ t \end{pmatrix} \times \begin{pmatrix} 0 \\ 1 \\ s \end{pmatrix} = \begin{pmatrix} -t \\ -s \\ 1 \end{pmatrix}$

In $(1,1,1)$ geldt: $s = t = 1$,

een normaalvector is dus $\begin{pmatrix} -1 \\ -1 \\ 1 \end{pmatrix}$ en de vergelijking van het raakvlak:

$$-(x-1) - (y-1) + (z-1) = 0$$

b) $A(S) = \iint_S |\underline{r}_s \times \underline{r}_t| \, ds\,dt = \iint_H \sqrt{t^2+s^2+1} \, ds\,dt$

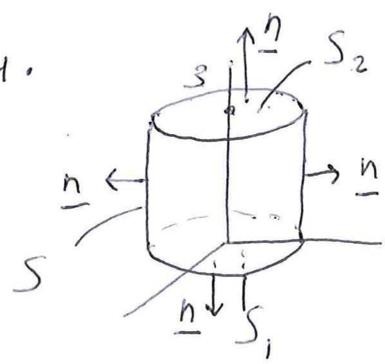
met $H = \{(s,t) \mid s^2+t^2 \leq 3\}$.

We gaan over op poolcoordinaten:

$$A(S) = \int_0^{2\pi} \int_0^{\sqrt{3}} \sqrt{r^2+1} \; r \, dr \, d\theta =$$

$$2\pi \left[\frac{1}{2} \cdot \frac{2}{3} (r^2+1)^{\frac{3}{2}} \right]_0^{\sqrt{3}} = \frac{2\pi}{3}(8-1) = \frac{14}{3}\pi.$$

4.



Maak een plaatje en volg de hint.

Neem $S_1 = \{(x,y,z) \mid z = 0, \; x^2+y^2 \leq 4\}$

en $S_2 = \{(x,y,z) \mid z = 3, \; x^2+y^2 \leq 4\}$.

4. Divergentiestelling:

$$\iint_S \underline{G} \cdot d\underline{S} + \iint_{S_1} \underline{G} \cdot d\underline{S} + \iint_{S_2} \underline{G} \cdot d\underline{S} = \iiint_E \operatorname{div} \underline{G}\, dV$$

met E het lichaam: $\{(x,y,z) \mid 0 \leq z \leq 3,\ x^2+y^2 \leq 4\}$.

Bereken dus $\iint_{S_1}$, $\iint_{S_2}$, $\iiint_E$.

$$\iint_{S_1} \underline{G} \cdot d\underline{S} = \iint_{S_1} \underline{G} \cdot \underline{n}\, dS = \iint_{S_1} \begin{pmatrix} - \\ - \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} dS =$$

$$-\iint_{S_1} dS = -A(S_1) = -4\pi$$

$$\iint_{S_2} \underline{G} \cdot d\underline{S} = \iint_{S_2} \underline{G} \cdot \underline{n}\, dS = \iint_{S_2} \begin{pmatrix} - \\ - \\ 10 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} =$$

$$10\, A(S_2) = 40\pi.$$

$$\operatorname{div} \underline{G} = -2z - 1 + 2z = -1.$$

$$\iiint_E \operatorname{div} \underline{G}\, dV = -\iiint_E dV = -(4\pi \cdot 3) = -12\pi$$

Voor de flux door S volgt nu:

$$\iint_S \underline{G} \cdot d\underline{S} = -12\pi + 4\pi - 40\pi = -48\pi$$