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studienummer / student number:
vak / course: Analyse 4
code / code:
datum / date:
opleiding / program: TA
aantal ingeleverde vellen / total number of sheets:
opgave nummer / question number:

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1) $\int_k y\,ds$ $\quad ds = v(t)dt = |r'(t)|\,dt$

$r(t) = \begin{pmatrix} t^2 \\ t \end{pmatrix}$ $\quad r'(t) = \begin{pmatrix} 2t \\ 1 \end{pmatrix}$ $\quad |r'(t)| = \sqrt{4t^2+1}$

$\rightarrow$ dus $x = t^2$, $y = t$

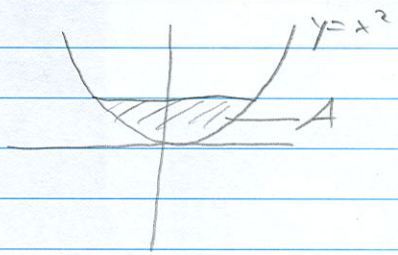
$$\int_{t=0}^{2} t\sqrt{4t^2+1}$$

$$= \left[\frac{1}{12}(4t^2+1)^{3/2}\right]_0^2 = \frac{1}{12}(17\sqrt{17}) - \frac{1}{12}$$

2) $\int_c P\,dx + Q\,dy = \int_c \underline{F}\cdot d\underline{r} = \iint_D \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\,dA$

$\frac{\partial Q}{\partial x} = 3x^2$

$\frac{\partial P}{\partial y} = x^2$



$$\int_{x=-1}^{1}\int_{y=x^2}^{1} 3x^2 - x^2\,dy\,dx = \int_{x=-1}^{1} [2x^2 y]_{y=x^2}^{1}\,dx$$

$$= \int_{x=-1}^{1} (2x^2 - 2x^4)\,dx = \left[\frac{2}{3}x^3 - \frac{2}{5}x^5\right]_{-1}^{1} = \left(\frac{2}{3} - \frac{2}{5}\right) - \left(-\frac{2}{3} + \frac{2}{5}\right)$$

$$= \frac{4}{3} - \frac{4}{5} = \frac{20}{15} - \frac{12}{15} = \frac{8}{15}$$

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3) $S: \quad \underline{r}(s,t) = \begin{pmatrix} st \\ s+t \\ s-t \end{pmatrix} \qquad s^2+t^2 \leq 6$

a) $st=2$

$s+t=3 \rightarrow s=3-t$
$s-t=-1 \rightarrow s=t-1$
$\Big\}\ 3-t=t-1 \Rightarrow 2t=4 \quad t=2$

$s \cdot t = 2 \Rightarrow 2s=2 \Rightarrow s=1$

$s=1$ en $t=2$ invullen geeft $(2,3,-1) \Rightarrow$ ligt op S

$$r_s \times r_t = \begin{pmatrix} t \\ 1 \\ 1 \end{pmatrix} \times \begin{pmatrix} s \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -2 \\ s+t \\ t-s \end{pmatrix}$$

voor $s=1$ en $t=2$ wordt dit $\begin{pmatrix} -2 \\ 3 \\ 1 \end{pmatrix}$

vergelijking raakvlak: $-2(x-2) + 3(y-3) + (z+1) = 0$

$-2x+4+3y-9+z+1=0$

$-2x+3y+z=4$

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3b) $A(S) = \iint_S |r_u \times r_v| \, du\,dv$

$r_s \times r_t = \begin{pmatrix} -2 \\ s+t \\ t-s \end{pmatrix}$ $\quad |r_s \times r_t| = \sqrt{4+(s+t)^2+(t-s)^2}$

$A(S) = \iint_S \sqrt{4+(s+t)^2+(t-s)^2}\, ds\, dt$

$= \iint_S \sqrt{4+s^2+2st+t^2+t^2-2st+s^2}\, ds\, dt$

$= \iint_S \sqrt{4+2(s^2+t^2)}\, ds\, dt$ $\quad$ a $s^2+t^2 \le 6$

pool coordinaten $\Rightarrow$ $s = r\cos\theta$

$t = r\sin\theta$ $\quad 0 \le r \le \sqrt{6}$

$= \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{6}} \sqrt{4+2r^2} \cdot r \, dr\, d\theta$

$= 2\pi \left[\frac{1}{6}(4+2r^2)^{3/2} \right]_{r=0}^{\sqrt{6}} = 2\pi \left(\left(\left(\frac{1}{6}(16)^{3/2}\right) - \left(\frac{1}{6}(4)^{3/2}\right) \right) \right)$

$= 2\pi \left(\frac{64}{6} - \frac{8}{6} \right) = \frac{56}{3}\pi$

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4a) $\underline{F} = \begin{pmatrix} 2x\sin y + 2yz \\ x^2\cos y + 2xz \\ 2xy + 1 \end{pmatrix}$

$$\text{curl } \underline{f} = \nabla \times \underline{F} = \begin{pmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} \end{pmatrix} \times \begin{pmatrix} 2x\sin y + 2yz \\ x^2\cos y + 2xz \\ 2xy + 1 \end{pmatrix} = \begin{pmatrix} 2x - 2x \\ 2y - 2y \\ (2x\cos y + 2z) - (2x\cos y + 2z) \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\text{div } \underline{F} = \nabla \cdot \underline{F} = 2\sin y + x^2 \cdot -\sin y + 0 = 2\sin y - x^2 \sin y$$

$$= (2 - x^2)\sin y$$

b) Ja, $\text{curl } \underline{F} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ $\Rightarrow$ conservatief

c) $\frac{\partial f}{\partial x} = 2x\sin y + 2yz$

↳ $f = x^2\sin y + 2xyz + h(y,z)$ uit $\underline{F}$

↳ $\frac{\partial f}{\partial y} = x^2\cos y + 2xz + h'_y(y,z) = x^2\cos y + 2xz$

$\Rightarrow h'_y(y,z) = 0 \quad h(y,z) = g(z)$

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4c) verrudg

$f = x^2 \sin y + 2xyz + g(z)$ uit f

$\rightarrow \frac{\partial f}{\partial z} = 2xy + g'(z) = 2xy + 1$

$g'(z) = 1$

$g(z) = z + C$

$f = x^2 \sin y + 2xyz + z + C$

d) $\int_k \underline{F}\, d\underline{r} = \int \nabla f\, d\underline{r} = f(\underline{r}(b)) - f(\underline{r}(a))$

↑ conservatief

$r(t) = \begin{pmatrix} t \\ \arctan t \\ \sqrt{1+3t^2} \end{pmatrix}$ $r(1) = \begin{pmatrix} 1 \\ \frac{1}{4}\pi \\ 2 \end{pmatrix}$ $r(0) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$

$f(r(1)) = (1)^2 \sin(\frac{1}{4}\pi) + 2(1)(\frac{1}{4}\pi)(2) + 2 + C$

$= \frac{1}{2}\sqrt{2} + \pi + 2 + C$

$f(r(0)) = 1 + C$

$\int_k \underline{F}\, d\underline{r} = (\frac{1}{2}\sqrt{2} + \pi + 2 + C) - (1 + C) = \frac{1}{2}\sqrt{2} + \pi + 1$

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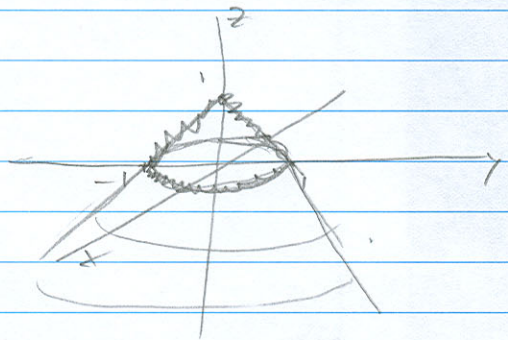
5) $\underline{G} = \begin{pmatrix} yz - 2xz \\ x - y + z \\ z^2 + 1 \end{pmatrix}$ $\quad S: z = 1 - \sqrt{x^2+y^2} \quad z \geq 0$

n heeft positief 3e getal
⇒ omhoog gericht dus voor de formules in de goede richting

a) $z = 1 - \sqrt{x^2+y^2} = 1 - r$

$z = 0 \Rightarrow \sqrt{x^2+y^2} = 1$
$z = 1 \Rightarrow \sqrt{x^2+y^2} = 0$
$z = -1 \Rightarrow \sqrt{x^2+y^2} = 2$

is een naar beneden gerichte kegel



S is stuk van kegel boven $z=0$

b) $\iint_S \underline{G}\, d\underline{S} = \iiint_E \operatorname{div} \underline{G}\, dV - \iint_D \underline{G}\, d\underline{S}$

S: "open" kegel
E: totale kegel
D: grondvlak

$\operatorname{div} \underline{G} = \nabla \cdot \underline{G} = -2z - 1 + 2z = -1$

$\iiint_E -1 \, dV = -1 \cdot$ inhoud kegel $= -1 \cdot \frac{1}{3}$ (inhoud cilinder)

$= -1 \cdot \frac{1}{3}(\pi r^2 h) = -1 \cdot \frac{1}{3}(\pi \cdot 1^2 \cdot 1)$

$= -\frac{1}{3}\pi$

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of dls je de inhoud van een kegel niet weet:

5b) vervolg

$$\iiint_E -1 \, dV \qquad \text{poolcoordinaten} \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$$

$$\int_{\theta=0}^{2\pi} \int_{r=0}^{1} \int_{z=0}^{1-r} -r \cdot dz\, dr\, d\theta$$

$$= -2\pi \int_{r=0}^{1} [rz]_{z=0}^{1-r} dr = -2\pi \int_{r=0}^{1} (r(1-r))\, dr$$

$$= -2\pi \int_{r=0}^{1} (r - r^2)\, dr = -2\pi \left[\tfrac{1}{2}r^2 - \tfrac{1}{3}r^3\right]_0^1 = -2\pi\left(\tfrac{1}{2} - \tfrac{1}{3}\right) = -\tfrac{1}{3}\pi$$

nog eraf

$$\iint_D \underline{G}\, d\underline{S} = \iint_D \underline{G}\cdot\underline{n}\, ds = \iint_D \underline{G}\cdot\underline{n}\, dA$$

(in xy-vlak dus $ds = dA$)

$$\underline{n} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\iint_D \underline{G}\cdot\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} dA = \iint_D (z^2+1)\, dA \quad \Rightarrow \text{poolcoordinaten} \begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = 0 \ (xy\text{-vlak}) \end{cases}$$

$$\iint_D dA \Rightarrow \text{oppervlak van gebied } D \Rightarrow \pi r^2 = \pi$$

Dus $\iint_S \underline{G}\, d\underline{S} = -\tfrac{1}{3}\pi - \pi = -\tfrac{4}{3}\pi$