PARTICLE PHYSICS AND COSMOLOGY

44

44.1. **(a) IDENTIFY** and **SET UP:** Use Eq.(37.36) to calculate the kinetic energy K.

EXECUTE: $K = mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}}-1\right) = 0.1547mc^2$

$m = 9.109\times10^{-31}$ kg, so $K = 1.27\times10^{-14}$ J

(b) IDENTIFY and **SET UP:** The total energy of the particles equals the sum of the energies of the two photons. Linear momentum must also be conserved.

EXECUTE: The total energy of each electron or positron is $E = K + mc^2 = 1.1547mc^2 = 9.46\times10^{-13}$ J. The total energy of the electron and positron is converted into the total energy of the two photons. The initial momentum of the system in the lab frame is zero (since the equal-mass particles have equal speeds in opposite directions), so the final momentum must also be zero. The photons must have equal wavelengths and must be traveling in opposite directions. Equal λ means equal energy, so each photon has energy 9.46×10^{-14} J.

(c) IDENTIFY and **SET UP:** Use Eq. (38.2) to relate the photon energy to the photon wavelength.

EXECUTE: $E = hc/\lambda$ so $\lambda = hc/E = hc/(9.46\times10^{-14}\ \text{J}) = 2.10$ pm

EVALUATE: The wavelength calculated in Example 44.1 is 2.43 pm. When the particles also have kinetic energy, the energy of each photon is greater, so its wavelength is less.

44.2. The total energy of the positron is

$$E = K + mc^2 = 5.00\ \text{MeV} + 0.511\ \text{MeV} = 5.51\ \text{MeV}.$$

We can calculate the speed of the positron from Eq.(37.38):

$$E = \frac{mc^2}{\sqrt{1-\dfrac{v^2}{c^2}}} \Rightarrow \frac{v}{c} = \sqrt{1-\left(\frac{mc^2}{E}\right)^2} = \sqrt{1-\left(\frac{0.511\ \text{MeV}}{5.51\ \text{MeV}}\right)^2} = 0.996.$$

44.3. **IDENTIFY** and **SET UP:** By momentum conservation the two photons must have equal and opposite momenta. Then $E = pc$ says the photons must have equal energies. Their total energy must equal the rest mass energy $E = mc^2$ of the pion. Once we have found the photon energy we can use $E = hf$ to calculate the photon frequency and use $\lambda = c/f$ to calculate the wavelength.

EXECUTE: The mass of the pion is $270m_e$, so the rest energy of the pion is $270(0.511\ \text{MeV}) = 138\ \text{MeV}$. Each photon has half this energy, or 69 MeV. $E = hf$ so $f = \dfrac{E}{h} = \dfrac{(69\times10^6\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 1.7\times10^{22}$ Hz

$\lambda = \dfrac{c}{f} = \dfrac{2.998\times10^8\ \text{m/s}}{1.7\times10^{22}\ \text{Hz}} = 1.8\times10^{-14}\ \text{m} = 18\ \text{fm}.$

EVALUATE: These photons are in the gamma ray part of the electromagnetic spectrum.

44.4. **(a)** The energy will be the proton rest energy, 938.3 MeV, corresponding to a frequency of 2.27×10^{23} Hz and a wavelength of 1.32×10^{-15} m.

(b) The energy of each photon will be $938.3\ \text{MeV} + 830\ \text{MeV} = 1768\ \text{MeV}$, with frequency 42.8×10^{22} Hz and wavelength 7.02×10^{-16} m.

44.5. **(a)** $\Delta m = m_{\pi+} - m_{\mu+} = 270\ m_e - 207\ m_e = 63\ m_e \Rightarrow E = 63(0.511\ \text{MeV}) = 32\ \text{MeV}.$

(b) A positive muon has less mass than a positive pion, so if the decay from muon to pion was to happen, you could always find a frame where energy was not conserved. This cannot occur.

44.6. **(a)** $\lambda = \dfrac{hc}{E} = \dfrac{hc}{m_\mu c^2} = \dfrac{h}{m_\mu c} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{(207)(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = 1.17\times10^{-14}\ \text{m} = 0.0117\ \text{pm}$

In this case, the muons are created at rest (no kinetic energy).

(b) Shorter wavelengths would mean higher photon energy, and the muons would be created with non-zero kinetic energy.

44.7. **IDENTIFY:** The energy released comes from the mass difference.

SET UP: The mass difference is the initial mass minus the final mass.

$$\Delta m = m_{\mu^-} - m_{e^-} - m_{e^+}$$

EXECUTE: Using the masses from Table 44.2, we have

$\Delta m = m_{\mu^-} - m_{e^-} - m_{e^+} = (105.7\ \text{MeV}/c^2) - (0.511\ \text{MeV}/c^2) - (0.511\ \text{MeV}/c^2) = 105\ \text{MeV}/c^2$

Multiplying these masses by c^2 gives E = 105 MeV.

EVALUATE: This energy is observed as kinetic energy of the electron and positron.

44.8. **IDENTIFY** and **SET UP:** Calculate the mass change in each reaction, using the atomic masses in Table 44.2. A mass change of 1 u is equivalent to an energy of 931.5 MeV.

EXECUTE: **(a)** and **(b)** Eq.(44.1): ${}^4_2\text{He} + {}^9_4\text{Be} \rightarrow {}^{12}_6\text{C} + {}^1_0\text{n}$

$\Delta M = m({}^4\text{He}) + m({}^9\text{Be}) - \left[m({}^{12}\text{C}) + m({}^1\text{n})\right]$

$\Delta M = 4.00260\ \text{u} + 9.01218\ \text{u} - 12.00000\ \text{u} - 1.00866\ \text{u} = 0.00612\ \text{u}$

The mass decreases and the energy liberated is 5.70 MeV. The reaction is exoergic.

Eq.(44.2): ${}^1_0\text{n} + {}^{10}_5\text{B} \rightarrow {}^7_3\text{Li} + {}^4_2\text{He}$

$\Delta M = m({}^1\text{n}) + m({}^{10}\text{B}) - \left[m({}^7\text{Li}) + m({}^4\text{He})\right]$

$\Delta M = 1.00866\ \text{u} + 10.01294\ \text{u} - 7.01600\ \text{u} - 4.00260\ \text{u} = 0.00300\ \text{u}$

The mass decreases and the energy liberated is 2.79 MeV. The reaction is exoergic.

(c) The reactants in the reactions of Eq.(44.1) have positive nuclear charges and a threshold kinetic energy is required for the reactants to overcome their Coulomb repulsion and get close enough for the reaction to occur. The neutron in Eq.(44.2) is neutral so there is no Coulomb repulsion and no threshold energy for this reaction.

44.9. **IDENTIFY:** The antimatter annihilates with an equal amount of matter.

SET UP: The energy of the matter is $E = (\Delta m)c^2$.

EXECUTE: Putting in the numbers gives

$$E = (\Delta m)c^2 = (400\ \text{kg} + 400\ \text{kg})(3.00\times10^8\ \text{m/s})^2 = 7.2\times10^{19}\ \text{J}.$$

This is about 70% of the annual energy use in the U.S.

EVALUATE: If this huge amount of energy were released suddenly, it would blow up the *Enterprise*! Getting useable energy from matter-antimatter annihiliation is not so easy to do!

44.10. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_a^2 = 2mc^2\left(E_m + mc^2\right)$ for two particles of equal mass when one is initially stationary. In this case, the initial kinetic energy (20.0 GeV = 20,000 MeV) is much more than the rest energy of the electron (0.511 MeV), so the formula for available energy reduces to $E_a = \sqrt{2mc^2E_m}$.

EXECUTE: **(a)** Using the formula for available energy gives

$$E_a = \sqrt{2mc^2E_m} = \sqrt{2(0.511\ \text{MeV})(20.0\ \text{GeV})} = 143\ \text{MeV}$$

(b) For colliding beams of equal mass, each particle has half the available energy, so each has 71.5 MeV. The *total* energy is twice this, or 143 MeV.

EVALUATE: Colliding beams provide considerably more available energy to do experiments than do beams hitting a stationary target. With a stationary electron target in part (a), we had to give the moving electron 20,000 MeV of energy to get the same available energy that we got with only 143 MeV of energy with the colliding beams.

44.11. **(a) IDENTIFY** and **SET UP:** Eq. (44.7) says $\omega = |q|B/m$ so $B = m\omega/|q|$. And since $\omega = 2\pi f$, this becomes $B = 2\pi mf/|q|$.

EXECUTE: A deuteron is a deuterium nucleus $\left({}^2_1\text{H}\right)$. Its charge is $q = +e$. Its mass is the mass of the neutral ${}^2_1\text{H}$ atom (Table 43.2) minus the mass of the one atomic electron:

$m = 2.014102\ \text{u} - 0.0005486\ \text{u} = 2.013553\ \text{u}(1.66054\times10^{-27}\ \text{kg}/1\ \text{u}) = 3.344\times10^{-27}\ \text{kg}$

$$B = \frac{2\pi mf}{|q|} = \frac{2\pi(3.344\times10^{-27}\ \text{kg})(9.00\times10^6\ \text{Hz})}{1.602\times10^{-19}\ \text{C}} = 1.18\ \text{T}$$

(b) Eq.(44.8): $K = \dfrac{q^2B^2R^2}{2m} = \dfrac{[(1.602\times10^{-19}\ \text{C})(1.18\ \text{T})(0.320\ \text{m})]^2}{2(3.344\times10^{-27}\ \text{kg})}$.

$K = 5.471\times10^{-13}\ \text{J} = (5.471\times10^{-13}\ \text{J})(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 3.42\ \text{MeV}$

$K = \frac{1}{2}mv^2$ so $v = \sqrt{\dfrac{2K}{m}} = \sqrt{\dfrac{2(5.471\times10^{-13}\ \text{J})}{3.344\times10^{-27}\ \text{kg}}} = 1.81\times10^7\ \text{m/s}$

EVALUATE: $v/c = 0.06,$ so it is ok to use the nonrelativistic expression for kinetic energy.

44.12. **(a)** $2f = \dfrac{\omega}{\pi} = \dfrac{eB}{m\pi} = 3.97\times10^7/\text{s}.$

(b) $\omega R = \dfrac{eBR}{m} = 3.12\times10^7\ \text{m/s}$

(c) For three-figure precision, the relativistic form of the kinetic energy must be used,

$eV = (\gamma-1)mc^2,$ so $eV = (\gamma-1)mc^2,$ so $V = \dfrac{(\gamma-1)mc^2}{e} = 5.11\times10^6\ \text{V}.$

44.13. **(a) IDENTIFY** and **SET UP:** The masses of the target and projectile particles are equal, so Eq. (44.10) can be used. $E_a^2 = 2mc^2(E_m + mc^2).$ E_a is specified; solve for the energy E_m of the beam particles.

EXECUTE: $E_m = \dfrac{E_a^2}{2mc^2} - mc^2$

The mass for the alpha particle can be calculated by subtracting two electron masses from the ${}^4_2\text{He}$ atomic mass:

$m = m_\alpha = 4.002603\ \text{u} - 2(0.0005486\ \text{u}) = 4.001506\ \text{u}$

Then $mc^2 = (4.001506\ \text{u})(931.5\ \text{MeV/u}) = 3.727\ \text{GeV}.$

$E_m = \dfrac{E_a^2}{2mc^2} - mc^2 = \dfrac{(16.0\ \text{GeV})^2}{2(3.727\ \text{GeV})} - 3.727\ \text{GeV} = 30.6\ \text{GeV}.$

(b) Each beam must have $\frac{1}{2}E_a = 8.0\ \text{GeV}.$

EVALUATE: For a stationary target the beam energy is nearly twice the available energy. In a colliding beam experiment all the energy is available and each beam needs to have just half the required available energy.

44.14. **(a)** $\gamma = \dfrac{1000\times10^3\ \text{MeV}}{938.3\ \text{MeV}} = 1065.8,$ so $v = 0.999999559c.$

(b) Nonrelativistic: $\omega = \dfrac{eB}{m} = 3.83\times10^8\ \text{rad/s}.$

Relativistic: $\omega = \dfrac{eB}{m}\dfrac{1}{\gamma} = 3.59\times10^5\ \text{rad/s}.$

44.15. **(a) IDENTIFY** and **SET UP:** For a proton beam on a stationary proton target and since E_a is much larger than the proton rest energy we can use Eq.(44.11): $E_a^2 = 2mc^2E_m.$

EXECUTE: $E_m = \dfrac{E_a^2}{2mc^2} = \dfrac{(77.4\ \text{GeV})^2}{2(0.938\ \text{GeV})} = 3200\ \text{GeV}$

(b) IDENTIFY and **SET UP:** For colliding beams the total momentum is zero and the available energy E_a is the total energy for the two colliding particles.

EXECUTE: For proton-proton collisions the colliding beams each have the same energy, so the total energy of each beam is $\frac{1}{2}E_a = 38.7\ \text{GeV}.$

EVALUATE: For a stationary target less than 3% of the beam energy is available for conversion into mass. The beam energy for a colliding beam experiment is a factor of (1/83) times smaller than the required energy for a stationary target experiment.

44.16. **IDENTIFY:** Only part of the initial kinetic energy of the moving electron is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy.

SET UP: To create the η^0, the minimum available energy must be equal to the rest mass energy of the products, which in this case is the η^0 plus two protons. In a collider, all of the initial energy is available, so the beam energy is the available energy.

EXECUTE: The minimum amount of available energy must be rest mass energy

$$E_a = 2m_p + m_\eta = 2(938.3\ \text{MeV}) + 547.3\ \text{MeV} = 2420\ \text{MeV}$$

Each incident proton has half of the rest mass energy, or 1210 MeV = 1.21 GeV.

EVALUATE: As we saw in problem 44.10, we would need much more initial energy if one of the initial protons were stationary. The result here (1.21 GeV) is the *minimum* amount of energy needed; the original protons could have more energy and still trigger this reaction.

44.17. Section 44.3 says $m(\mathrm{Z}^0) = 91.2\ \mathrm{GeV}/c^2$.

$$E = 91.2\times10^9\ \mathrm{eV} = 1.461\times10^{-8}\ \mathrm{J};\ m = E/c^2 = 1.63\times10^{-25}\ \mathrm{kg};\ m(\mathrm{Z}^0)/m(\mathrm{p}) = 97.2$$

44.18. **(a)** We shall assume that the kinetic energy of the Λ^0 is negligible. In that case we can set the value of the photon's energy equal to Q:

$$Q = (1193-1116)\ \mathrm{MeV} = 77\ \mathrm{MeV} = E_{\text{photon}}.$$

(b) The momentum of this photon is

$$p = \frac{E_{\text{photon}}}{c} = \frac{(77\times10^6\ \mathrm{eV})(1.60\times10^{-18}\ \mathrm{J/eV})}{(3.00\times10^8\ \mathrm{m/s})} = 4.1\times10^{-20}\ \mathrm{kg\cdot m/s}$$

To justify our original assumption, we can calculate the kinetic energy of a Λ^0 that has this value of momentum

$$K_{\Lambda^0} = \frac{p^2}{2m} = \frac{E^2}{2mc^2} = \frac{(77\ \mathrm{MeV})^2}{2(1116\ \mathrm{MeV})} = 2.7\ \mathrm{MeV} << Q = 77\ \mathrm{MeV}.$$

Thus, we can ignore the momentum of the Λ^0 without introducing a large error.

44.19. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: The mass decrease is $m(\Sigma^+) - m(\mathrm{p}) - m(\pi^0)$ and the energy released is

$mc^2(\Sigma^+) - mc^2(\mathrm{p}) - mc^2(\pi^0) = 1189\ \mathrm{MeV} - 938.3\ \mathrm{MeV} - 135.0\ \mathrm{MeV} = 116\ \mathrm{MeV}$. (The mc^2 values for each particle were taken from Table 44.3.)

EVALUATE: The mass of the decay products is less than the mass of the original particle, so the decay is energetically allowed and energy is released.

44.20. **IDENTIFY:** If the initial and final rest mass energies were equal, there would be no left over energy for kinetic energy. Therefore the kinetic energy of the products is the difference between the mass energy of the initial particles and the final particles.

SET UP: The difference in mass is $\Delta m = M_{\Omega^-} - m_{\Lambda^0} - m_{\mathrm{K}^-}$.

EXECUTE: Using Table 44.3, the energy difference is

$$E = (\Delta m)c^2 = 1672\ \mathrm{MeV} - 1116\ \mathrm{MeV} - 494\ \mathrm{MeV} = 62\ \mathrm{MeV}$$

EVALUATE: There is less rest mass energy after the reaction than before because 62 MeV of the initial energy was converted to kinetic energy of the products.

44.21. Conservation of lepton number.

(a) $\mu^- \to \mathrm{e}^- + \nu_\mathrm{e} + \overline{\nu}_\mu \Rightarrow L_\mu : +1 \neq -1,\ L_\mathrm{e} : 0 \neq +1+1$, so lepton numbers are not conserved.

(b) $\tau^- \to \mathrm{e}^- + \overline{\nu}_\mathrm{e} + \nu_\tau \Rightarrow L_\mathrm{e} : 0 = +1-1;\ L_\tau : +1 = +1$, so lepton numbers are conserved.

(c) $\pi^+ \to \mathrm{e}^+ + \gamma$. Lepton numbers are not conserved since just one lepton is produced from zero original leptons.

(d) $\mathrm{n} \to \mathrm{p} + \mathrm{e}^- + \overline{\gamma}_\mathrm{e} \Rightarrow L_\mathrm{e} : 0 = +1-1$, so the lepton numbers are conserved.

44.22. **IDENTIFY** and **SET UP:** p and n have baryon number $+1$ and $\overline{\mathrm{p}}$ has baryon number -1. e^+, e^-, $\overline{\nu}_\mathrm{e}$ and γ all have baryon number zero. Baryon number is conserved if the total baryon number of the products equals the total baryon number of the reactants.

EXECUTE: **(a)** reactants: $B = 1+1 = 2$. Products: $B = 1+0 = 1$. Not conserved.

(b) reactants: $B = 1+1 = 2$. Products: $B = 0+0 = 0$. Not conserved.

(c) reactants: $B = +1$. Products: $B = 1+0+0 = +1$. Conserved.

(d) reactants: $B = 1-1 = 0$. Products: $B = 0$. Conserved.

44.23. **IDENTIFY** and **SET UP:** Compare the sum of the strangeness quantum numbers for the particles on each side of the decay equation. The strangeness quantum numbers for each particle are given Table 44.3.

EXECUTE: **(a)** $\mathrm{K}^+ \to \mu^+ + \nu_\mu;\ S_{\mathrm{K}+} = +1,\ S_{\mu+} = 0,\ S_{\nu_\mu} = 0$

$S = 1$ initially; $S = 0$ for the products; S is <u>not conserved</u>

(b) $\mathrm{n} + \mathrm{K}^+ \to \mathrm{p} + \pi^0;\ S_\mathrm{n} = 0,\ S_{\mathrm{K}^+} = +1,\ S_\mathrm{p} = 0,\ S_{\pi^0} = 0$

$S = 1$ initially; $S = 0$ for the products; S is <u>not conserved</u>

(c) $\mathrm{K}^+ + \mathrm{K}^- \to \pi^0 + \pi^0;\ S_{\mathrm{K}^+} = +1;\ S_{\mathrm{K}^-} = -1;\ S_{\pi^0} = 0$

$S = +1-1 = 0$ initially; $S = 0$ for the products; S is <u>conserved</u>

(d) $p + K^- \to \Lambda^0 + \pi^0$; $S_p = 0, S_{K^-} = -1, S_{\Lambda^0} = -1, S_{\pi^0} = 0.$

$S = -1$ initially; $S = -1$ for the products; S is conserved

EVALUATE: Strangeness is not a conserved quantity in weak interactions and strangeness non-conserving reactions or decays can occur.

44.24. **(a)** Using the values of the constants from Appendix F,

$$\frac{e^2}{4\pi\epsilon_0\hbar c} = 7.29660475\times10^{-3} = \frac{1}{137.050044}, \text{ or } 1/137 \text{ to three figures.}$$

(b) From Section 38.5, $v_1 = \dfrac{e^2}{2\epsilon_0 h}$. But notice this is just $\left(\dfrac{e^2}{4\pi\epsilon_0\hbar c}\right)c$, as claimed.

44.25. $\left[\dfrac{f^2}{\hbar c}\right] = \dfrac{(\text{J}\cdot\text{m})}{(\text{J}\cdot\text{s})(\text{m}\cdot\text{s}^{-1})} = 1$ and thus $\dfrac{f^2}{\hbar c}$ is dimensionless. (Recall f^2 has units of energy times distance.)

44.26. **(a)** The diagram is given in Figure 44.26. The Ω^- particle has $Q = -1$ (as its label suggests) and $S = -3$. Its appears as a "hole"in an otherwise regular lattice in the $S-Q$ plane. The mass difference between each S row is around 145 MeV (or so). This puts the Ω^- mass at about the right spot. As it turns out, all the other particles on this lattice had been discovered already and it was this "hole" and mass regularity that led to an accurate prediction of the properties of the Ω !

(b) See diagram. Use quark charges $\boldsymbol{u} = +\dfrac{2}{3}, \boldsymbol{d} = \dfrac{-1}{3}$, and $\boldsymbol{s} = \dfrac{-1}{3}$ as a guide.

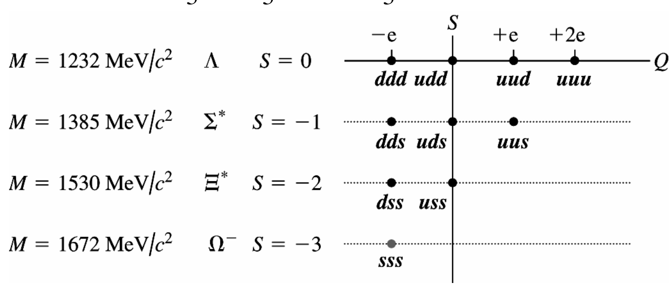

Figure 44.26

44.27. **IDENTIFY** and **SET UP:** Each value for the combination is the sum of the values for each quark. Use Table 44.4.

EXECUTE: **(a)** ***uds***

$Q = \frac{2}{3}e - \frac{1}{3}e - \frac{1}{3}e = 0$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = 1$

$S = 0 + 0 - 1 = -1$

$C = 0 + 0 + 0 = 0$

(b) $c\bar{u}$

The values for $\bar{u}$ are the negative for those for u.

$Q = \frac{2}{3}e - \frac{2}{3}e = 0$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = +1 + 0 = +1$

(c) ***ddd***

$Q = -\frac{1}{3}e - \frac{1}{3}e - \frac{1}{3}e = -e$

$B = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = +1$

$S = 0 + 0 + 0 = 0$

$C = 0 + 0 + 0 = 0$

(d) $d\bar{c}$

$Q = -\frac{1}{3}e - \frac{2}{3}e = -e$

$B = \frac{1}{3} - \frac{1}{3} = 0$

$S = 0 + 0 = 0$

$C = 0 - 1 = -1$

EVALUATE: The charge, baryon number, strangeness and charm quantum numbers of a particle are determined by the particle's quark composition.

44.28. $(m_\gamma - 2m_\tau)c^2 = (9460\text{ MeV} - 2(1777\text{ MeV})) = 5906\text{ MeV}$ (see Sections 44.3 and 44.4 for masses).

44.29. **(a)** The antiparticle must consist of the antiquarks so $\overline{\text{n}} = \overline{\boldsymbol{u}}\overline{\boldsymbol{d}}\overline{\boldsymbol{d}}$.
(b) So $\text{n} = \boldsymbol{udd}$ is not its own antiparticle.
(c) $\psi = \boldsymbol{c}\overline{\boldsymbol{c}}$ so $\overline{\psi} = \overline{\boldsymbol{c}}\boldsymbol{c} = \psi$ so the ψ is its own antiparticle.

44.30. **(a)** $S = 1$ indicates the presence of one $\overline{s}$ antiquark and no $\boldsymbol{s}$ quark. To have baryon number 0 there can be only one other quark, and to have net charge $+e$ that quark must be a $\boldsymbol{u}$, and the quark content is $\boldsymbol{u}\overline{\boldsymbol{s}}$.
(b) The particle has an $\overline{s}$ antiquark, and for a baryon number of –1 the particle must consist of three antiquarks. For a net charge of $-e$, the quark content must be $\overline{\boldsymbol{d}}\overline{\boldsymbol{d}}\,\overline{\boldsymbol{s}}$.
(c) $S = -2$ means that there are two $\boldsymbol{s}$ quarks, and for baryon number 1 there must be one more quark. For a charge of 0 the third quark must be a $\boldsymbol{u}$ quark and the quark content is $\boldsymbol{uss}$.

44.31. **IDENTIFY:** A proton is made up of $\boldsymbol{uud}$ quarks and a neutron consists of $\boldsymbol{udd}$ quarks.
SET UP: If a proton decays by β^+ decay, we have $\text{p} \rightarrow \text{e}^+ + \text{n} + \nu_e$ (both charge and lepton number are conserved).
EVALUATE: Since a proton consists of $\boldsymbol{uud}$ quarks and a neutron is $\boldsymbol{udd}$ quarks, it follows that in β^+ decay a $\boldsymbol{u}$ quark changes to a $\boldsymbol{d}$ quark.

44.32. **(a)** Using the definition of z from Example 44.9 we have that

$$1 + z = 1 + \frac{(\lambda_0 - \lambda_s)}{\lambda_0} = \frac{\lambda_0}{\lambda_s}.$$

Now we use Eq.(44.13) to obtain $1 + z = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+v/c}{1-v/c}} = \sqrt{\frac{1+\beta}{1-\beta}}.$

(b) Solving the above equation for β we obtain $\beta = \frac{(1+z)^2 - 1}{(1+z)^2 + 1} = \frac{1.5^2 - 1}{1.5^2 + 1} = 0.3846.$

Thus, $v = 0.3846\ c = 1.15 \times 10^8\text{ m/s}.$

(c) We can use Eq.(44.15) to find the distance to the given galaxy,

$$r = \frac{v}{H_0} = \frac{(1.15 \times 10^8\text{ m/s})}{(7.1 \times 10^4\ (\text{m/s})/\text{Mpc})} = 1.6 \times 10^3\text{ Mpc}$$

44.33. **(a) IDENTIFY and SET UP:** Use Eq.(44.14) to calculate v.

EXECUTE: $v = \left[\frac{(\lambda_0/\lambda_s)^2 - 1}{(\lambda_0/\lambda_s)^2 + 1}\right]c = \left[\frac{(658.5\text{ nm}/590\text{ nm})^2 - 1}{(658.5\text{ nm}/590\text{ nm})^2 + 1}\right]c = 0.1094c$

$v = (0.1094)(2.998 \times 10^8\text{ m/s}) = 3.28 \times 10^7\text{ m/s}$

(b) IDENTIFY and SET UP: Use Eq.(44.15) to calculate r.

EXECUTE: $r = \frac{v}{H_0} = \frac{3.28 \times 10^4\text{ km/s}}{(71\text{ (km/s)/Mpc})(1\text{ Mpc}/3.26\text{ Mly})} = 1510\text{ Mly}$

EVALUATE: The red shift $\lambda_0/\lambda_S - 1$ for this galaxy is 0.116. It is therefore about twice as far from earth as the galaxy in Examples 44.9 and 44.10, that had a red shift of 0.053.

44.34. From Eq.(44.15), $r = \frac{c}{H_0} = \frac{3.00 \times 10^8\text{ m/s}}{20(\text{km/s})/\text{Mly}} = 1.5 \times 10^4\text{ Mly}.$

(b) This distance represents looking back in time so far that the light has not been able to reach us.

44.35. **(a) IDENTIFY and SET UP:** Hubble's law is Eq.(44.15), with $H_0 = 71$ (km/s)/(Mpc). 1 Mpc = 3.26 Mly.

EXECUTE: $r = 5210\text{ Mly}$ so $v = H_0 r = ((71\text{ km/s})/\text{Mpc})(1\text{ Mpc}/3.26\text{ Mly})(5210\text{ Mly}) = 1.1 \times 10^5\text{ km/s}$

(b) IDENTIFY and SET UP: Use v from part (a) in Eq. (44.13).

EXECUTE: $\frac{\lambda_0}{\lambda_S} = \sqrt{\frac{c+v}{c-v}} = \sqrt{\frac{1+v/c}{1-v/c}}$

$\frac{v}{c} = \frac{1.1 \times 10^8\text{ m/s}}{2.9980 \times 10^8\text{ m/s}} = 0.367$ so $\frac{\lambda_0}{\lambda_S} = \sqrt{\frac{1+0.367}{1-0.367}} = 1.5$

EVALUATE: The galaxy in Examples 44.9 and 44.10 is 710 Mly away so has a smaller recession speed and redshift than the galaxy in this problem.

44.36. **IDENTIFY and SET UP:** $m_\text{H} = 1.67 \times 10^{-27}\text{ kg}$. The ideal gas law says $pV = nRT$. Normal pressure is $1.013 \times 10^5\text{ Pa}$ and normal temperature is about $27\ ^\circ\text{C} = 300\text{ K}$. 1 mole is 6.02×10^{23} atoms.

EXECUTE: **(a)** $\dfrac{6.3\times10^{-27}\ \text{kg/m}^3}{1.67\times10^{-27}\ \text{kg/atom}} = 3.8\ \text{atoms/m}^3$

(b) $V = (4\ \text{m})(7\ \text{m})(3\ \text{m}) = 84\ \text{m}^3$ and $(3.8\ \text{atoms/m}^3)(84\ \text{m}^3) = 320$ atoms

(c) With $p = 1.013\times10^5$ Pa , $V = 84\ \text{m}^3$, $T = 300$ K the ideal gas law gives the number of moles to be

$$n = \frac{pV}{RT} = \frac{(1.013\times10^5\ \text{Pa})(84\ \text{m}^3)}{(8.3145\ \text{J/mol}\cdot\text{K})(300\ \text{K})} = 3.4\times10^3\ \text{moles}$$

$(3.4\times10^3\ \text{moles})(6.02\times10^{23}\ \text{atoms/mol}) = 2.0\times10^{27}$ atoms

EVALUATE: The average density of the universe is very small. Interstellar space contains a very small number of atoms per cubic meter, compared to the number of atoms per cubit meter in ordinary material on the earth, such as air.

44.37. **IDENTIFY** and **SET UP:** Find the energy equivalent of the mass decrease.

EXECUTE: **(a)** $\text{p} + {}^2_1\text{H} \to {}^3_2\text{He}$ or can write as ${}^1_1\text{H} + {}^2_1\text{H} \to {}^3_2\text{He}$

If neutral atom masses are used then the masses of the two atomic electrons on each side of the reaction will cancel.

Taking the atomic masses from Table 43.2, the mass decrease is $m\left({}^1_1\text{H}\right) + m\left({}^2_1\text{H}\right) - m\left({}^3_2\text{He}\right) =$ 1.007825 u + 2.014102 u − 3.016029 u = 0.005898 u. The energy released is the energy equivalent of this mass decrease: $(0.005898\ \text{u})(931.5\ \text{MeV/u}) = 5.494\ \text{MeV}$

(b) ${}^1_0\text{n} + {}^3_2\text{He} \to {}^4_2\text{He}$

If neutral helium masses are used then the masses of the two atomic electrons on each side of the reaction equation will cancel. The mass decrease is $m\left({}^1_0\text{n}\right) + m\left({}^3_2\text{He}\right) - m\left({}^4_2\text{He}\right) = 1.008665$ u + 3.016029 u − 4.002603 u = 0.022091 u. The energy released is the energy equivalent of this mass decrease: $(0.022091\ \text{u})(931.l5\ \text{MeV/u}) = 20.58\ \text{MeV}$

EVALUATE: These are important nucleosynthesis reactions, discussed in Section 44.7.

44.38. $3m({}^4\text{He}) - m({}^{12}\text{C}) = 7.80\times10^{-3}$ u, or 7.27 MeV.

44.39. $\Delta m = m_\text{e} + m_\text{p} - m_\text{n} - m_{\nu_\text{e}}$ so assuming $m_{\nu_\text{e}} \approx 0$,

$\Delta m = 0.0005486\ \text{u} + 1.007276\ \text{u} - 1.008665\ \text{u} = -8.40\times10^{-4}\ \text{u}$

$\Rightarrow E = (\Delta m)c^2 = (-8.40\times10^{-4}\ \text{u})(931.5\ \text{MeV/u}) = -0.783\ \text{MeV}$ and is endoergic.

44.40. $m_{{}^{12}_6\text{C}} + m_{{}^4_2\text{He}} - m_{{}^{16}_8\text{O}} = 7.69\times10^{-3}$ u, or 7.16 MeV, an exoergic reaction.

44.41. **IDENTIFY** and **SET UP:** The Wien displacement law (Eq.38.30) sys $\lambda_\text{m}T$ equals a constant. Use this to relate $\lambda_{\text{m},1}$ at T_1 to $\lambda_{\text{m},2}$ at T_2.

EXECUTE: $\lambda_{\text{m},1}T_1 = \lambda_{\text{m},2}T_2$

$$\lambda_{\text{m},1} = \lambda_{\text{m},2}\left(\frac{T_2}{T_1}\right) = 1.062\times10^{-3}\ \text{m}\left(\frac{2.728\ \text{K}}{3000\ \text{K}}\right) = 966\ \text{nm}$$

EVALUATE: The peak wavelength was much less when the temperature was much higher.

44.42. **(a)** The dimensions of $\hbar$ are energy times time, the dimensions of G are energy times time per mass squared, and so the dimensions of $\sqrt{\hbar G/c^3}$ are

$$\left[\frac{(\text{E}\cdot\text{T})(\text{E}\cdot\text{L}/\text{M}^2)}{(\text{L}/\text{T})^3}\right]^{1/2} = \left[\frac{\text{E}}{\text{M}}\right]\left[\frac{\text{T}^2}{\text{L}}\right] = \left[\frac{\text{L}}{\text{T}}\right]^2\left[\frac{\text{T}^2}{\text{L}}\right] = \text{L}.$$

(b) $\left(\dfrac{\hbar G}{c^3}\right)^{1/2} = \left(\dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)}{2\pi(3.00\times10^8\ \text{m/s})^3}\right)^{1/2} = 1.616\times10^{-35}\ \text{m}.$

44.43. **IDENTIFY** and **SET UP:** For colliding beams the available energy is twice the beam energy. For a fixed-target experiment only a portion of the beam energy is available energy (Eqs.44.9 and 44.10).

EXECUTE: **(a)** $E_\text{a} = 2(7.0\ \text{TeV}) = 14.0\ \text{TeV}$

(b) Need $E_\text{a} = 14.0\ \text{TeV} = 14.0\times10^6\ \text{MeV}$. Since the target and projectile particles are both protons Eq. (44.10) can be used: $E_\text{a}^2 = 2mc^2(E_m + mc^2)$

$$E_m = \frac{E_\text{a}^2}{2mc^2} - mc^2 = \frac{(14.0\times10^6\ \text{MeV})^2}{2(938.3\ \text{MeV})} - 938.3\ \text{MeV} = 1.0\times10^{11}\ \text{MeV} = 1.0\times10^5\ \text{TeV}.$$

EVALUATE: This shows the great advantage of colliding beams at relativistic energies.

44.44. $K+m_{\text{p}}c^2=\dfrac{hc}{\lambda},\ K=\dfrac{hc}{\lambda}-m_{\text{p}}c^2=652\text{ MeV}.$

44.45. **IDENTIFY** and **SET UP:** Section 44.3 says the strong interaction is 100 times as strong as the electromagnetic interaction and that the weak interaction is 10^{-9} times as strong as the strong interaction. The Coulomb force is $F_{\text{e}}=\dfrac{kq_1q_2}{r^2}$ and the gravitational force is $F_{\text{g}}=G\dfrac{m_1m_2}{r^2}$.

EXECUTE: **(a)** $F_{\text{e}}=\dfrac{(9.0\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(1\times10^{-15}\text{ m})^2}=200\text{ N}$

$F_{\text{g}}=\dfrac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.67\times10^{-27}\text{ kg})^2}{(1\times10^{-15}\text{ m})^2}=2\times10^{-34}\text{ N}$

(b) $F_{\text{str}}\approx100F_{\text{e}}\approx2\times10^4\text{ N}$. $F_{\text{weak}}\approx10^{-9}F_{\text{str}}\approx2\times10^{-5}\text{ N}$

(c) $F_{\text{str}}>F_{\text{e}}>F_{\text{weak}}>F_{\text{g}}$

(d) $F_{\text{e}}\approx1\times10^{36}F_{\text{g}}$. $F_{\text{str}}\approx100F_{\text{e}}\approx1\times10^{38}F_{\text{g}}$. $F_{\text{weak}}\approx10^{-9}F_{\text{str}}\approx1\times10^{29}F_{\text{g}}$

EVALUATE: The gravity force is much weaker than any of the other three forces. Gravity is important only when one very massive object is involved.

44.46. In Eq.(44.9), $E_{\text{a}}=(m_{\Sigma^0}+m_{\text{K}^0})c^2$, and with $M=m_{\text{p}}, m=m_{\pi^-}$ and $E_m=(m_{\pi^-})c^2+K$,

$$K=\frac{E_{\text{a}}^2-(m_\pi-c^2)^2-(m_{\text{p}}c^2)^2}{2m_{\text{p}}c^2}-(m_{\pi^-})c^2$$

$$K=\frac{(1193\text{ MeV}+497.7\text{ MeV})^2-(139.6\text{ MeV})^2-(938.3\text{ MeV})^2}{2(938.3\text{ MeV})}-139.6\text{ MeV}=904\text{ MeV}.$$

44.47. **IDENTIFY:** With a stationary target, only part of the initial kinetic energy of the moving proton is available. Momentum conservation tells us that there must be nonzero momentum after the collision, which means that there must also be left over kinetic energy. Therefore not all of the initial energy is available.

SET UP: The available energy is given by $E_{\text{a}}^2=2mc^2\left(E_m+mc^2\right)$ for two particles of equal mass when one is initially stationary. The *minimum* available energy must be equal to the rest mass energies of the products, which in this case is two protons, a K^+ and a K^-. The available energy must be at least the sum of the final rest masses.

EXECUTE: The minimum amount of available energy must be

$E_{\text{a}}=2m_{\text{p}}+m_{\text{K}^+}+m_{\text{K}^-}=2(938.3\text{ MeV})+493.7\text{ MeV}+493.7\text{ MeV}=2864\text{ MeV}=2.864\text{ GeV}$

Solving the available energy formula for E_m gives $E_{\text{a}}^2=2mc^2\left(E_m+mc^2\right)$ and

$$E_m=\frac{E_{\text{a}}^2}{2mc^2}-mc^2=\frac{(2864\text{ MeV})^2}{2(938.3\text{ MeV})}-938.3\text{ MeV}=3432.6\text{ MeV}$$

Recalling that E_m is the *total* energy of the proton, including its rest mass energy (RME), we have

$$K=E_m-RME=3432.6\text{ MeV}-938.3\text{ MeV}=2494\text{ MeV}=2.494\text{ GeV}$$

Therefore the threshold kinetic energy is $K=2494$ MeV = 2.494 GeV.

EVALUATE: Considerably less energy would be needed if the experiment were done using colliding beams of protons.

44.48. **(a)** The decay products must be neutral, so the only possible combinations are $\pi^0\pi^0\pi^0$ *or* $\pi^0\pi^+\pi^-$

(b) $m_{\eta_0}-3m_{\pi^0}=142.3\text{ MeV}/c^2$, so the kinetic energy of the π^0 mesons is 142.3 MeV. For the other reaction, $K=(m_{\eta_0}-m_{\pi^0}-m_{\pi^+}-m_{\pi^-})c^2=133.1\text{ MeV}$.

44.49. **IDENTIFY** and **SET UP:** Apply conservation of linear momentum to the collision. A photon has momentum $p=h/\lambda$, in the direction it is traveling. The energy of a photon is $E=pc=\dfrac{hc}{\lambda}$. All the mass of the electron and positron is converted to the total energy of the two photons, according to $E=mc^2$. The mass of an electron and of a positron is $m_{\text{e}}=9.11\times10^{-31}\text{ kg}$

EXECUTE: **(a)** In the lab frame the initial momentum of the system is zero, since the electron and positron have equal speeds in opposite directions. According to momentum conservation, the final momentum of the system must also be zero. A photon has momentum, so the momentum of a single photon is not zero.

(b) For the two photons to have zero total momentum they must have the same magnitude of momentum and move in opposite directions. Since $E=pc$, equal p means equal E.

(c) $2E_{\text{ph}} = 2m_{\text{e}}c^2$ so $E_{\text{ph}} = m_{\text{e}}c^2$

$$E_{\text{ph}} = \frac{hc}{\lambda} \text{ so } \frac{hc}{\lambda} = m_{\text{e}}c^2 \text{ and } \lambda = \frac{h}{m_{\text{e}}c} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})} = 2.43\ \text{pm}$$

These are gamma ray photons.

EVALUATE: The total charge of the electron/positron system is zero and the photons have no charge, so charge is conserved in the particle-antiparticle annihilation.

44.50. **(a)** If the π^- decays, it must end in an electron and neutrinos. The rest energy of π^- (139.6 MeV) is shared between the electron rest energy (0.511 MeV) and kinetic energy (assuming the neutrino masses are negligible). So the energy released is 139.6 MeV – 0.511 MeV = 139.1 MeV.

(b) Conservation of momentum leads to the neutrinos carrying away most of the energy.

44.51. **(a)** The baryon number is 0, the charge is $+e$, the strangeness is 1, all lepton numbers are zero, and the particle is K^+.

(b) The baryon number is 0, the charge is $-e$, the strangeness is 0, all lepton numbers are zero, and the particle is π^-.

(c) The baryon numbers is –1, the charge is 0, the strangeness is zero, all lepton numbers are 0, and the particle is an antineutron.

(d) The baryon number is 0, the charge is $+e$, the strangeness is 0, the muonic lepton number is –1, all other lepton numbers are 0, and the particle is μ^+.

44.52. $$\Delta t = 7.6\times10^{-21}\ \text{s} \Rightarrow \Delta E = \frac{\hbar}{\Delta t} = \frac{1.054\times10^{-34}\ \text{J}\cdot\text{s}}{7.6\times10^{-21}\ \text{s}} = 1.39\times10^{-14}\ \text{J} = 87\ \text{keV}.$$

$$\frac{\Delta E}{m_\psi c^2} = \frac{0.087\ \text{MeV}}{3097\ \text{MeV}} = 2.8\times10^{-5}.$$

44.53. $$\frac{\hbar}{\Delta E} = \frac{(1.054\times10^{-34}\ \text{J}\cdot\text{s})}{(4.4\times10^{6}\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})} = 1.5\times10^{-22}\ \text{s}.$$

44.54. **IDENTIFY** and **SET UP:** $\phi \to \text{K}^+ + \text{K}^-$. The total energy released is the energy equivalent of the mass decrease.

(a) EXECUTE: The mass decrease is $m(\phi) - m(\text{K}^+) - m(\text{K}^-)$. The energy equivalent of the mass decrease is $mc^2(\phi) - mc^2(\text{K}^+) - mc^2(\text{K}^-)$. The rest mass energy mc^2 for the ϕ meson is given in Problem 44.53, and the values for $\text{K}^+$and K^- are given in Table 44.3. The energy released then is $1019.4\ \text{MeV} - 2(493.7\ \text{MeV}) =$ $32.0\ \text{MeV}$. The K^+ gets half this, 16.0 MeV.

EVALUATE: **(b)** Does the decay $\phi \to \text{K}^+ + \text{K}^- + \pi^0$ occur? The energy equivalent of the $\text{K}^+ + \text{K}^- + \pi^0$ mass is 493.7 MeV + 493.7 MeV + 135.0 MeV = 1122 MeV. This is greater than the energy equivalent of the ϕ mass. The mass of the decay products would be greater than the mass of the parent particle; the decay is energetically forbidden.

(c) Does the decay $\phi \to \text{K}^+ + \pi^-$ occur? The reaction $\phi \to \text{K}^+ + \text{K}^-$ is observed. K^+ has strangeness $+1$ and K^- has strangeness -1, so the total strangeness of the decay products is zero. If strangeness must be conserved we deduce that the ϕ particle has strangeness zero. π^- has strangeness 0, so the product $\text{K}^+ + \pi^-$ has strangeness -1. The decay $\phi \to \text{K}^+ + \pi^-$ violates conservation of strangeness. Does the decay $\phi \to \text{K}^+ + \mu^-$ occur? μ^- has strangeness 0, so this decay would also violate conservation of strangeness.

44.55. **(a)** The number of protons in a kilogram is

$$(1.00\ \text{kg})\left(\frac{6.023\times10^{23}\ \text{molecules/mol}}{18.0\times10^{-3}\ \text{kg/mol}}\right)(2\ \text{protons/molecule}) = 6.7\times10^{25}.$$

Note that only the protons in the hydrogen atoms are considered as possible sources of proton decay. The energy per decay is $m_{\text{p}}c^2 = 938.3\ \text{MeV} = 1.503\times10^{-10}\ \text{J}$, and so the energy deposited in a year, per kilogram, is

$$(6.7\times10^{25})\left(\frac{\ln(2)}{1.0\times10^{18}\ \text{y}}\right)(1\ \text{y})\,(1.50\times10^{-10}\ \text{J}) = 7.0\times10^{-3}\ \text{Gy} = 0.70\ \text{rad}$$

(b) For an RBE of unity, the equivalent dose is (1) (0.70 rad) = 0.70 rem.

44.56. **IDENTIFY** and **SET UP:** The total released energy is the equivalent of the mass decrease. Use conservation of linear momentum to relate the kinetic energies of the decay particles.

EXECUTE: **(a)** The energy equivalent of the mass decrease is

$$mc^2(\Xi^-) - mc^2(\Lambda^0) - mc^2(\pi^-) = 1321\ \text{MeV} - 1116\ \text{MeV} - 139.6\ \text{MeV} = 65\ \text{MeV}$$

(b) The Ξ^- is at rest means that the linear momentum is zero. Conservation of linear momentum then says that the Λ^0 and π^- must have equal and opposite momenta:

$$m_{\Lambda^0} v_{\Lambda^0} = m_{\pi^-} v_{\pi^-}$$

$$v_{\pi^-} = \left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right) v_{\Lambda^0}$$

Also, the sum of the kinetic energies of the Λ^0 and π^- must equal the total kinetic energy $K_{\text{tot}} = 65\text{ MeV}$ calculated in part (a):

$$K_{\text{tot}} = K_{\Lambda^0} + K_{\pi^-}$$

$$K_{\Lambda^0} + \tfrac{1}{2} m_{\pi^-} v_{\pi^-}^2 = K_{\text{tot}}$$

Use the momentum conservation result:

$$K_{\Lambda^0} + \tfrac{1}{2} m_{\pi^-} \left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right)^2 v_{\Lambda^0}^2 = K_{\text{tot}}$$

$$K_{\Lambda^0} + \left(\frac{m_{\Lambda^0}}{m_{\pi^-}}\right)\left(\tfrac{1}{2} m_{\Lambda^0} v_{\Lambda^0}^2\right) = K_{\text{tot}}$$

$$K_{\Lambda^0}\left(1 + \frac{m_{\Lambda^0}}{m_{\pi^-}}\right) = K_{\text{tot}}$$

$$K_{\Lambda^0} = \frac{K_{\text{tot}}}{1 + m_{\Lambda^0}/m_{\pi^-}} = \frac{65\text{ MeV}}{1 + (1116\text{ MeV})/(139.6\text{ MeV})} = 7.2\text{ MeV}$$

$K_{\Lambda^0} + K_{\pi^-} = K_{\text{tot}}$ so $K_{\pi^-} = K_{\text{tot}} - K_{\Lambda^0} = 65\text{ MeV} - 7.2\text{ MeV} = 57.8\text{ MeV}$

The fraction for the Λ^0 is $\dfrac{7.2\text{ MeV}}{65\text{ MeV}} = 11\%$.

The fraction for the π^- is $\dfrac{57.8\text{ MeV}}{65\text{ MeV}} = 89\%$.

EVALUATE: The lighter particle carries off more of the kinetic energy that is released in the decay than the heavier particle does.

44.57. **(a)** For this model, $\dfrac{dR}{dt} = HR$, so $\dfrac{dR/dt}{R} = \dfrac{HR}{R} = H$, presumed to be the same for all points on the surface.

(b) For constant θ, $\dfrac{dr}{dt} = \dfrac{dR}{dt}\theta = HR\theta = Hr$.

(c) See part (a), $H_0 = \dfrac{dR/dt}{R}$.

(d) The equation $\dfrac{dR}{dt} = H_0 R$ is a differential equation, the solution to which, for constant H_0, is $R(t) = R_0 e^{H_0 t}$, where R_0 is the value of R at $t = 0$. This equation may be solved by separation of variables, as $\dfrac{dR/dt}{R} = \dfrac{d}{dt}\ln(R) = H_0$ and integrating both sides with respect to time.

(e) A constant H_0 would mean a constant critical density, which is inconsistent with uniform expansion.

44.58. From Problem 44.57, $r = R\theta \Rightarrow R = \dfrac{r}{\theta}$. So $\dfrac{dR}{dt} = \dfrac{1}{\theta}\dfrac{dr}{dt} - \dfrac{r}{\theta^2}\dfrac{d\theta}{dt} = \dfrac{1}{\theta}\dfrac{dr}{dt}$ since $\dfrac{d\theta}{dt} = 0$.

So $\dfrac{1}{R}\dfrac{dR}{dt} = \dfrac{1}{R\theta}\dfrac{dr}{dt} = \dfrac{1}{r}\dfrac{dr}{dt} \Rightarrow v = \dfrac{dr}{dt} = \left(\dfrac{1}{R}\dfrac{dR}{dt}\right) r = H_0 r$. Now $\dfrac{dv}{d\theta} = 0 = \dfrac{d}{d\theta}\left(\dfrac{r}{R}\dfrac{dR}{dt}\right) = \dfrac{d}{d\theta}\left(\theta \dfrac{dR}{dt}\right)$

$\Rightarrow \theta \dfrac{dR}{dt} = K$ where K is a constant. $\Rightarrow \dfrac{dR}{dt} = \dfrac{K}{\theta} \Rightarrow R = \left(\dfrac{K}{\theta}\right) t$ since $\dfrac{d\theta}{dt} = 0 \Rightarrow H_0 = \dfrac{1}{R}\dfrac{dR}{dt} = \dfrac{\theta}{Kt}\dfrac{K}{\theta} = \dfrac{1}{t}$. So the current value of the Hubble constant is $\dfrac{1}{T}$ where T is the present age of the universe.

44.59. **(a)** For mass m, in Eq. (37.23) $u = -v_{\text{cm}}$, $v' = v_0$, and so $v_m = \dfrac{v_0 - v_{\text{cm}}}{1 - v_0 v_{\text{cm}}/c^2}$. For mass M, $u = -v_{\text{cm}}$, $v' = 0$, so $v_M = -v_{\text{cm}}$.

(b) The condition for no net momentum in the center of mass frame is $m\gamma_m v_m + M\gamma_M v_M = 0,$ where γ_m and γ_M correspond to the velocities found in part (a). The algebra reduces to $\beta_m\gamma_m = (\beta_0 - \beta')\gamma_0\gamma_M$, where $\beta_0 = \frac{v_0}{c}$, $\beta' = \frac{v_{\text{cm}}}{c}$, and the condition for no net momentum becomes $m(\beta_0 - \beta')\gamma_0\gamma_M = M\beta'\gamma_M$, or

$$\beta' = \frac{\beta_0}{1+\frac{M}{m\gamma_0}} = \beta_0 \frac{m}{m + M\sqrt{1-\beta_0^2}}\text{ . } v_{\text{cm}} = \frac{mv_0}{m + M\sqrt{1-(v_0/c)^2}}.$$

(c) Substitution of the above expression into the expressions for the velocities found in part (a) gives the relatively simple forms $v_m = v_0\gamma_0 \frac{M}{m + M\gamma_0}, v_M = -v_0\gamma_0\frac{m}{m\gamma_0 + M}$. After some more algebra,

$$\gamma_m = \frac{m + M\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}, \gamma_M = \frac{M + m\gamma_0}{\sqrt{m^2 + M^2 + 2mM\gamma_0}}$$

, from which $m\gamma_m + M\gamma_M = \sqrt{m^2 + M^2 + 2mM\gamma_0}$. This last expression, multiplied by c^2, is the available energy E_a in the center of mass frame, so that

$$E_\text{a}^2 = (m^2 + M^2 + 2mM\gamma_0)c^4 = (mc^2)^2 + (Mc^2)^2 + (2Mc^2)(m\gamma_0 c^2) = (mc^2)^2 + (Mc^2)^2 + 2Mc^2 E_m,$$ which is Eq.(44.9).

44.60. $\Lambda^0 \to \text{n} + \pi^0$

(a) $E = (\Delta m)c^2 = (m_{\Lambda^0})c^2 - (m_\text{n})c^2(m_{\pi^0})c^2 = 1116\text{ MeV} - 939.6\text{ MeV} - 135.0\text{ MeV} = 41.4\text{ MeV}$

(b) Using conservation of momentum and kinetic energy; we know that the momentum of the neutron and pion must have the same magnitude, $p_n = p_\pi$.

$$K_\text{n} = E_\text{n} - m_\text{n}c^2 = \sqrt{(m_\text{n}c^2)^2 + (p_\text{n}c)^2} - m_\text{n}c^2 = \sqrt{(m_\text{n}c^2)^2 + (p_\text{n}c)^2} - m_\text{n}c^2$$

$$K_\text{n} = \sqrt{(m_\text{n}c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_\text{n}c^2 = K_\pi + K_\text{n} = K_\pi + \sqrt{(m_\text{n}c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi} - m_\text{n}c^2 = E.$$

$(m_\text{n}c^2)^2 + K_\pi^2 + 2m_\pi c^2 K_\pi = E^2 + (m_\text{n}c^2)^2 + K_\pi^2 + 2Em_\pi c^2 - 2EK_\pi - 2m_\text{n}c^2 K_\pi$. Collecting terms we find:

$K_\pi(2m_\pi c^2 + 2E + 2m_\text{n}c^2) = E^2 + 2Em_\text{n}c^2$

$$\Rightarrow K_\pi = \frac{(41.4\text{ MeV})^2 + 2(41.4\text{ MeV})(939.6\text{ MeV})}{2(135.0\text{ MeV}) + 2(41.4\text{ MeV}) + 2(939.6\text{ MeV})} = 35.62\text{ MeV}.$$

So the fractional energy carried by the pion is $\frac{35.62}{41.4} = 0.86$, and that of the neutron is 0.14.