Mech 2 : ~~20 aug~~ 2008
13 juni

Opgave 1)

a) $I_c = \frac{1}{2} m R^2$ t.o.v. z-as ; $R = 2m$, $m = 4kg$

b)

→ $5 \hat{i} N$ F_1

← $-4 \hat{i} N$ F_2

→ $2 \hat{i} N$ F_3

rechts : pos
links : neg

• $\bar{F}_{res} = \Sigma \bar{F}_i = 5\hat{i}N + 2\hat{i}N - 4\hat{i}N = \underline{\underline{3\hat{i}N}}$

• $\bar{M}_{res,c} = ?$

$$\bar{M}_{res,c} = \bar{M}_{1,c} + \bar{M}_{2,c} + \bar{M}_{3,c}$$
$$= r_{1\to c} \times \bar{F}_1 + r_{2\to c} \times \bar{F}_2 + r_{3\to c} \times \bar{F}_3$$
$$= 2\hat{j}_m \times 5\hat{i}_N + \hat{j}_m \times -4\hat{i}_N + -2\hat{j}_m \times 2\hat{i}_N$$
$$= 10(-k) + -4(-k) + -4(-k)$$
$$= -10k + 4k + 4k$$
$$= -2\hat{k} \; Nm$$

c) $\bar{F}_{res}$, werklijn = ?

$\bar{M}_{res,c}$ = $\bar{F}_{res} \times r_{res\to c}$

$-2\hat{k}_m = 3\hat{i}_N \times r_{res\to c}$

$r_{res\to c} = \frac{3}{2}\hat{j}$, dus snijdt y-as in $1\frac{1}{2}$ (punt)

// met $\bar{F}_1$

d) I. $S(t) = S_0 + V_0 t + \frac{1}{2} a t^2$

II. $\varphi(t) = \varphi_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

III. $L_c(t) = \underbrace{rmv}_{0 \text{ (gen traml)}} + I\omega$

I: $S_0 = 0$; $V_0 = 0$; $\bar{a}_c = \frac{F_{res}}{m} = \frac{3\ N}{4\ kg} = \frac{3}{4}\ m/s^2$

$$\boxed{S(t) = \frac{3}{8} t^2}\ m$$

II: $\varphi_0 = 0$; $\omega_0 = 0$;

$\varphi(t) = \frac{1}{2} \alpha t^2$

$M = I\alpha \Leftrightarrow \alpha = \frac{M}{I} = \frac{(-)\ 2\ Nm}{\frac{1}{2} \cdot 4 \cdot 4} = (-)\frac{1}{4}\ rad/s^2$

$$\boxed{\varphi(t) = \frac{1}{8} t^2}\ rad$$

III: $L_c(t) = I\omega$

$= I(\alpha t) = \frac{1}{2} \cdot 4 \cdot 4 \left((-)\frac{1}{4} t\right) = (-)\ 2t$

$$\boxed{L_c(t) = (-)\ 2t}$$; gen traml dus $rmv = 0$

$kg\,m^2\,rad/s$

Opg 2) m kogel = m staaf = m ; blijft staan dus mech energie vóór ≠ mech energie na

a) – wet v behoud v Impuls ; $\Sigma p_v = \Sigma p_{na}$
– wet v behoud v Impulsmoment ; $\Sigma L_{voor} = \Sigma L_{na}$ } geen externe krachten

– wet v behoud v Energie **niet**, omdat de kogel blijft staan ($\Sigma u_v \neq \Sigma u_{na}$)

b) T.o.v de grond waarop de balk staat : (oorsprong)

$r_{\bar{c}} = \frac{\Sigma m_i r_i}{\Sigma m_i} = \frac{m \cdot \frac{1}{2}l + m \cdot \frac{1}{4}l}{2m} = \frac{m\left(\frac{1}{2}l + \frac{1}{4}l\right)}{m\ (2)} = \frac{3/4\ l}{2} = \frac{3}{8}$

in de y-ri

c) $\Sigma p_{voor} = \Sigma p_{na}$

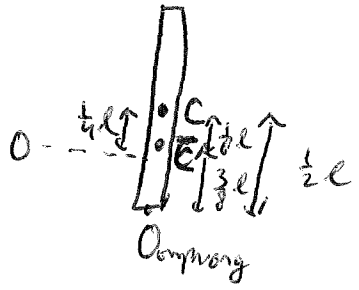
$m_{kogel} V_{kogel} = (m_{kogel} + m_{balk}) V_{\bar{c}}$

$m\ V = 2m V_{\bar{c}}$

$$\boxed{V_{\bar{c}} = \frac{V}{2}}$$

$\Sigma L_{\text{voor}} = \Sigma L_{\text{na}}$

$m_{\text{kogel}} v_{\text{kogel}} = r m v_{(\text{kogel + balk})} + I \omega_{(\text{kogel + balk})}$



Gebruik Steiner:

$I_{\tilde{C}} = I_C + m s^2$ → kogel + balk (2m); → distance kogel → $\tilde{C}$ = $\frac{1}{8}\ell$

$= \frac{1}{12} m\ell^2 + (2m)\left(\frac{1}{8}\ell\right)^2$

$= \frac{1}{12} m\ell^2 + \frac{1}{32} m\ell^2$

$= \frac{11}{96} m\ell^2$

e) $\omega_{\tilde{C}} = ?$ $\quad \vec{L}_{\tilde{C}}^{\text{na}} = I_{\tilde{C}}\, \omega_{\tilde{C}}$

$\omega_{\tilde{C}} = \frac{L_{\tilde{C}}}{I_{\tilde{C}}}$

Voor de botsing: $\vec{L}_{\tilde{C}} = \vec{L}_{\text{kogel}} = r m v = \frac{1}{8} \ell m v$

$I_{\tilde{C}} = \frac{11}{96} m\ell^2$

$\omega_{\tilde{C}} = \frac{\frac{1}{8}\ell m v}{\frac{11}{96} m\ell^2} = \frac{12}{11} v/\ell$

Opgave 3

Dit is een heel wiskundige som (neem niet te serieus)

g) $I_{C,z} = \frac{1}{6} m a^2$

b) $T = 2\pi\sqrt{\frac{I}{s m g}}$ (uit Formule blad)

$I = \frac{1}{6} m a^2$

→ D

s = distance tot mmp(C)

m = massa

$g = 10\ m/s^2$

$T_A = 2\pi\sqrt{\frac{\frac{1}{6} m a^2}{\ell m g}}$

$= 2\pi\sqrt{\frac{1}{6} a^2 / \ell g}$???

c) $T_A'(\ell) = 0$

(4)

d) $T_A = 2\pi \sqrt{\frac{1}{6}a^2 / lg}$

$T_{Mathematisch} = 2\pi \sqrt{\frac{l}{g}}$

$l = l + a$

$= 2\pi \sqrt{\frac{l+a}{g}}$

Opg 3) Dit is en heel wiskundige som (neem met te serieuz)

a) $I_{C,z} = \frac{1}{6} m a^2$

b) $T_A = 2\pi \sqrt{\frac{I_A}{smg}}$; $I_A = I_C + ms^2$; s = distance tot mmp (C) ↳ steiner (Starr lichaam).

$I_A = \frac{1}{6} m a^2 + m l^2$

$T_A = 2\pi \sqrt{\frac{m(\frac{1}{6}a^2 + l^2)}{l m g}} = 2\pi \sqrt{\frac{\frac{1}{6}a^2 + l^2}{lg}}$

c) $T_A{}'(l) = 0$; $\left(2\pi \sqrt{\frac{\frac{1}{6}a^2 + l^2}{lg}}\right)' = 0$; $\frac{dT}{dl} = 0$

Teveel werk

d) $T_A = 2\pi \sqrt{\frac{\frac{1}{6}a^2 + l^2}{lg}}$

$T_{mathematisch} = 2\pi \sqrt{\frac{l}{g}}$ met $l = l + a$

$= 2\pi \sqrt{\frac{l+a}{g}}$

Gelijkstellen, $\frac{\frac{1}{6}a^2 + l^2}{lg} = \frac{l+a}{g}$

$l = \ldots$ (teveel werk)