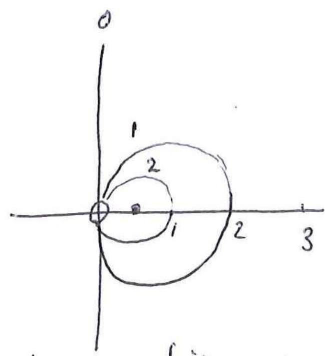
Analyse 3 Wi1300TA, 15 april 2011

1. a) $f(x,y) = \frac{2x}{x^2+y^2}$, $(x,y) \neq (0,0)$.

hoogte 0: $x = 0$

" 1: $2x = x^2+y^2 \to (x-1)^2+y^2 = 1$

" 2: $2x = 2(x^2+y^2) \to (x-\frac{1}{2})^2+y^2 = \frac{1}{4}$



b). Limiet bestaat niet, verschillende hooglijnen komen bij (0,0) aan.

of b.v.: langs x-as: $y=0$ $\quad f(x,0) = \frac{2}{x}$

$\lim\limits_{x\to 0} f(x,0)$ bestaat niet en dus ook $\lim\limits_{(x,y)\to(0,0)} f(x,y)$ bestaat niet.

c) $f(1,2) = \frac{2}{5}$

$f_x(x,y) = \frac{2(x^2+y^2)-4x^2}{(x^2+y^2)^2} \qquad f_x(1,2) = \frac{6}{25}$

$f_y(x,y) = \frac{-4xy}{(x^2+y^2)^2} \qquad f_y(1,2) = \frac{-8}{25}$

raakvlak: $z = \frac{2}{5} + \frac{6}{25}(x-1) - \frac{8}{25}(y-2)$

2 a) $f(x,y) = y\ln x + \frac{1}{x} - y$

$\begin{cases} f_x = \frac{y}{x} - \frac{1}{x^2} \\ f_y = \ln x - 1 \end{cases} \qquad \nabla f(1,2) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \underline{u} = \frac{\underline{v}}{|\underline{v}|} = \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix}$

$D f_{\underline{u}}(1,2) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} 3/5 \\ 4/5 \end{pmatrix} = \frac{3}{5} - \frac{4}{5} = -\frac{1}{5}$

b) $\begin{cases} \frac{y}{x} - \frac{1}{x^2} = 0 \\ \ln x - 1 = 0 \end{cases} \to x = e \quad \frac{y}{e} - \frac{1}{e^2} = 0 \to y = \frac{1}{e}$

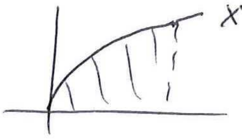
Dus $(e, \frac{1}{e})$ is het enige stationaire punt.

c) $f_{xx} = -\frac{y}{x^2} + \frac{2}{x^3}$ | $f_{xx}(e, \frac{1}{e}) = \frac{1}{e^3}$ | $D = f_{xx} f_{yy} - f_{xy}^2 = -\frac{1}{e^2} < 0$

$f_{yy} = 0$ | $f_{yy}(e, \frac{1}{e}) = 0$ | Dus zadelpunt.

$f_{xy} = \frac{1}{x}$ | $f_{xy}(e, \frac{1}{e}) = \frac{1}{e}$

3. $y \geq 0 \quad y^2 \leq x \leq 1$



$$\iint_D y \sin(x^2)\, dA = \int_0^1 \int_{y^2}^1 y \sin(x^2)\, dx\, dy$$ primitiveren lukt niet.

$$\int_0^1 \int_0^{\sqrt{x}} y \sin(x^2)\, dy\, dx = \int_0^1 \left[\tfrac{1}{2} y^2\right]_0^{\sqrt{x}} \sin(x^2)\, dx =$$

$$\int_0^1 \tfrac{1}{2} x \sin(x^2)\, dx = \left[-\tfrac{1}{4}\cos(x^2)\right]_0^1 = -\tfrac{1}{4}\cos(1) + \tfrac{1}{4}$$

4. $2x - 1 = u \rightarrow x = \tfrac{1}{2}(1+u)$

$y - 2 = v \rightarrow y = 2 + v$

$$J = \begin{vmatrix} 1/2 & 0 \\ 0 & 1 \end{vmatrix} = 1/2$$

$(2x-1)^2 + (y-2)^2 = 1$ wordt $u^2 + v^2 = 1$ $\quad H = \{(u,v) \mid u^2 + v^2 \leq 1\}$

$$\iint_G (4x + 2y)\, dA = \iint_H \big((2+2u) + (4+2v)\big) \tfrac{1}{2}\, du\, dv =$$

↑ poolcoordinaten $u = r\cos\theta$, $v = r\sin\theta$

$$\int_0^{2\pi} \int_0^1 (6 + 2r\cos\theta + 2r\sin\theta)\, \tfrac{1}{2} r\, dr\, d\theta = \ldots = 3\pi$$

5. $E = \{(x,y,z) \mid 0 \leq z \leq 1 + x^2,\ 1 \leq x^2 + y^2 \leq 4\}$.

cilindercoordinaten:

$$\iiint_E \frac{1}{x^2+y^2}\, dV = \int_{\theta=0}^{2\pi} \int_{r=1}^{2} \int_{z=0}^{1+r^2\cos^2\theta} \frac{1}{r^2}\, r\, dz\, dr\, d\theta =$$

$$\int_0^{2\pi} \int_1^2 \frac{1 + r^2\cos^2\theta}{r}\, dr\, d\theta = \int_0^{2\pi} \left[\ln r + \tfrac{1}{2} r^2 \cos^2\theta\right]_{r=1}^{2} d\theta =$$

$$\int_0^{2\pi} \left(\ln 2 + 2\cos^2\theta - \tfrac{1}{2}\cos^2\theta\right) d\theta = \ldots = 2\pi \ln 2 + \tfrac{3}{2}\pi$$

6. bolcoordinaten: $\begin{cases} 0 \le \varphi \le \frac{\pi}{4} \\ 0 \le \rho \le 2 \\ 0 \le \theta \le 2\pi . \end{cases}$

$$\int_{\theta=0}^{2\pi} \int_{\varphi=0}^{\frac{\pi}{4}} \int_{\rho=0}^{2} \rho^2 \cos^2\varphi \ \rho \ \rho^2 \sin\varphi \ d\rho \, d\varphi \, d\theta =$$

$$2\pi \int_0^{\frac{\pi}{4}} \cos^2\varphi \sin\varphi \, d\varphi \int_0^2 \rho^5 d\rho = 2\pi \left[-\tfrac{1}{3}\cos^3\varphi \right]_0^{\frac{\pi}{4}} \left[\tfrac{1}{6}\rho^6 \right]_0^2 =$$

$$\frac{2\pi}{3}\left(-\left(\tfrac{1}{2}\sqrt{2}\right)^3 + 1 \right) \frac{32}{3} = \frac{64}{9}\left(1 - \tfrac{1}{4}\sqrt{2}\right)\pi .$$