THE WAVE NATURE OF PARTICLES

39

39.1. IDENTIFY and SET UP: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. For an electron, $m = 9.11\times10^{-31}$ kg. For a proton, $m = 1.67\times10^{-27}$ kg.

EXECUTE: **(a)** $\lambda = \dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(4.70\times10^{6}\ \text{m/s})} = 1.55\times10^{-10}\ \text{m} = 0.155\ \text{nm}$

(b) λ is proportional to $\dfrac{1}{m}$, so $\lambda_{\text{p}} = \lambda_{\text{e}}\left(\dfrac{m_{\text{e}}}{m_{\text{p}}}\right) = (1.55\times10^{-10}\ \text{m})\left(\dfrac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 8.46\times10^{-14}\ \text{m}$.

39.2. IDENTIFY and SET UP: For a photon, $E = \dfrac{hc}{\lambda}$. For an electron or proton, $p = \dfrac{h}{\lambda}$ and $E = \dfrac{p^2}{2m}$, so $E = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** $E = \dfrac{hc}{\lambda} = \dfrac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{0.20\times10^{-9}\ \text{m}} = 6.2\ \text{keV}$

(b) $E = \dfrac{h^2}{2m\lambda^2} = \left(\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{0.20\times10^{-9}\ \text{m}}\right)^2 \dfrac{1}{2(9.11\times10^{-31}\ \text{kg})} = 6.03\times10^{-18}\ \text{J} = 38\ \text{eV}$

(c) $E_{\text{p}} = E_{\text{e}}\left(\dfrac{m_{\text{e}}}{m_{\text{p}}}\right) = (38\ \text{eV})\left(\dfrac{9.11\times10^{-31}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}}\right) = 0.021\ \text{eV}$

EVALUATE: For a given wavelength a photon has much more energy than an electron, which in turn has more energy than a proton.

39.3. **(a)** $\lambda = \dfrac{h}{p} \Rightarrow p = \dfrac{h}{\lambda} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(2.80\times10^{-10}\ \text{m})} = 2.37\times10^{-24}\ \text{kg}\cdot\text{m/s}.$

(b) $K = \dfrac{p^2}{2m} = \dfrac{(2.37\times10^{-24}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 3.08\times10^{-18}\ \text{J} = 19.3\ \text{eV}.$

39.4. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(6.64\times10^{-27}\ \text{kg})\,(4.20\times10^{6}\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}} = 7.02\times10^{-15}\ \text{m}.$$

39.5. IDENTIFY and SET UP: The de Broglie wavelength is $\lambda = \dfrac{h}{p} = \dfrac{h}{mv}$. In the Bohr model, $mvr_n = n(h/2\pi)$, so $mv = nh/(2\pi r_n)$. Combine these two expressions and obtain an equation for λ in terms of n. Then

$\lambda = h\left(\dfrac{2\pi r_n}{nh}\right) = \dfrac{2\pi r_n}{n}.$

EXECUTE: **(a)** For $n = 1$, $\lambda = 2\pi r_1$ with $r_1 = a_0 = 0.529\times10^{-10}$ m, so $\lambda = 2\pi(0.529\times10^{-10}\ \text{m}) = 3.32\times10^{-10}$ m

$\lambda = 2\pi r_1$; the de Broglie wavelength equals the circumference of the orbit.

(b) For $n = 4$, $\lambda = 2\pi r_4/4$.

$r_n = n^2 a_0$ so $r_4 = 16a_0$.

$\lambda = 2\pi(16a_0)/4 = 4(2\pi a_0) = 4(3.32\times10^{-10}\ \text{m}) = 1.33\times10^{-9}$ m

$\lambda = 2\pi r_4/4$; the de Broglie wavelength is $\dfrac{1}{n} = \dfrac{1}{4}$ times the circumference of the orbit.

EVALUATE: As n increases the momentum of the electron increases and its de Broglie wavelength decreases. For any n, the circumference of the orbits equals an integer number of de Broglie wavelengths.

39.6. **(a)** For a nonrelativistic particle, $K=\dfrac{p^2}{2m}$, so $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{2Km}}$.

(b) $(6.63\times10^{-34}\ \text{J}\cdot\text{s})\big/\sqrt{2(800\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})(9.11\times10^{-31}\ \text{kg})}=4.34\times10^{-11}\ \text{m}$.

39.7. **IDENTIFY:** A person walking through a door is like a particle going through a slit and hence should exhibit wave properties.

SET UP: The de Broglie wavelength of the person is $\lambda = h/mv$.

EXECUTE: **(a)** Assume m = 75 kg and v = 1.0 m/s.

$$\lambda = h/mv = (6.626\times10^{-34}\ \text{J}\cdot\text{s})/[(75\ \text{kg})(1.0\ \text{m/s})] = 8.8\times10^{-36}\ \text{m}$$

EVALUATE: **(b)** A typical doorway is about 1 m wide, so the person's de Broglie wavelength is much too small to show wave behavior through a "slit" that is about 10^{35} times as wide as the wavelength. Hence ordinary objects do not show wave behavior in everyday life.

39.8. Combining Equations 37.38 and 37.39 gives $p=mc\sqrt{\gamma^2-1}$.

(a) $\lambda=\dfrac{h}{p}=(h/mc)\big/\sqrt{\gamma^2-1}=4.43\times10^{-12}$ m. (The incorrect nonrelativistic calculation gives 5.05×10^{-12} m.)

(b) $(h/mc)\big/\sqrt{\gamma^2-1}=7.07\times10^{-13}$ m.

39.9. **IDENTIFY** and **SET UP:** A photon has zero mass and its energy and wavelength are related by Eq.(38.2). An electron has mass. Its energy is related to its momentum by $E=p^2/2m$ and its wavelength is related to its momentum by Eq.(39.1).

EXECUTE: **(a)** photon

$$E=\frac{hc}{\lambda}\ \text{so}\ \lambda=\frac{hc}{E}=\frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{(20.0\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}=62.0\ \text{nm}$$

electron

$$E=p^2/(2m)\ \text{so}\ p=\sqrt{2mE}=\sqrt{2(9.109\times10^{-31}\ \text{kg})(20.0\ \text{eV})(1.602\times10^{-19}\ \text{J/eV})}=2.416\times10^{-24}\ \text{kg}\cdot\text{m/s}$$

$$\lambda=h/p=0.274\ \text{nm}$$

(b) photon $E=hc/\lambda=7.946\times10^{-19}\ \text{J}=4.96\ \text{eV}$

electron $\lambda=h/p$ so $p=h/\lambda=2.650\times10^{-27}\ \text{kg}\cdot\text{m/s}$

$$E=p^2/(2m)=3.856\times10^{-24}\ \text{J}=2.41\times10^{-5}\ \text{eV}$$

(c) EVALUATE: You should use a probe of wavelength approximately 250 nm. An electron with $\lambda=250$ nm has much less energy than a photon with $\lambda=250$ nm, so is less likely to damage the molecule. Note that $\lambda=h/p$ applies to all particles, those with mass and those with zero mass. $E=hf=hc/\lambda$ applies only to photons and $E=p^2/2m$ applies only to particles with mass.

39.10. **IDENTIFY:** Any moving particle has a de Broglie wavelength. The speed of a molecule, and hence its de Broglie wavelength, depends on the temperature of the gas.

SET UP: The average kinetic energy of the molecule is K_{av} = 3/2 kT, and the de Broglie wavelength is $\lambda = h/mv = h/p$.

EXECUTE: **(a)** Combining K_{av} = 3/2 kT and $K = p^2/2m$ gives 3/2 $kT = p_{av}^2/2m$ and $p_{av} = \sqrt{3mkT}$. The de Broglie wavelength is $\lambda=\dfrac{h}{p}=\dfrac{h}{\sqrt{3mkT}}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{3\left(2\times1.67\times10^{-27}\ \text{kg}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)\left(273\ \text{K}\right)}}=1.08\times10^{-10}\ \text{m}$.

(b) For an electron, $\lambda = h/p = h/mv$ gives

$$v=\frac{h}{m\lambda}=\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.08\times10^{-10}\ \text{m}\right)}=6.75\times10^{6}\ \text{m/s}$$

This is about 2% the speed of light, so we do *not* need to use relativity.

(c) For photon:

$$E = hc/\lambda = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(1.08\times10^{-10}\ \text{m}) = 1.84\times10^{-15}\ \text{J}$$

For the H_2 molecule: $K_{av} = (3/2)kT = 3/2\,(1.38\times10^{-23}\ \text{J/K})(273\ \text{K}) = 5.65\times10^{-21}$ J

For the electron: $K = ½\, mv^2 = ½\,(9.11\times10^{-31}\ \text{kg})(6.73\times10^{6}\ \text{m/s})^2 = 2.06\times10^{-17}$ J

EVALUATE: The photon has about 100 times more energy than the electron and 300,000 times more energy than the H_2 molecule. This shows that photons of a given wavelength will have much more energy than particles of the same wavelength.

39.11. **IDENTIFY** and **SET UP:** Use Eq.(39.1).

EXECUTE: $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(5.00\times10^{-3}\ \text{kg})(340\ \text{m/s})} = 3.90\times10^{-34}\ \text{m}$

EVALUATE: This wavelength is extremely short; the bullet will not exhibit wavelike properties.

39.12. **(a)** $\lambda = h/mv \rightarrow v = h/m\lambda$

Energy conservation: $e\Delta V = \dfrac{1}{2}mv^2$

$$\Delta V = \frac{mv^2}{2e} = \frac{m\left(\dfrac{h}{m\lambda}\right)^2}{2e} = \frac{h^2}{2em\lambda^2} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.60\times10^{-19}\ \text{C})\,(9.11\times10^{-31}\ \text{kg})\,(0.15\times10^{-9}\ \text{m})^2} = 66.9\ \text{V}$$

(b) $E_{\text{photon}} = hf = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(3.0\times10^{8}\ \text{m/s})}{0.15\times10^{-9}\ \text{m}} = 1.33\times10^{-15}\ \text{J}$

$e\Delta V = K = E_{\text{photon}}$ and $\Delta V = \dfrac{E_{\text{photon}}}{e} = \dfrac{1.33\times10^{-15}\ \text{J}}{1.6\times10^{-19}\ \text{C}} = 8310\ \text{V}$

39.13. **(a)** $\lambda = 0.10\ \text{nm}$. $p = mv = h/\lambda$ so $v = h/(m\lambda) = 7.3\times10^{6}\ \text{m/s}$.

(b) $E = \dfrac{1}{2}mv^2 = 150\ \text{eV}$

(c) $\text{E} = hc/\lambda = 12\ \text{KeV}$

(d) The electron is a better probe because for the same λ it has less energy and is less damaging to the structure being probed.

39.14. **IDENTIFY:** The electrons behave like waves and are diffracted by the slit.

SET UP: We use conservation of energy to find the speed of the electrons, and then use this speed to find their de Broglie wavelength, which is $\lambda = h/mv$. Finally we know that the first dark fringe for single-slit diffraction occurs when $a \sin\theta = \lambda$.

EXECUTE: **(a)** Use energy conservation to find the speed of the electron: $\tfrac{1}{2} mv^2 = eV$.

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\left(1.60\times10^{-19}\ \text{C}\right)(100\ \text{V})}{9.11\times10^{-31}\ \text{kg}}} = 5.93\times10^{6}\ \text{m/s}$$

which is about 2% the speed of light, so we can ignore relativity.

(b) First find the de Broglie wavelength:

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.93\times10^{6}\ \text{m/s}\right)} = 1.23\times10^{-10}\ \text{m} = 0.123\ \text{nm}$$

For the first single slit dark fringe, we have $a \sin\theta = \lambda$, which gives

$$a = \frac{\lambda}{\sin\theta} = \frac{1.23\times10^{-10}\ \text{m}}{\sin(11.5°)} = 6.16\times10^{-10}\ \text{m} = 0.616\ \text{nm}$$

EVALUATE: The slit width is around 5 times the de Broglie wavelength of the electron, and both are much smaller than the wavelength of visible light.

39.15. For $m = 1$, $\lambda = d\sin\theta = \dfrac{h}{\sqrt{2mE}}$.

$$E = \frac{h^2}{2md^2\sin^2\theta} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(1.675\times10^{-27}\ \text{kg})\,(9.10\times10^{-11}\ \text{m})^2\sin^2(28.6°)} = 6.91\times10^{-20}\ \text{J} = 0.432\ \text{eV}.$$

39.16. Intensity maxima occur when $d\sin\theta = m\lambda$. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2ME}}$ so $d\sin\theta = \dfrac{mh}{\sqrt{2ME}}$. (Careful! Here, m is the order of the maxima, whereas M is the mass of the incoming particle.)

(a) $d = \dfrac{mh}{\sqrt{2ME}\sin\theta} = \dfrac{(2)(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(188\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}\sin(60.6°)}$

$= 2.06\times10^{-10}\ \text{m} = 0.206\ \text{nm}.$

(b) $m = 1$ also gives a maximum.

$$\theta = \arcsin\left(\frac{(1)\,(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})\,(188\ \text{eV})\,(1.60\times10^{-19}\ \text{J/eV})}\,(2.06\times10^{-10}\ \text{m})}\right) = 25.8°.$$

This is the only other one. If we let $m \geq 3$, then there are no more maxima.

(c) $E = \dfrac{m^2h^2}{2Md^2\sin^2\theta} = \dfrac{(1)^2(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times10^{-31}\text{ kg})(2.60\times10^{-10}\text{ m})^2\sin^2(60.6°)}$

$= 7.49\times10^{-18}\text{ J} = 46.8\text{ eV}.$

Using this energy, if we let $m = 2$, then $\sin\theta > 1$. Thus, there is no $m = 2$ maximum in this case.

39.17. The condition for a maximum is $d\sin\theta = m\lambda$. $\lambda = \dfrac{h}{p} = \dfrac{h}{Mv}$, so $\theta = \arcsin\left(\dfrac{mh}{dMv}\right)$. (Careful! Here, *m* is the order of the maximum, whereas *M* is the incoming particle mass.)

(a) $m = 1 \Rightarrow \theta_1 = \arcsin\left(\dfrac{h}{dMv}\right)$

$= \arcsin\left(\dfrac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(1.60\times10^{-6}\text{ m})(9.11\times10^{-31}\text{ kg})(1.26\times10^{4}\text{ m/s})}\right) = 2.07°.$

$m = 2 \Rightarrow \theta_2 = \arcsin\left(\dfrac{(2)(6.63\times10^{-34}\text{ J}\cdot\text{s})}{(1.60\times10^{-6}\text{ m})(9.11\times10^{-31}\text{ kg})(1.26\times10^{4}\text{ m/s})}\right) = 4.14°.$

(b) For small angles (in radians!) $y \cong D\theta$, so $y_1 \approx (50.0\text{ cm})(2.07°)\left(\dfrac{\pi\text{ radians}}{180°}\right) = 1.81\text{ cm}$,

$y_2 \approx (50.0\text{ cm})(4.14°)\left(\dfrac{\pi\text{ radians}}{180°}\right) = 3.61\text{ cm}$ and $y_2 - y_1 = 3.61\text{ cm} - 1.81\text{ cm} = 1.81\text{ cm}.$

39.18. **IDENTIFY:** Since we know only that the mosquito is somewhere in the room, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x\Delta p_x \geq \hbar$.

EXECUTE: **(a)** You know the mosquito is somewhere in the room, so the maximum uncertainty in its horizontal position is $\Delta x = 5.0$ m.

(b) The uncertainty principle gives $\Delta x\Delta p_x \geq \hbar$, and $\Delta p_x = m\Delta v_x$ since we know the mosquito's mass. This gives $\Delta x\, m\Delta v_x \geq \hbar$, which we can solve for Δv_x to get the minimum uncertainty in v_x.

$$\Delta v_x = \frac{\hbar}{m\Delta x} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(1.5\times10^{-6}\text{ kg})(5.0\text{ m})} = 1.4\times10^{-29}\text{ m/s}$$

which is hardly a serious impediment!

EVALUATE: For something as "large" as a mosquito, the uncertainty principle places a negligible limitation on our ability to measure its speed.

39.19. **(a) IDENTIFY** and **SET UP:** Use $\Delta x\Delta p_x \geq h/2\pi$ to calculate Δx and obtain Δv_x from this.

EXECUTE: $\Delta p_x \geq \dfrac{h}{2\pi\Delta x} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{2\pi(1.00\times10^{-6}\text{ m})} = 1.055\times10^{-28}\text{ kg}\cdot\text{m/s}$

$\Delta v_x = \dfrac{\Delta p_x}{m} = \dfrac{1.055\times10^{-28}\text{ kg}\cdot\text{m/s}}{1200\text{ kg}} = 8.79\times10^{-32}\text{ m/s}$

(b) EVALUATE: Even for this very small Δx the minimum Δv_x required by the Heisenberg uncertainty principle is very small. The uncertainty principle does not impose any practical limit on the simultaneous measurements of the positions and velocities of ordinary objects.

39.20. **IDENTIFY:** Since we know that the marble is somewhere on the table, there is an uncertainty in its position. The Heisenberg uncertainty principle tells us that there is therefore an uncertainty in its momentum.

SET UP: The uncertainty principle is $\Delta x\Delta p_x \geq \hbar$.

EXECUTE: **(a)** Since the marble is somewhere on the table, the maximum uncertainty in its horizontal position is $\Delta x = 1.75$ m.

(b) Following the same procedure as in part (b) of problem 39.18, the minimum uncertainty in the horizontal velocity of the marble is

$$\Delta v_x = \frac{\hbar}{m\Delta x} = \frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{(0.0100\text{ kg})(1.75\text{ m})} = 6.03\times10^{-33}\text{ m/s}$$

(c) The uncertainty principle tells us that we cannot know that the marble's horizontal velocity is *exactly* zero, so the smallest we could measure it to be is 6.03×10^{-33} m/s, from part (b). The longest time it could remain on the table is the time to travel the full width of the table (1.75 m), so $t = x/v_x$ = (1.75 m)/(6.03 × 10^{-33} m/s) = 2.90 × 10^{32} s = 9.20 × 10^{24} years

Since the universe is about 14 × 10^9 years old, this time is about

$$\frac{9.0\times 10^{24}\text{ yr}}{14\times 10^{9}\text{ yr}} \approx 6\times 10^{14} \text{ times the age of the universe! Don't hold your breath!}$$

EVALUATE: For household objects, the uncertainty principle places a negligible limitation on our ability to measure their speed.

39.21. Heisenberg's Uncertainty Principles tells us that $\Delta x \Delta p_x \geq \dfrac{h}{2\pi}$. We can treat the standard deviation as a direct measure of uncertainty. Here $\Delta x\Delta p_x = (1.2\times 10^{-10}\text{ m})\,(3.0\times 10^{-25}\text{ kg}\cdot\text{m/s}) = 3.6\times 10^{-35}\text{ J}\cdot\text{s}$ but $\dfrac{h}{2\pi} = 1.05\times 10^{-34}\text{ J}\cdot\text{s}$

Therefore $\Delta x\Delta p_x < \dfrac{h}{2\pi}$ so the claim is *not valid*.

39.22. **(a)** $(\Delta x)\,(m\Delta v_x) \geq h/2\pi$, and setting $\Delta v_x = (0.010)v_x$ and the product of the uncertainties equal to $h/2\pi$ (for the minimum uncertainty) gives $v_x = h/(2\pi m(0.010)\Delta x) = 57.9\text{ m/s}$.

(b) Repeating with the proton mass gives 31.6 mm/s.

39.23. $\Delta E > \dfrac{h}{2\pi\Delta t} = \dfrac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})}{2\pi(5.2\times 10^{-3}\text{ s})} = 2.03\times 10^{-32}\text{ J} = 1.27\times 10^{-13}\text{ eV}.$

39.24. **IDENTIFY** and **SET UP:** The Heisenberg Uncertainty Principle says $\Delta x\Delta p_x \geq \dfrac{h}{2\pi}$. The minimum allowed $\Delta x\Delta p_x$ is $h/2\pi$. $\Delta p_x = m\Delta v_x$.

EXECUTE: **(a)** $m\Delta x\Delta v_x = \dfrac{h}{2\pi}$. $\Delta v_x = \dfrac{h}{2\pi m\Delta x} = \dfrac{6.63\times 10^{-34}\text{ J}\cdot\text{s}}{2\pi(1.67\times 10^{-27}\text{ kg})(2.0\times 10^{-12}\text{ m})} = 3.2\times 10^{4}\text{ m/s}$

(b) $\Delta x = \dfrac{h}{2\pi m\Delta v_x} = \dfrac{6.63\times 10^{-34}\text{ J}\cdot\text{s}}{2\pi(9.11\times 10^{-31}\text{ kg})(0.250\text{ m/s})} = 4.6\times 10^{-4}\text{ m}$

39.25. $\Delta E\Delta t = \dfrac{h}{2\pi}$. $\Delta E = \dfrac{h}{2\pi\Delta t} = \dfrac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})}{2\pi(7.6\times 10^{-21}\text{ s})} = 1.39\times 10^{-14}\text{ J} = 8.69\times 10^{4}\text{ eV} = 0.0869\text{ MeV}.$

$$\frac{\Delta E}{E} = \frac{0.0869\text{ MeV}/c^2}{3097\text{ MeV}/c^2} = 2.81\times 10^{-5}.$$

39.26. $\Delta E\Delta t = \dfrac{h}{2\pi}$. $\Delta E = \Delta mc^2$. $\Delta m = 2.06\times 10^{9}\text{ eV}/c^2 = 3.30\times 10^{-10}\text{ J}/c^2$.

$\Delta t = \dfrac{h}{2\pi\Delta mc^2} = \dfrac{6.63\times 10^{-34}\text{ J}\cdot\text{s}}{2\pi(3.30\times 10^{-10}\text{ J})} = 3.20\times 10^{-25}\text{ s}.$

39.27. **IDENTIFY** and **SET UP:** For a photon $E_{\text{ph}} = \dfrac{hc}{\lambda} = \dfrac{1.99\times 10^{-25}\text{ J}\cdot\text{m}}{\lambda}$. For an electron $E_{\text{e}} = \dfrac{p^2}{2m} = \dfrac{1}{2m}\left(\dfrac{h}{\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2}$.

EXECUTE: **(a)** photon $E_{\text{ph}} = \dfrac{1.99\times 10^{-25}\text{ J}\cdot\text{m}}{10.0\times 10^{-9}\text{ m}} = 1.99\times 10^{-17}\text{ J}$

electron $E_{\text{e}} = \dfrac{(6.63\times 10^{-34}\text{ J}\cdot\text{s})^2}{2(9.11\times 10^{-31}\text{ kg})(10.0\times 10^{-9}\text{ m})^2} = 2.41\times 10^{-21}\text{ J}$

$\dfrac{E_{\text{ph}}}{E_{\text{e}}} = \dfrac{1.99\times 10^{-17}\text{ J}}{2.41\times 10^{-21}\text{ J}} = 8.26\times 10^{3}$

(b) The electron has much less energy so would be less damaging.

EVALUATE: For a particle with mass, such as an electron, $E \sim \lambda^{-2}$. For a massless photon $E \sim \lambda^{-1}$.

39.28. **(a)** $eV = K = \dfrac{p^2}{2m} = \dfrac{(h/\lambda)^2}{2m}$, so $V = \dfrac{(h/\lambda)^2}{2me} = 419\text{ V}$.

(b) The voltage is reduced by the ratio of the particle masses, $(419\text{ V})\dfrac{9.11\times 10^{-31}\text{ kg}}{1.67\times 10^{-27}\text{ kg}} = 0.229\text{ V}$.

39.29. **IDENTIFY** and **SET UP:** $\psi(x) = A\sin kx$. The position probability density is given by $|\psi(x)|^2 = A^2\sin^2 kx$.

EXECUTE: **(a)** The probability is highest where $\sin kx = 1$ so $kx = 2\pi x/\lambda = n\pi/2$, $n = 1, 3, 5, \ldots$
$x = n\lambda/4$, $n = 1, 3, 5, \ldots$ so $x = \lambda/4, 3\lambda/4, 5\lambda/4, \ldots$

(b) The probability of finding the particle is zero where $|\psi|^2 = 0$, which occurs where $\sin kx = 0$ and
$kx = 2\pi x/\lambda = n\pi,\ n = 0,\ 1,\ 2,\ldots$
$x = n\lambda/2,\ n = 0,\ 1,\ 2,\ldots$ so $x = 0,\ \lambda/2,\ \lambda,\ 3\lambda/2,\ldots$

EVALUATE: The situation is analogous to a standing wave, with the probability analogous to the square of the amplitude of the standing wave.

39.30. $\Psi^* = \psi^* \sin\omega t$, so $|\Psi|^2 = |\Psi^*\Psi| = \psi^*\psi\sin^2\omega t = |\psi|^2\sin^2\omega t$. $|\Psi|^2$ is not time-independent, so Ψ is not the wavefunction for a stationary state.

39.31. IDENTIFY: To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2\, dV$ is the probability that the particle is found in volume dV. Since the particle must be *somewhere*, ψ must have the property that $\int|\psi|^2\, dV = 1$ when the integral is taken over all space.

EXECUTE: **(a)** In one dimension, as we have here, the integral discussed above is of the form $\int_{-\infty}^{\infty} |\psi(x)|^2\, dx = 1$.

(b) Using the result from part (a), we have $\int_{-\infty}^{\infty}\left(e^{ax}\right)^2 dx = \int_{-\infty}^{\infty} e^{2ax} dx = \left.\dfrac{e^{2ax}}{2a}\right|_{-\infty}^{\infty} = \infty$. Hence this wave function cannot be normalized and therefore cannot be a valid wave function.

(c) We only need to integrate this wave function of 0 to ∞ because it is zero for $x < 0$. For normalization we have
$1 = \int_{-\infty}^{\infty} |\psi|^2\, dx = \int_0^{\infty}\left(Ae^{-bx}\right)^2 dx = \int_0^{\infty} A^2 e^{-2bx} dx = \left.\dfrac{A^2 e^{-2bx}}{-2b}\right|_0^{\infty} = \dfrac{A^2}{2b}$, which gives $\dfrac{A^2}{2b} = 1$, so $A = \sqrt{2b}$.

EVALUATE: If b were positive, the given wave function could not be normalized, so it would not be allowable.

39.32. **(a)** The uncertainty in the particle position is proportional to the width of $\psi(x)$, and is inversely proportional to $\sqrt{\alpha}$. This can be seen by either plotting the function for different values of α, finding the expectation value $\langle x^2\rangle = \int \psi^2 x^2 dx$ for the normalized wave function or by finding the full width at half-maximum. The particle's uncertainty in position decreases with increasing α. The dependence of the expectation value $\langle x^2\rangle$ on α may be found by considering

$$\langle x^2\rangle = \frac{\int_{-\infty}^{\infty} x^2 e^{-2\alpha x^2} dx}{\int_{-\infty}^{\infty} e^{-2\alpha x^2} dx} = -\frac{1}{2}\frac{\partial}{\partial\alpha}\ln\left[\int_{-\infty}^{\infty} e^{-2\alpha x^2} dx\right] = -\frac{1}{2}\frac{\partial}{\partial\alpha}\ln\left[\frac{1}{\sqrt{2\alpha}}\int_{-\infty}^{\infty} e^{-u^2} du\right] = \frac{1}{4\alpha},$$

where the substitution $u = \sqrt{\alpha}x$ has been made.

(b) Since the uncertainty in position decreases, the uncertainty in momentum must increase.

39.33. $f(x,y) = \left(\dfrac{x - iy}{x + iy}\right)$ and $f^*(x,y) = \left(\dfrac{x + iy}{x - iy}\right) \Rightarrow |f|^2 = f\,f^* = \left(\dfrac{x - iy}{x + iy}\right)\cdot\left(\dfrac{x + iy}{x - iy}\right) = 1.$

39.34. The same. $|\psi(x,y,z)|^2 = \psi^*(x,y,z)\psi(x,y,z)$

$|\psi(x,y,z)e^{i\phi}|^2 = (\psi^*(x,y,z)e^{-i\phi})(\psi(x,y,z)e^{+i\phi}) = \psi^*(x,y,z)\psi(x,y,z).$

The complex conjugate means convert all i's to–i's and vice-versa. $e^{i\phi}\cdot e^{-i\phi} = 1.$

39.35. IDENTIFY: To describe a real situation, a wave function must be normalizable.

SET UP: $|\psi|^2\, dV$ is the probability that the particle is found in volume dV. Since the particle must be *somewhere*, ψ must have the property that $\int|\psi|^2\, dV = 1$ when the integral is taken over all space.

EXECUTE: **(a)** For normalization of the one-dimensional wave function, we have
$1 = \int_{-\infty}^{\infty} |\psi|^2\, dx = \int_{-\infty}^{0}\left(Ae^{bx}\right)^2 dx + \int_0^{\infty}\left(Ae^{-bx}\right)^2 dx = \int_{-\infty}^{0} A^2 e^{2bx} dx + \int_0^{\infty} A^2 e^{-2bx} dx.$

$1 = A^2\left\{\left.\dfrac{e^{2bx}}{2b}\right|_{-\infty}^{0} + \left.\dfrac{e^{-2bx}}{-2b}\right|_0^{\infty}\right\} = \dfrac{A^2}{b}$, which gives $A = \sqrt{b} = \sqrt{2.00\ \text{m}^{-1}} = 1.41\ \text{m}^{-1/2}$

(b) The graph of the wavefunction versus x is given in Figure 39.35.

(c) (i) $P = \int_{-0.500\ \text{m}}^{+5.00\ \text{m}} |\psi|^2\, dx = 2\int_0^{+5.00\ \text{m}} A^2 e^{-2bx} dx$, where we have used the fact that the wave function is an even function of x. Evaluating the integral gives

$$P = \frac{-A^2}{b}\left(e^{-2b(0.500\ \text{m})} - 1\right) = \frac{-(2.00\,\text{m}^{-1})}{2.00\,\text{m}^{-1}}\left(e^{-2.00} - 1\right) = 0.865$$

There is a little more than an 86% probability that the particle will be found within 50 cm of the origin.

(ii) $P = \int_{-\infty}^{0}\left(Ae^{bx}\right)^2 dx = \int_{-\infty}^{0} A^2e^{2bx}dx = \frac{A^2}{2b} = \frac{2.00\ \text{m}^{-1}}{2(2.00\ \text{m}^{-1})} = \frac{1}{2} = 0.500$

There is a 50-50 chance that the particle will be found to the left of the origin, which agrees with the fact that the wave function is symmetric about the y-axis.

(iii) $P = \int_{0.500\ \text{m}}^{1.00\ \text{m}} A^2e^{-2bx}dx$

$$= \frac{A^2}{-2b}\left(e^{-2(2.00\ \text{m}^{-1})(1.00\ \text{m})} - e^{-2(2.00\ \text{m}^{-1})(0.500\ \text{m})}\right) = -\frac{1}{2}\left(e^{-4} - e^{-2}\right) = 0.0585$$

EVALUATE: There is little chance of finding the particle in regions where the wave function is small.

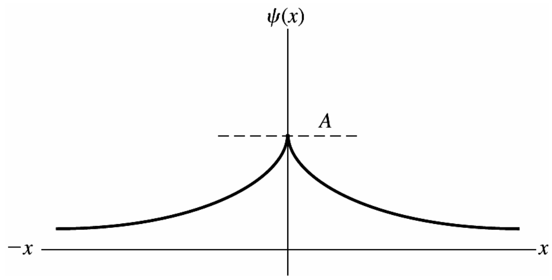

Figure 39.35

39.36. Eq. (39.18): $\frac{-\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = E\psi$. Let $\psi = A\psi_1 + B\psi_2$

$\Rightarrow \frac{-\hbar^2}{2m}\frac{d^2}{dx^2}(A\psi_1 + B\psi_2) + U(A\psi_1 + B\psi_2) = E(A\psi_1 + B\psi_2)$

$\Rightarrow A\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_1}{dx^2} + U\psi_1 - E\psi_1\right) + B\left(-\frac{\hbar^2}{2m}\frac{d^2\psi_2}{dx^2} + U\psi_2 - E\psi_2\right) = 0.$

But each of ψ_1 and ψ_2 satisfy Schrödinger's equation separately so the equation still holds true, for any A or B.

39.37. $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U\psi = BE_1\psi_1 + CE_2\psi_2$. If ψ were a solution with energy E, then $BE_1\psi_1 + CE_2\psi_2 = BE\psi_1 + CE\psi_2$ or $B(E_1 - E)\psi_1 = C(E - E_2)\psi_2$. This would mean that ψ_1 is a constant multiple of ψ_2, and ψ_1 and ψ_2 would be wave functions with the same energy. However, $E_1 \neq E_2$, so this is not possible, and ψ cannot be a solution to Eq. (39.18).

39.38. **(a)** $\lambda = \frac{h}{\sqrt{2mK}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(40\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = 1.94\times10^{-10}\ \text{m}.$

(b) $\frac{R}{v} = \frac{R}{\sqrt{2E/m}} = \frac{(2.5\ \text{m})(9.11\times10^{-31}\ \text{kg})^{1/2}}{\sqrt{2(40\ \text{eV})(1.6\times10^{-19}\ \text{J/eV})}} = 6.67\times10^{-7}\ \text{s}.$

(c) The width w is $w = 2R\frac{\lambda}{a}$' and $w = \Delta v_y t = \Delta p_y t/m$, where t is the time found in part (b) and a is the slit width. Combining the expressions for w, $\Delta p_y = \frac{2m\lambda R}{at} = 2.65\times10^{-28}\ \text{kg}\cdot\text{m/s}.$

(d) $\Delta y = \frac{h}{2\pi\Delta p_y} = 0.40\ \mu\text{m}$, which is the same order of magnitude.

39.39. **(a)** $E = hc/\lambda = 12\ \text{eV}$

(b) Find E for an electron with $\lambda = 0.10\times10^{-6}\ \text{m}$. $\lambda = h/p$ so $p = h/\lambda = 6.626\times10^{-27}\ \text{kg}\cdot\text{m/s}$.

$E = p^2/(2m) = 1.5\times10^{-4}\ \text{eV}$. $E = q\Delta V$ so $\Delta V = 1.5\times10^{-4}\ \text{V}$

$v = p/m = (6.626\times10^{-27}\ \text{kg}\cdot\text{m/s})/(9.109\times10^{-31}\ \text{kg}) = 7.3\times10^3\ \text{m/s}$

(c) Same λ so same p. $E = p^2/(2m)$ but now $m = 1.673\times10^{-27}\ \text{kg}$ so $E = 8.2\times10^{-8}\ \text{eV}$ and $\Delta V = 8.2\times10^{-8}\ \text{V}$.

$v = p/m = (6.626\times10^{-27}\ \text{kg}\cdot\text{m/s})/(1.673\times10^{-27}\ \text{kg}) = 4.0\ \text{m/s}$

39.40. **(a)** Single slit diffraction: $a\sin\theta = m\lambda$. $\lambda = a\sin\theta = (150\times10^{-9}\ \text{m})\sin20° = 5.13\times10^{-8}\ \text{m}$

$$\lambda = h/mv \rightarrow v = h/m\lambda.\ \ v = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(5.13\times10^{-8}\ \text{m})} = 1.42\times10^4\ \text{m/s}$$

(b) $a\sin\theta_2 = 2\lambda$. $\sin\theta_2 = \pm2\frac{\lambda}{a} = \pm2\left(\frac{5.13\times10^{-8}\ \text{m}}{150\times10^{-9}\ \text{m}}\right) = \pm0.684$. $\theta_2 = \pm43.2°$

39.41. **IDENTIFY:** The electrons behave like waves and produce a double-slit interference pattern after passing through the slits.

SET UP: The first angle at which destructive interference occurs is given by $d \sin \theta = \lambda/2$. The de Broglie wavelength of each of the electrons is $\lambda = h/mv$.

EXECUTE: **(a)** First find the wavelength of the electrons. For the first dark fringe, we have $d \sin \theta = \lambda/2$, which gives (1.25 nm)(sin 18.0°) = $\lambda/2$, and λ = 0.7725 nm. Now solve the de Broglie wavelength equation for the speed of the electron:

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(0.7725\times10^{-9}\ \text{m})} = 9.42 \times 10^5\ \text{m/s}$$

which is about 0.3% the speed of light, so they are *nonrelativistic*.

(b) Energy conservation gives $eV = \frac{1}{2} mv^2$ and

$$V = mv^2/2e = (9.11 \times 10^{-31}\ \text{kg})(9.42 \times 10^5\ \text{m})^2/[2(1.60 \times 10^{-19}\ \text{C})] = 2.52\ \text{V}$$

EVALUATE: The hole must be much smaller than the wavelength of visible light for the electrons to show diffraction.

39.42. **IDENTIFY:** The alpha particles and protons behave as waves and exhibit circular-aperture diffraction after passing through the hole.

SET UP: For a round hole, the first dark ring occurs at the angle θ for which $\sin\theta = 1.22\lambda/D$, where D is the diameter of the hole. The de Broglie wavelength for a particle is $\lambda = h/p = h/mv$.

EXECUTE: Taking the ratio of the sines for the alpha particle and proton gives

$$\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{1.22\lambda_\alpha}{1.22\lambda_\text{p}} = \frac{\lambda_\alpha}{\lambda_\text{p}}$$

The de Broglie wavelength gives $\lambda_\text{p} = h/p_\text{p}$ and $\lambda_\alpha = h/p_\alpha$, so $\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{h/p_\alpha}{h/p_\text{p}} = \frac{p_\text{p}}{p_\alpha}$. Using $K = p^2/2m$, we have $p = \sqrt{2mK}$. Since the alpha particle has twice the charge of the proton and both are accelerated through the same potential difference, $K_\alpha = 2K_\text{p}$. Therefore $p_\text{p} = \sqrt{2m_\text{p}K_\text{p}}$ and $p_\alpha = \sqrt{2m_\alpha K_\alpha} = \sqrt{2m_\alpha(2K_\text{p})} = \sqrt{4m_\alpha K_\text{p}}$. Substituting these quantities into the ratio of the sines gives

$$\frac{\sin\theta_\alpha}{\sin\theta_\text{p}} = \frac{p_\text{p}}{p_\alpha} = \frac{\sqrt{2m_\text{p}K_\text{p}}}{\sqrt{4m_\alpha K_\text{p}}} = \sqrt{\frac{m_\text{p}}{2m_\alpha}}$$

Solving for $\sin\theta_\alpha$ gives $\sin\theta_\alpha = \sqrt{\frac{1.67\times10^{-27}\ \text{kg}}{2(6.64\times10^{-27}\ \text{kg})}}\sin15.0°$ and $\theta_\alpha = 5.3°$.

EVALUATE: Since $\sin\theta$ is inversely proportional to the mass of the particle, the larger-mass alpha particles form their first dark ring at a smaller angle than the ring for the lighter protons.

39.43. **IDENTIFY:** Both the electrons and photons behave like waves and exhibit single-slit diffraction after passing through their respective slits.

SET UP: The energy of the photon is $E = hc/\lambda$ and the de Broglie wavelength of the electron is $\lambda = h/mv = h/p$. Destructive interference for a single slit first occurs when $a \sin\theta = \lambda$.

EXECUTE: **(a)** For the photon: $\lambda = hc/E$ and $a \sin\theta = \lambda$. Since the a and θ are the same for the photons and electrons, they must both have the same wavelength. Equating these two expressions for λ gives $a \sin\theta = hc/E$. For the electron, $\lambda = h/p = \frac{h}{\sqrt{2mK}}$ and $a \sin\theta = \lambda$. Equating these two expressions for λ gives $a \sin\theta = \frac{h}{\sqrt{2mK}}$.

Equating the two expressions for $a\sin\theta$ gives $hc/E = \frac{h}{\sqrt{2mK}}$, which gives $E = c\sqrt{2mK} = (4.05\times10^{-7}\ \text{J}^{1/2})\sqrt{K}$

(b) $\frac{E}{K} = \frac{c\sqrt{2mK}}{K} = \sqrt{\frac{2mc^2}{K}}$. Since $v << c$, $mc^2 > K$, so the square root is >1. Therefore $E/K > 1$, meaning that the photon has more energy than the electron.

EVALUATE: As we have seen in Problem 39.10, when a photon and a particle have the same wavelength, the photon has more energy than the particle.

39.44. According to Eq.(35.4) $\lambda = \frac{d\sin\theta}{m} = \frac{(40.0\times10^{-6}\ \text{m})\sin(0.0300\ \text{rad})}{2} = 600\ \text{nm}$. The velocity of an electron with this wavelength is given by Eq.(39.1)

$$v = \frac{p}{m} = \frac{h}{m\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(600\times10^{-9}\ \text{m})} = 1.21\times10^3\ \text{m/s}.$$

Since this velocity is much smaller than c we can calculate the energy of the electron classically

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(1.21\times10^3\ \text{m/s})^2 = 6.70\times10^{-25}\ \text{J} = 4.19\ \mu\text{eV}.$$

39.45. The de Broglie wavelength of the blood cell is

$$\lambda = \frac{h}{mv} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.00\times10^{-14}\ \text{kg})(4.00\times10^{-3}\ \text{m/s})} = 1.66\times10^{-17}\text{m}.$$

We need not be concerned about wave behavior.

39.46. **(a)** $\lambda = \frac{h}{p} = \frac{h\left(1-\frac{v^2}{c^2}\right)^{1/2}}{mv} \Rightarrow \lambda^2 m^2 v^2 = h^2\left(1-\frac{v^2}{c^2}\right) = h^2 - \frac{h^2 v^2}{c^2} \Rightarrow \lambda^2 m^2 v^2 + h^2\frac{v^2}{c^2} = h^2$

$$\Rightarrow v^2 = \frac{h^2}{\left(\lambda^2 m^2 + \frac{h^2}{c^2}\right)} = \frac{c^2}{\left(\frac{\lambda^2 m^2 c^2}{h^2}+1\right)} \Rightarrow v = \frac{c}{\left(1+\left(\frac{mc\lambda}{h}\right)^2\right)^{1/2}}.$$

(b) $v = \frac{c}{\left(1+\left(\frac{\lambda}{(h/mc)}\right)^2\right)^{1/2}} \approx c\left(1-\frac{1}{2}\left(\frac{mc\lambda}{h}\right)^2\right) = (1-\Delta)c. \quad \Delta = \frac{m^2c^2\lambda^2}{2h^2}.$

(c) $\lambda = 1.00\times10^{-15}\,\text{m} << \frac{h}{mc}.$

$$\Delta = \frac{(9.11\times10^{-31}\ \text{kg})^2(3.00\times10^8\ \text{m/s})^2(1.00\times10^{-15}\text{m})^2}{2(6.63\times10^{-34}\ \text{J}\cdot\text{s})^2} = 8.50\times10^{-8}$$

$\Rightarrow v = (1-\Delta)c = (1-8.50\times10^{-8})c.$

39.47. **(a)** Recall $\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{h}{\sqrt{2mq\Delta V}}$. So for an electron:

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{C})(125\ \text{V})}} \Rightarrow \lambda = 1.10\times10^{-10}\ \text{m}.$$

(b) For an alpha particle: $\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2(6.64\times10^{-27}\ \text{kg})2(1.60\times10^{-19}\ \text{C})(125\ \text{V})}} = 9.10\times10^{-13}\ \text{m}.$

39.48. **IDENTIFY** and **SET UP:** The minimum uncertainty product is $\Delta x \Delta p_x = \frac{h}{2\pi}$. $\Delta x = r_1$, where r_1 is the radius of the $n=1$ Bohr orbit. In the $n=1$ Bohr orbit, $mv_1r_1 = \frac{h}{2\pi}$ and $p_1 = mv_1 = \frac{h}{2\pi r_1}$.

EXECUTE: $\Delta p_x = \frac{h}{2\pi\Delta x} = \frac{h}{2\pi r_1} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(0.529\times10^{-10}\ \text{m})} = 2.0\times10^{-24}\ \text{kg}\cdot\text{m/s}$. This is the same as the magnitude of the momentum of the electron in the $n=1$ Bohr orbit.

EVALUATE: Since the momentum is the same order of magnitude as the uncertainty in the momentum, the uncertainty principle plays a large role in the structure of atoms.

39.49. **IDENTIFY** and **SET UP:** Combining the two equations in the hint gives $PC = \sqrt{K(K+2mc^2)}$ and $\lambda = \frac{hc}{\sqrt{K(K+2mc^2)}}$.

EXECUTE: **(a)** With $K = 3mc^2$ this becomes $\lambda = \frac{hc}{\sqrt{3mc^2(3mc^2+2mc^2)}} = \frac{h}{\sqrt{15}mc}$.

(b) (i) $K = 3mc^2 = 3(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 2.456\times10^{-13}\ \text{J} = 1.53\ \text{MeV}$

$$\lambda = \frac{h}{\sqrt{15}mc} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{15}(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})} = 6.26\times10^{-13}\ \text{m}$$

(ii) K is proportional to m, so for a proton $K = (m_p/m_e)(1.53\ \text{MeV}) = 1836(1.53\ \text{MeV}) = 2810\ \text{MeV}$

λ is proportional to $1/m$, so for a proton $\lambda = (m_e/m_p)(6.26\times10^{-13}\ \text{m}) = (1/1836)(6.626\times10^{-13}\ \text{m}) = 3.41\times10^{-16}\ \text{m}$

EVALUATE: The proton has a larger rest mass energy so its kinetic energy is larger when $K = 3mc^2$. The proton also has larger momentum so has a smaller λ.

39.50. **(a)** $\dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(5.0\times10^{-15}\ \text{m})}=2.1\times10^{-20}\ \text{kg}\cdot\text{m/s}.$

(b) $K=\sqrt{(pc)^2+(mc^2)^2}-mc^2=1.3\times10^{-13}\ \text{J}=0.82\ \text{MeV}.$

(c) The result of part (b), about $1\ \text{MeV}=1\times10^6\ \text{eV}$, is many orders of magnitude larger than the potential energy of an electron in a hydrogen atom.

39.51. **(a) IDENTIFY** and **SET UP:** $\Delta x\Delta p_x\geq h/2\pi$

Estimate Δx as $\Delta x\approx5.0\times10^{-15}$ m.

EXECUTE: Then the minimum allowed Δp_x is $\Delta p_x\approx\dfrac{h}{2\pi\Delta x}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(5.0\times10^{-15}\ \text{m})}=2.1\times10^{-20}\ \text{kg}\cdot\text{m/s}$

(b) IDENTIFY and **SET UP:** Assume $p\approx2.1\times10^{-20}\ \text{kg}\cdot\text{m/s}$. Use Eq.(37.39) to calculate *E*, and then $K=E-mc^2$.

EXECUTE: $E=\sqrt{(mc^2)^2+(pc)^2}$

$mc^2=(9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2=8.187\times10^{-14}\ \text{J}$

$pc=(2.1\times10^{-20}\ \text{kg}\cdot\text{m/s})(2.998\times10^8\ \text{m/s})=6.296\times10^{-12}\ \text{J}$

$E=\sqrt{(8.187\times10^{-14}\ \text{J})^2+(6.296\times10^{-12}\ \text{J})^2}=6.297\times10^{-12}\ \text{J}$

$K=E-mc^2=6.297\times10^{-12}\ \text{J}-8.187\times10^{-14}\ \text{J}=6.215\times10^{-12}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J})=39\ \text{MeV}$

(c) IDENTIFY and **SET UP:** The Coulomb potential energy for a pair of point charges is given by Eq.(23.9). The proton has charge $+e$ and the electron has charge $-e$.

EXECUTE: $U=-\dfrac{ke^2}{r}=-\dfrac{(8.988\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.602\times10^{-19}\ \text{C})^2}{5.0\times10^{-15}\ \text{m}}=-4.6\times10^{-14}\ \text{J}=-0.29\ \text{MeV}$

EVALUATE: The kinetic energy of the electron required by the uncertainty principle would be much larger than the magnitude of the negative Coulomb potential energy. The total energy of the electron would be large and positive and the electron could not be bound within the nucleus.

39.52. **(a)** Take the direction of the electron beam to be the *x*-direction and the direction of motion perpendicular to the beam to be the *y*-direction. $\Delta v_y=\dfrac{\Delta p_y}{m}=\dfrac{h}{2\pi m\Delta y}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(9.11\times10^{-31}\ \text{kg})(0.50\times10^{-3}\ \text{m})}=0.23\ \text{m/s}$

(b) The uncertainty Δr in the position of the point where the electrons strike the screen is

$$\Delta r=\Delta v_y t=\frac{\Delta p_y}{m}\frac{x}{v_x}=\frac{h}{2\pi m\Delta y}\frac{x}{\sqrt{2K/m}}=9.56\times10^{-10}\ \text{m},$$

(c) This is far too small to affect the clarity of the picture.

39.53. **IDENTIFY** and **SET UP:** $\Delta E\Delta t\geq\dfrac{h}{2\pi}$. Take the minimum uncertainty product, so $\Delta E=\dfrac{h}{2\pi\Delta t}$, with $\Delta t=8.4\times10^{-17}\ \text{s}$. $m=264m_\text{e}$. $\Delta m=\dfrac{\Delta E}{c^2}$.

EXECUTE: $\Delta E=\dfrac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(8.4\times10^{-17}\ \text{s})}=1.26\times10^{-18}\ \text{J}$. $\Delta m=\dfrac{1.26\times10^{-18}\ \text{J}}{(3.00\times10^8\ \text{m/s})^2}=1.4\times10^{-35}\ \text{kg}$.

$\dfrac{\Delta m}{m}=\dfrac{1.4\times10^{-35}\ \text{kg}}{(264)(9.11\times10^{-31}\ \text{kg})}=5.8\times10^{-8}$

39.54. **IDENTIFY:** The insect behaves like a wave as it passes through the hole in the screen.

SET UP: **(a)** For wave behavior to show up, the wavelength of the insect must be of the order of the diameter of the hole. The de Broglie wavelength is $\lambda=h/mv$.

EXECUTE: The de Broglie wavelength of the insect must be of the order of the diameter of the hole in the screen, so $\lambda\approx5.00$ mm. The de Broglie wavelength gives

$$v=\frac{h}{m\lambda}=\frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.25\times10^{-6}\ \text{kg}\right)\left(0.00400\ \text{m}\right)}=1.33\times10^{-25}\ \text{m/s}$$

(b) $t=x/v=(0.000500\ \text{m})/(1.33\times10^{-25}\ \text{m/s})=3.77\times10^{21}\ \text{s}=1.4\times10^{10}\ \text{yr}$

The universe is about 14 billion years old (1.4×10^{10} yr), so this time would be about 85,000 times the age of the universe.

EVALUATE: Don't expect to see a diffracting insect! Wave behavior of particles occurs only at the very small scale.

39.55. **IDENTIFY** and **SET UP:** Use Eq.(39.1) to relate your wavelength and speed.

EXECUTE: **(a)** $\lambda=\dfrac{h}{mv}$, so $v=\dfrac{h}{m\lambda}=\dfrac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(60.0\ \text{kg})(1.0\ \text{m})}=1.1\times10^{-35}\ \text{m/s}$

(b) $t = \dfrac{\text{distance}}{\text{velocity}} = \dfrac{0.80\ \text{m}}{1.1\times10^{-35}\ \text{m/s}} = 7.3\times10^{34}\ \text{s}(1\ \text{y}/3.156\times10^{7}\ \text{s}) = 2.3\times10^{27}\ \text{y}$

Since you walk through doorways much more quickly than this, you will not experience diffraction effects.

EVALUATE: A 1 kg object moving at 1 m/s has a de Broglie wavelength $\lambda = 6.6\times10^{-34}$ m, which is exceedingly small. An object like you has a very, very small λ at ordinary speeds and does not exhibit wavelike properties.

39.56. **(a)** $E = 2.58\ \text{eV} = 4.13\times10^{-19}$ J, with a wavelength of $\lambda = \dfrac{hc}{E} = 4.82\times10^{-7}\ \text{m} = 482\ \text{nm}$

(b) $\Delta E = \dfrac{h}{2\pi\Delta t} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{2\pi(1.64\times10^{-7}\ \text{s})} = 6.43\times10^{-28}\ \text{J} = 4.02\times10^{-9}\ \text{eV}.$

(c) $\lambda E = hc$, so $(\Delta\lambda)E + \lambda\Delta E = 0$, and $|\Delta E/E| = |\Delta\lambda/\lambda|$, so

$$\Delta\lambda = \lambda|\Delta E/E| = (4.82\times10^{-7}\ \text{m})\left(\frac{6.43\times10^{-28}\ \text{J}}{4.13\times10^{-19}\ \text{J}}\right) = 7.50\times10^{-16}\ \text{m} = 7.50\times10^{-7}\ \text{nm}.$$

39.57. **IDENTIFY:** The electrons behave as waves whose wavelength is equal to the de Broglie wavelength.

SET UP: The de Broglie wavelength is $\lambda = h/mv$, and the energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: **(a)** Use the de Broglie wavelength to find the speed of the electron.

$$v = \frac{h}{m\lambda} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})(1.00\times10^{-9}\ \text{m})} = 7.27\times10^{5}\ \text{m/s}$$

which is much less than the speed of light, so it is nonrelativistic.

(b) Energy conservation gives $eV = \tfrac{1}{2}mv^2$.

$$V = mv^2/2e = (9.11\times10^{-31}\ \text{kg})(7.27\times10^{5}\ \text{m/s})^2/[2(1.60\times10^{-19}\ \text{C})] = 1.51\ \text{V}$$

(c) $K = eV = e(1.51\ \text{V}) = 1.51$ eV, which is about ¼ the potential energy of the NaCl crystal, so the electron would not be too damaging.

(d) $E = hc/\lambda = (4.136\times10^{-15}\ \text{eV s})(3.00\times10^{8}\ \text{m/s})/(1.00\times10^{-9}\ \text{m}) = 1240\ \text{eV}$

which would certainly destroy the molecules under study.

EVALUATE: As we have seen in Problems 39.10 and 39.43, when a particle and a photon have the same wavelength, the photon has much more energy.

39.58. $\sin\theta' = \dfrac{\lambda'}{\lambda}\sin\theta$, and $\lambda' = (h/p') = (h/\sqrt{2mE'})$, and so $\theta' = \arcsin\left(\dfrac{h}{\lambda\sqrt{2mE'}}\sin\theta\right)$.

$$\theta' = \arcsin\left(\frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\sin 35.8^\circ}{(3.00\times10^{-11}\ \text{m})\sqrt{2(9.11\times10^{-31}\ \text{kg})(4.50\times10^{+3})(1.60\times10^{-19}\ \text{J/eV})}}\right) = 20.9^\circ$$

39.59. **(a)** The maxima occur when $2d\sin\theta = m\lambda$ as described in Section 38.7.

(b) $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE}}$. $\lambda = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{\sqrt{2(9.11\times10^{-37}\ \text{kg})(71.0\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})}} = 1.46\times10^{-10}\text{m} = 0.146\ \text{nm}$.

$\theta = \sin^{-1}\left(\dfrac{m\lambda}{2d}\right)$ (Note: This m is the order of the maximum, not the mass.)

$$\Rightarrow \sin^{-1}\left(\frac{(1)(1.46\times10^{-10}\ \text{m})}{2(9.10\times10^{-11}\ \text{m})}\right) = 53.3^\circ.$$

(c) The work function of the metal acts like an attractive potential increasing the kinetic energy of incoming electrons by $e\phi$. An increase in kinetic energy is an increase in momentum that leads to a smaller wavelength. A smaller wavelength gives a smaller angle θ (see part (b)).

39.60. **(a)** Using the given approximation, $E = \dfrac{1}{2}\left((h/x)^2/m + kx^2\right)$, $(dE/dx) = kx - (h^2/mx^3)$, and the minimum energy occurs when $kx = (h^2/mx^3)$, or $x^2 = \dfrac{h}{\sqrt{mk}}$. The minimum energy is then $h\sqrt{k/m}$.

(b) They are the same.

39.61. **(a) IDENTIFY** and **SET UP:** $U = A|x|$. Eq.(7.17) relates force and potential. The slope of the function $A|x|$ is not continuous at $x = 0$ so we must consider the regions $x > 0$ and $x < 0$ separately.

EXECUTE: For $x > 0$, $|x| = x$ so $U = Ax$ and $F = -\dfrac{d(Ax)}{dx} = -A$. For $x < 0$, $|x| = -x$ so $U = -Ax$ and $F = -\dfrac{d(-Ax)}{dx} = +A$. We can write this result as $F = -A|x|/x$, valid for all x except for $x = 0$.

(b) IDENTIFY and SET UP: Use the uncertainty principle, expressed as $\Delta p\Delta x \approx h,$ and as in Problem 39.50 estimate Δp by p and Δx by x. Use this to write the energy E of the particle as a function of x. Find the value of x that gives the minimum E and then find the minimum E.

EXECUTE: $E = K + U = \dfrac{p^2}{2m} + A|x|$

$px \approx h,$ so $p \approx h/x$

Then $E \approx \dfrac{h^2}{2mx^2} + A|x|.$

For $x > 0, E = \dfrac{h^2}{2mx^2} + Ax.$

To find the value of x that gives minimum E set $\dfrac{dE}{dx} = 0.$

$$0 = \frac{-2h^2}{2mx^3} + A$$

$x^3 = \dfrac{h^2}{mA}$ and $x = \left(\dfrac{h^2}{mA}\right)^{1/3}$

With this x the minimum E is

$$E = \frac{h^2}{2m}\left(\frac{mA}{h^2}\right)^{2/3} + A\left(\frac{h^2}{mA}\right)^{1/3} = \frac{1}{2}h^{2/3}m^{-1/3}A^{2/3} + h^{2/3}m^{-1/3}A^{2/3}$$

$$E = \tfrac{3}{2}\left(\frac{h^2A^2}{m}\right)^{1/3}$$

EVALUATE: The potential well is shaped like a V. The larger A is the steeper the slope of U and the smaller the region to which the particle is confined and the greater is its energy. Note that for the x that minimizes E, $2K = U$.

39.62. For this wave function, $\Psi^* = \psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t}$, so

$$\Psi^2 = \Psi^*\Psi = (\psi_1^* e^{i\omega_1 t} + \psi_2^* e^{i\omega_2 t})(\psi_1 e^{-i\omega_1 t} + \psi_2 e^{-i\omega_2 t}) = \psi_1^*\psi_1 + \psi_2^*\psi_2 + \psi_1^*\psi_2 e^{i(\omega_1-\omega_2)t} + \psi_2^*\psi_1 e^{i(\omega_2-\omega_1)t}.$$

The frequencies ω_1 and ω_2 are given as not being the same, so $|\Psi|^2$ is not time-independent, and Ψ is not the wave function for a stationary state.

39.63. The time-dependent equation, with the separated form for $\Psi(x,t)$ as given becomes

$$i\hbar\psi(-i\omega) = \left(-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + U(x)\psi\right).$$

Since ψ is a solution of the time-independent solution with energy E, the term in parenthesis is $E\psi$, and so $\omega\hbar = E,$ and $\omega = (E/\hbar).$

39.64. **(a)** $\omega = 2\pi f = \dfrac{2\pi E}{h} = \dfrac{E}{\hbar}.$ $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi}{h}p = \dfrac{p}{\hbar}.$ $\hbar\omega = E = K = \dfrac{p^2}{2m} = \dfrac{(\hbar k)^2}{2m} \Rightarrow \omega = \dfrac{\hbar k^2}{2m}.$

(b) From Problem 39.63 the time-dependent Schrödinger's equation is $-\dfrac{\hbar^2}{2m}\dfrac{\partial^2\psi(x,t)}{\partial x^2} +$ $U(x)\psi(x,t) = i\hbar\dfrac{\partial\psi(x,t)}{\partial t}$. $U(x) = 0$ for a free particle, so $\dfrac{\partial^2\psi(x,t)}{\partial x^2} = -\dfrac{2mi}{\hbar}\dfrac{\partial\psi(x,t)}{\partial t}.$

Try $\psi(x,t) = \cos(kx - \omega t)$:

$$\frac{\partial\psi}{\partial t}(x,t) = A\omega\sin(kx - \omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = -Ak\sin(kx-\omega t) \text{ and } \frac{\partial^2\psi}{\partial x^2} = Ak^2\cos(kx-\omega t).$$

Putting this into the Schrödinger's equation, $Ak^2\cos(kx-\omega t) = -\left(\dfrac{2mi}{\hbar}\right)A\omega\sin(kx-\omega t).$

This is not generally true for all x and t so is not a solution.

(c) Try $\psi(x,t) = A\sin(kx-\omega t)$:

$$\frac{\partial\psi(x,t)}{\partial t} = -A\omega\cos(kx-\omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = Ak\cos(kx-\omega t) \text{ and } \frac{\partial^2\psi(x,t)}{\partial x^2} = -Ak^2\sin(kx-\omega t).$$

Again, $-Ak^2\sin(kx-\omega t) = -\left(\frac{2mi}{\hbar}\right)A\omega\cos(kx-\omega t)$ is not generally true for *all* x and t so is not a solution.

(d) Try $\psi(x,t) = A\cos(kx-\omega t) + B\sin(kx-\omega t)$:

$$\frac{\partial\psi(x,t)}{\partial t} = +A\omega\sin(kx-\omega t) - B\omega\cos(kx-\omega t)$$

$$\frac{\partial\psi(x,t)}{\partial x} = -Ak\sin(kx-\omega t) + Bk\cos(kx-\omega t) \text{ and } \frac{\partial^2\psi(x,t)}{\partial x^2} = -Ak^2\cos((kx-\omega t) - Bk^2\sin(kx-\omega t).$$

Putting this into the Schrödinger's equation,

$$-Ak^2\cos(kx-\omega t) - Bk^2\sin(kx-\omega t) = -\frac{2mi}{\hbar}(+A\omega\sin(kx-\omega t) - B\omega\cos(kx-\omega t)).$$

Recall that $\omega = \frac{\hbar k^2}{2m}$. Collect sin and cos terms.

$(A+iB)k^2\cos(kx-\omega t) + (iA-B)k^2\sin(kx-\omega t) = 0.$ This is only true if $B = iA$.

39.65. **(a) IDENTIFY** and **SET UP:** Let the *y*-direction be from the thrower to the catcher, and let the *x*-direction be horizontal and perpendicular to the *y*-direction. A cube with volume $V = 125\text{ cm}^3 = 0.125\times10^{-3}\text{ m}^3$ has side length $l = V^{1/3} = (0.125\times10^{-3}\text{ m}^3)^{1/3} = 0.050\text{ m}$. Thus estimate Δx as $\Delta x \approx 0.050\text{ m}$. Use the uncertainty principle to estimate Δp_x.

EXECUTE: $\Delta x\Delta p_x \geq h/2\pi$ then gives $\Delta p_x \approx \frac{h}{2\pi\Delta x} = \frac{0.0663\text{ J}\cdot\text{s}}{2\pi(0.050\text{ m})} = 0.21\text{ kg}\cdot\text{m/s}$

(The value of *h* in this other universe has been used.)

(b) IDENTIFY and **SET UP:** $\Delta x = (\Delta v_x)t$ is the uncertainty in the *x*-coordinate of the ball when it reaches the catcher, where *t* is the time it takes the ball to reach the second student. Obtain Δv_x from Δp_x.

EXECUTE: The uncertainty in the ball's horizontal velocity is $\Delta v_x = \frac{\Delta p_x}{m} = \frac{0.21\text{ kg}\cdot\text{m/s}}{0.25\text{ kg}} = 0.84\text{ m/s}$

The time it takes the ball to travel to the second student is $t = \frac{12\text{ m}}{6.0\text{ m/s}} = 2.0\text{ s}$. The uncertainty in the *x*-coordinate of the ball when it reaches the second student that is introduced by Δv_x is $\Delta x = (\Delta v_x)t = (0.84\text{ m/s})(2.0\text{ s}) = 1.7\text{ m}$. The ball could miss the second student by about 1.7 m.

EVALUATE: A game of catch would be very different in this universe. We don't notice the effects of the uncertainty principle in everyday life because *h* is so small.

39.66. **(a)** $|\psi^2| = A^2x^2e^{-2(\alpha x^2+\beta y^2+\gamma z^2)}$. To save some algebra, let $u = x^2$, so that $|\psi|^2 = ue^{-2\alpha u}f(y,z)$.

$\frac{\partial}{\partial u}|\psi|^2 = (1-2\alpha u)|\psi|^2$; the maximum occurs at $u_0 = \frac{1}{2\alpha}$, $x_0 = \pm\frac{1}{\sqrt{2\alpha}}$.

(b) ψ vanishes at $x = 0$, so the probability of finding the particle in the $x = 0$ plane is zero. The wave function vanishes for $x = \pm\infty$.

39.67. **(a) IDENTIFY** and **SET UP:** The probability is $P = |\psi|^2\,dV$ with $dV = 4\pi r^2dr$

EXECUTE: $|\psi|^2 = A^2e^{-2\alpha r^2}$ so $P = 4\pi A^2r^2e^{-2\alpha r^2}\,dr$

(b) IDENTIFY and **SET UP:** P is maximum where $\frac{dP}{dr} = 0$

EXECUTE: $\frac{d}{dr}(r^2e^{-2\alpha r^2}) = 0$

$2re^{-2\alpha r^2} - 4\alpha r^3e^{-2\alpha r^2} = 0$ and this reduces to $2r - 4\alpha r^3 = 0$

$r = 0$ is a solution of the equation but corresponds to a minimum not a maximum. Seek r not equal to 0 so divide by r and get $2 - 4\alpha r^2 = 0$

This gives $r = \frac{1}{\sqrt{2\alpha}}$ (We took the positive square root since r must be positive.)

EVALUATE: This is different from the value of r, $r = 0$, where $|\psi|^2$ is a maximum. At $r = 0$, $|\psi|^2$ has a maximum but the volume element $dV = 4\pi r^2 dr$ is zero here so P does not have a maximum at $r = 0$.

39.68. **(a)** $B(k) = e^{-\alpha^2 k^2}$ $\quad B(0) = B_{\text{max}} = 1$

$$B(k_{\text{h}}) = \frac{1}{2} = e^{-\alpha^2 k_{\text{h}}^2} \Rightarrow \ln(1/2) = -\alpha^2 k_{\text{h}}^2 \Rightarrow k_{\text{h}} = \frac{1}{\alpha}\sqrt{\ln(2)} = \omega_k.$$

(b) Using integral tables: $\psi(x) = \int_0^\infty e^{-\alpha^2 k^2} \cos kx dk = \frac{\sqrt{\pi}}{2\alpha}(e^{-x^2/4\alpha^2})$. $\psi(x)$ is a maximum when $x = 0$.

(c) $\psi(x_{\text{h}}) = \frac{\sqrt{\pi}}{4\alpha}$ when $e^{-x_{\text{h}}^2/4\alpha^2} = \frac{1}{2} \Rightarrow \frac{-x_{\text{h}}^2}{4\alpha^2} = \ln(1/2) \Rightarrow x_{\text{h}} = 2\alpha\sqrt{\ln 2} = \omega_{\text{x}}$

(d) $\omega_p \omega_x = \left(\frac{h\omega_k}{2\pi}\right)\omega_x = \frac{h}{2\pi}\left(\frac{1}{\alpha}\sqrt{\ln 2}\right)\left(2\alpha\sqrt{\ln 2}\right) = \frac{h}{2\pi}(2\ln 2) = \frac{h\ln 2}{\pi}.$

39.69. **(a)** $\psi(x) = \int_0^\infty B(k)\cos kx dk = \int_0^{k_0}\left(\frac{1}{k_0}\right)\cos kx dk = \left.\frac{\sin kx}{k_0 x}\right|_0^{k_0} = \frac{\sin k_0 x}{k_0 x}$

(b) $\psi(x)$ has a maximum value at the origin $x = 0$. $\psi(x_0) = 0$ when $k_0 x_0 = \pi$ so $x_0 = \frac{\pi}{k_0}$. Thus the width of this function $w_x = 2x_0 = \frac{2\pi}{k_0}$. If $k_0 = \frac{2\pi}{L}, w_x = L$. $B(k)$ versus k is graphed in Figure 39.69a. The graph of $\psi(x)$ versus x is in Figure 39.69b.

(c) If $k_0 = \frac{\pi}{L} w_x = 2L$.

(d) $w_p w_x = \left(\frac{hw_k}{2\pi}\right)\left(\frac{2\pi}{k_0}\right) = \frac{hw_k}{k_0} = \frac{hk_0}{k_0} = h$. The uncertainty principle states that $w_p w_x \geq \frac{h}{2\pi}$. For us, no matter what k_0 is, $w_p w_x = h$, which is greater than $\frac{h}{2\pi}$.

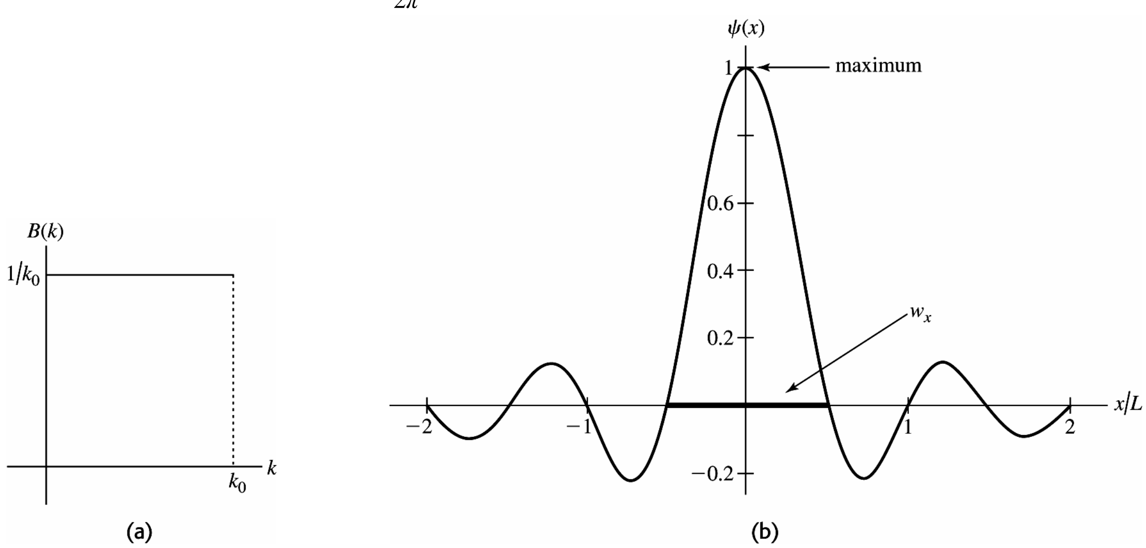

Figure 39.69

39.70. **(a)** For a standing wave, $n\lambda = 2L$, and $E_n = \frac{p^2}{2m} = \frac{(h/\lambda)^2}{2m} = \frac{n^2 h^2}{8mL^2}$.

(b) With $L = a_0 = 0.5292 \times 10^{-10}$ m, $E_1 = 2.15 \times 10^{-17}$ J $= 134$ eV.

39.71. Time of flight of the marble, from a free-fall kinematic equation is just $t=\sqrt{\dfrac{2y}{g}}=\sqrt{\dfrac{2(25.0\text{ m})}{9.81\text{ m/s}^2}}=2.26\text{ s}$.

$$\Delta x_f=\Delta x_i+(\Delta v_x)t=\Delta x_i+\left(\frac{\Delta p_x}{m}\right)t=\frac{ht}{2\pi\Delta x_i m}+\Delta x_i$$

To minimize Δx_f with respect to Δx_i, $\dfrac{d(\Delta x_f)}{d(\Delta x_i)}=0=\dfrac{-ht}{2\pi m(\Delta x_i)^2}+1$

$$\Rightarrow \Delta x_i(\text{min})=\sqrt{\left(\frac{ht}{2\pi m}\right)}$$

$$\Rightarrow \Delta x_f(\text{min})=\sqrt{\frac{ht}{2\pi m}}+\sqrt{\frac{ht}{2\pi m}}=\sqrt{\frac{2ht}{\pi m}}=\sqrt{\frac{2(6.63\times10^{-34}\text{ J}\cdot\text{s})(2.26\text{ s})}{\pi(0.0200\text{ kg})}}=2.18\times10^{-16}\text{ m}=2.18\times10^{-7}\text{ nm.}$$