38

PHOTONS, ELECTRONS, AND ATOMS

38.1. IDENTIFY and SET UP: The stopping potential V_0 is related to the frequency of the light by $V_0 = \frac{h}{e}f - \frac{\phi}{e}$. The slope of V_0 versus f is h/e. The value f_{th} of f when $V_0 = 0$ is related to ϕ by $\phi = hf_{th}$.

EXECUTE: **(a)** From the graph, $f_{th} = 1.25\times10^{15}$ Hz. Therefore, with the value of h from part (b), $\phi = hf_{th} = 4.8$ eV.

(b) From the graph, the slope is 3.8×10^{-15} V·s. $h = (e)(\text{slope}) = (1.60\times10^{-16}\ \text{C})(3.8\times10^{-15}\ \text{V}\cdot\text{s}) = 6.1\times10^{-34}\ \text{J}\cdot\text{s}$

(c) No photoelectrons are produced for $f < f_{th}$.

(d) For a different metal f_{th} and ϕ are different. The slope is h/e so would be the same, but the graph would be shifted right or left so it has a different intercept with the horizontal axis.

EVALUATE: As the frequency f of the light is increased above f_{th} the energy of the photons in the light increases and more energetic photons are produced. The work function we calculated is similar to that for gold or nickel.

38.2. IDENTIFY and SET UP: $c = f\lambda$ relates frequency and wavelength and $E = hf$ relates energy and frequency for a photon. $c = 3.00\times10^{8}$ m/s. $1\ \text{eV} = 1.60\times10^{-16}$ J.

EXECUTE: **(a)** $f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{505\times10^{-9}\ \text{m}} = 5.94\times10^{14}\ \text{Hz}$

(b) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(5.94\times10^{14}\ \text{Hz}) = 3.94\times10^{-19}\ \text{J} = 2.46\ \text{eV}$

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(3.94\times10^{-19}\ \text{J})}{9.5\times10^{-15}\ \text{kg}}} = 9.1\ \text{mm/s}$

38.3. $f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{5.20\times10^{-7}\ \text{m}} = 5.77\times10^{14}\ \text{Hz}$

$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{5.20\times10^{-7}\ \text{m}} = 1.28\times10^{-27}\ \text{kg}\cdot\text{m/s}$

$E = pc = (1.28\times10^{-27}\ \text{kg}\cdot\text{m/s})(3.00\times10^{8}\ \text{m/s}) = 3.84\times10^{-19}\ \text{J} = 2.40\ \text{eV}.$

38.4. IDENTIFY and SET UP: $P_{av} = \frac{\text{energy}}{t}$. $1\ \text{eV} = 1.60\times10^{-19}$ J. For a photon, $E = hf = \frac{hc}{\lambda}$. $h = 6.63\times10^{-34}$ J·s.

EXECUTE: **(a)** $\text{energy} = P_{av}t = (0.600\ \text{W})(20.0\times10^{-3}\ \text{s}) = 1.20\times10^{-2}\ \text{J} = 7.5\times10^{16}\ \text{eV}$

(b) $E = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{652\times10^{-9}\ \text{m}} = 3.05\times10^{-19}\ \text{J} = 1.91\ \text{eV}$

(c) The number of photons is the total energy in a pulse divided by the energy of one photon:

$\frac{1.20\times10^{-2}\ \text{J}}{3.05\times10^{-19}\ \text{J/photon}} = 3.93\times10^{16}\ \text{photons}.$

EVALUATE: The number of photons in each pulse is very large.

38.5. IDENTIFY and SET UP: Eq.(38.2) relates the photon energy and wavelength. $c = f\lambda$ relates speed, frequency and wavelength for an electromagnetic wave.

EXECUTE: **(a)** $E = hf$ so $f = \frac{E}{h} = \frac{(2.45\times10^{6}\ \text{eV})(1.602\times10^{-19}\ \text{J/1 eV})}{6.626\times10^{-34}\ \text{J}\cdot\text{s}} = 5.92\times10^{20}\ \text{Hz}$

(b) $c = f\lambda$ so $\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\ \text{m/s}}{5.92\times10^{20}\ \text{Hz}} = 5.06\times10^{-13}\ \text{m}$

(c) EVALUATE: λ is comparable to a nuclear radius. Note that in doing the calculation the energy in MeV was converted to the SI unit of Joules.

38.6. **IDENTIFY** and **SET UP:** $\lambda_{th} = 272\text{ nm}$. $c = f\lambda$. $\frac{1}{2}mv_{max}^2 = hf - \phi$. At the threshold frequency, f_{th}, $v_{max} \to 0$. $h = 4.136\times10^{-15}\text{ eV}\cdot\text{s}$.

EXECUTE: **(a)** $f_{th} = \dfrac{c}{\lambda_{th}} = \dfrac{3.00\times10^8\text{ m/s}}{272\times10^{-9}\text{ m}} = 1.10\times10^{15}\text{ Hz}$.

(b) $\phi = hf_{th} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(1.10\times10^{15}\text{ Hz}) = 4.55\text{ eV}$.

(c) $\frac{1}{2}mv_{max}^2 = hf - \phi = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(1.45\times10^{15}\text{ Hz}) - 4.55\text{ eV} = 6.00\text{ eV} - 4.55\text{ eV} = 1.45\text{ eV}$

EVALUATE: The threshold wavelength depends on the work function for the surface.

38.7. **IDENTIFY** and **SET UP:** Eq.(38.3): $\frac{1}{2}mv_{max}^2 = hf - \phi = \frac{hc}{\lambda} - \phi$. Take the work function ϕ from Table 38.1. Solve for v_{max}. Note that we wrote f as c/λ.

EXECUTE: $\frac{1}{2}mv_{max}^2 = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^8\text{ m/s})}{235\times10^{-9}\text{ m}} - (5.1\text{ eV})(1.602\times10^{-19}\text{ J/1 eV})$

$\frac{1}{2}mv_{max}^2 = 8.453\times10^{-19}\text{ J} - 8.170\times10^{-19}\text{ J} = 2.83\times10^{-20}\text{ J}$

$v_{max} = \sqrt{\dfrac{2(2.83\times10^{-20}\text{ J})}{9.109\times10^{-31}\text{ kg}}} = 2.49\times10^5\text{ m/s}$

EVALUATE: The work function in eV was converted to joules for use in Eq.(38.3). A photon with $\lambda = 235\text{ nm}$ has energy greater then the work function for the surface.

38.8. **IDENTIFY** and **SET UP:** $\phi = hf_{th} = \dfrac{hc}{\lambda_{th}}$. The minimum ϕ corresponds to the minimum λ.

EXECUTE: $\phi = \dfrac{hc}{\lambda_{th}} = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^8\text{ m/s})}{700\times10^{-9}\text{ m}} = 1.77\text{ eV}$

38.9. **IDENTIFY** and **SET UP:** $c = f\lambda$. The source emits $(0.05)(75\text{ J}) = 3.75\text{ J}$ of energy as visible light each second. $E = hf$, with $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$.

EXECUTE: **(a)** $f = \dfrac{c}{\lambda} = \dfrac{3.00\times10^8\text{ m/s}}{600\times10^{-9}\text{ m}} = 5.00\times10^{14}\text{ Hz}$

(b) $E = hf = (6.63\times10^{-34}\text{ J}\cdot\text{s})(5.00\times10^{14}\text{ Hz}) = 3.32\times10^{-19}\text{ J}$. The number of photons emitted per second is $\dfrac{3.75\text{ J}}{3.32\times10^{-19}\text{ J/photon}} = 1.13\times10^{19}$ photons.

(c) No. The frequency of the light depends on the energy of each photon. The number of photons emitted per second is proportional to the power output of the source.

38.10. **IDENTIFY:** In the photoelectric effect, the energy of the photon is used to eject an electron from the surface, and any excess energy goes into kinetic energy of the electron.

SET UP: The energy of a photon is $E = hf$, and the work function is given by $\phi = hf_0$, where f_0 is the threshold frequency.

EXECUTE: (a) From the graph, we see that $K_{max} = 0$ when $\lambda = 250$ nm, so the threshold wavelength is 250 nm. Calling f_0 the threshold frequency, we have

$$f_0 = c/\lambda_0 = (3.00 \times 10^8\text{ m/s})/(250\text{ nm}) = 1.2 \times 10^{15}\text{ Hz}.$$

(b) $\phi = hf_0 = (4.136 \times 10^{-15}\text{ eV}\cdot\text{s})(1.2 \times 10^{15}\text{ Hz}) = 4.96\text{ eV} = 5.0\text{ eV}$

(c) The graph (see Figure 38.10) is linear for $\lambda < \lambda_0$ $(1/\lambda > 1/\lambda_0)$, and linear graphs are easier to interpret than curves.

EVALUATE: If the wavelength of the light is longer than the threshold wavelength (that is, if $1/\lambda < 1/\lambda_0$), the kinetic energy of the electrons is really not defined since no photoelectrons are ejected from the metal.

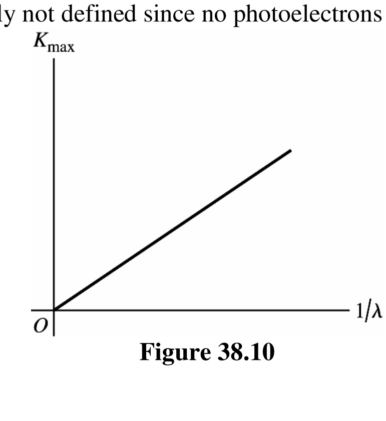

Figure 38.10

38.11. **IDENTIFY:** Protons have mass and photons are massless.

(a) SET UP: For a particle with mass, $K = p^2/2m.$

EXECUTE: $p_2 = 2p_1$ means $K_2 = 4K_1.$

(b) SET UP: For a photon, $E = pc.$

EXECUTE: $p_2 = 2p_1$ means $E_2 = 2E_1.$

EVALUATE: The relation between E and p is different for particles with mass and particles without mass.

38.12. **IDENTIFY** and **SET UP:** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$, where V_0 is the stopping potential. The stopping potential in volts equals eV_0 in electron volts. $\frac{1}{2}mv_{\text{max}}^2 = hf - \phi$.

EXECUTE: **(a)** $eV_0 = \frac{1}{2}mv_{\text{max}}^2$ so

$eV_0 = hf - \phi = \dfrac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{250\times10^{-9}\text{ m}} - 2.3\text{ eV} = 4.96\text{ eV} - 2.3\text{ eV} = 2.7\text{ eV}$. The stopping potential is 2.7 electron volts.

(b) $\frac{1}{2}mv_{\text{max}}^2 = 2.7\text{ eV}$

(c) $v_{\text{max}} = \sqrt{\dfrac{2(2.7\text{ eV})(1.60\times10^{-19}\text{ J/eV})}{9.11\times10^{-31}\text{ kg}}} = 9.7\times10^{5}\text{ m/s}$

38.13. **(a) IDENTIFY:** First use Eq.(38.4) to find the work function ϕ.

SET UP: $eV_0 = hf - \phi$ so $\phi = hf - eV_0 = \dfrac{hc}{\lambda} - eV_0$

EXECUTE: $\phi = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{254\times10^{-9}\text{ m}} - (1.602\times10^{-19}\text{ C})(0.181\text{ V})$

$\phi = 7.821\times10^{-19}\text{ J} - 2.900\times10^{-20}\text{ J} = 7.531\times10^{-19}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 4.70\text{ eV}$

IDENTIFY and **SET UP:** The threshold frequency f_{th} is the smallest frequency that still produces photoelectrons. It corresponds to $K_{\text{max}} = 0$ in Eq.(38.3), so $hf_{\text{th}} = \phi.$

EXECUTE: $f = \dfrac{c}{\lambda}$ says $\dfrac{hc}{\lambda_{\text{th}}} = \phi$

$\lambda_{\text{th}} = \dfrac{hc}{\phi} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{7.531\times10^{-19}\text{ J}} = 2.64\times10^{-7}\text{ m} = 264\text{ nm}$

(b) EVALUATE: As calculated in part (a), $\phi = 4.70$ eV. This is the value given in Table 38.1 for copper.

38.14. **IDENTIFY** and **SET UP:** A photon has zero rest mass, so its energy and momentum are related by Eq.(37.40). Eq.(38.5) then relates its momentum and wavelength.

EXECUTE: **(a)** $E = pc = (8.24\times10^{-28}\text{ kg}\cdot\text{m/s})(2.998\times10^{8}\text{ m/s}) = 2.47\times10^{-19}\text{ J} =$ $(2.47\times10^{-19}\text{ J})(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 1.54\text{ eV}$

(b) $p = \dfrac{h}{\lambda}$ so $\lambda = \dfrac{h}{p} = \dfrac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{8.24\times10^{-28}\text{ kg}\cdot\text{m/s}} = 8.04\times10^{-7}\text{ m} = 804\text{ nm}$

EVALUATE: This wavelength is longer than visible wavelengths; it is in the infrared region of the electromagnetic spectrum. To check our result we could verify that the same E is given by Eq.(38.2), using the λ we have calculated.

38.15. **IDENTIFY** and **SET UP:** Balmer's formula is $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{n^2}\right)$. For the H_γ spectral line $n = 5$. Once we have λ, calculate f from $f = c/\lambda$ and E from Eq.(38.2).

EXECUTE: **(a)** $\dfrac{1}{\lambda} = R\left(\dfrac{1}{2^2} - \dfrac{1}{5^2}\right) = R\left(\dfrac{25-4}{100}\right) = R\left(\dfrac{21}{100}\right).$

Thus $\lambda = \dfrac{100}{21R} = \dfrac{100}{21(1.097\times10^{7})}\text{ m} = 4.341\times10^{-7}\text{ m} = 434.1\text{ nm}.$

(b) $f = \dfrac{c}{\lambda} = \dfrac{2.998\times10^{8}\text{ m/s}}{4.341\times10^{-7}\text{ m}} = 6.906\times10^{14}\text{ Hz}$

(c) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(6.906\times10^{14}\ \text{Hz}) = 4.576\times10^{-19}\ \text{J} = 2.856\ \text{eV}$

EVALUATE: Section 38.3 shows that the longest wavelength in the Balmer series (H_α) is 656 nm and the shortest is 365 nm. Our result for H_γ falls within this range. The photon energies for hydrogen atom transitions are in the eV range, and our result is of this order.

38.16. **IDENTIFY** and **SET UP:** For the Lyman series the final state is $n = 1$ and the wavelengths are given by $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right),\ n = 2,3,....$ For the Paschen series the final state is $n = 3$ and the wavelengths are given by $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right),\ n = 4,5,....$ $R = 1.097\times10^7\ \text{m}^{-1}$. The longest wavelength is for the smallest *n* and the shortest wavelength is for $n \to \infty$.

EXECUTE: Lyman Longest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \frac{3R}{4}$. $\lambda = \frac{4}{3(1.097\times10^7\ \text{m}^{-1})} = 121.5\ \text{nm}$.

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) = R$. $\lambda = \frac{1}{1.097\times10^7\ \text{m}^{-1}} = 91.16\ \text{nm}$

Paschen Longest: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = \frac{7R}{144}$. $\lambda = \frac{144}{7(1.097\times10^7\ \text{m}^{-1})} = 1875\ \text{nm}$.

Shortest: $\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right) = \frac{R}{9}$.

38.17. **(a)** $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times10^8\ \text{m/s})}{8.60\times10^{-7}\ \text{m}} = 2.31\times10^{-19}\ \text{J} = 1.44\ \text{eV}.$

So the internal energy of the atom increases by 1.44 eV to $E = -6.52\ \text{eV} + 1.44\ \text{eV} = -5.08\ \text{eV}.$

(b) $E_\gamma = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times10^8\ \text{m/s})}{4.20\times10^{-7}\ \text{m}} = 4.74\times10^{-19}\ \text{J} = 2.96\ \text{eV}.$

So the final internal energy of the atom decreases to $E = -2.68\ \text{eV} - 2.96\ \text{eV} = -5.64\ \text{eV}.$

38.18. **IDENTIFY** and **SET UP:** The ionization threshold is at $E = 0$. The energy of an absorbed photon equals the energy gained by the atom and the energy of an emitted photon equals the energy lost by the atom.

EXECUTE: **(a)** $\Delta E = 0 - (-20\ \text{eV}) = 20\ \text{eV}$

(b) When the atom in the $n = 1$ level absorbs a 18 eV photon, the final level of the atom is $n = 4$. The possible transitions from $n = 4$ and corresponding photon energies are $n = 4 \to n = 3,\ 3\ \text{eV}$; $n = 4 \to n = 2,\ 8\ \text{eV}$; $n = 4 \to n = 1,\ 18\ \text{eV}$. Once the atom has gone to the $n = 3$ level, the following transitions can occur: $n = 3 \to n = 2,\ 5\ \text{eV}$; $n = 3 \to n = 1,\ 15\ \text{eV}$. Once the atom has gone to the $n = 2$ level, the following transition can occur: $n = 2 \to n = 1,\ 10\ \text{eV}$. The possible energies of emitted photons are: 3 eV, 5 eV, 8 eV, 10 eV, 15 eV, and 18 eV.

(c) There is no energy level 8 eV higher in energy than the ground state, so the photon cannot be absorbed.

(d) The photon energies for $n = 3 \to n = 2$ and for $n = 3 \to n = 1$ are 5 eV and 15 eV. The photon energy for $n = 4 \to n = 3$ is 3 eV. The work function must have a value between 3 eV and 5 eV.

38.19. **IDENTIFY** and **SET UP:** The wavelength of the photon is related to the transition energy $E_\text{i} - E_\text{f}$ of the atom by $E_\text{i} - E_\text{f} = \frac{hc}{\lambda}$ where $hc = 1.240\times10^{-6}\ \text{eV}\cdot\text{m}$.

EXECUTE: **(a)** The minimum energy to ionize an atom is when the upper state in the transition has $E = 0$, so $E_1 = -17.50\ \text{eV}$. For $n = 5 \to n = 1$, $\lambda = 73.86\ \text{nm}$ and $E_5 - E_1 = \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{73.86\times10^{-9}\ \text{m}} = 16.79\ \text{eV}$. $E_5 = -17.50\ \text{eV} + 16.79\ \text{eV} = -0.71\ \text{eV}$. For $n = 4 \to n = 1$, $\lambda = 75.63\ \text{nm}$ and $E_4 = -1.10\ \text{eV}$. For $n = 3 \to n = 1$, $\lambda = 79.76\ \text{nm}$ and $E_3 = -1.95\ \text{eV}$. For $n = 2 \to n = 1$, $\lambda = 94.54\ \text{nm}$ and $E_2 = -4.38\ \text{eV}$.

(b) $E_\text{i} - E_\text{f} = E_4 - E_2 = -1.10\ \text{eV} - (-4.38\ \text{eV}) = 3.28\ \text{eV}$ and $\lambda = \frac{hc}{E_\text{i} - E_\text{f}} = \frac{1.240\times10^{-6}\ \text{eV}\cdot\text{m}}{3.28\ \text{eV}} = 378\ \text{nm}$

EVALUATE: The $n = 4 \to n = 2$ transition energy is smaller than the $n = 4 \to n = 1$ transition energy so the wavelength is longer. In fact, this wavelength is longer than for any transition that ends in the $n = 1$ state.

38.20. **(a)** Equating initial kinetic energy and final potential energy and solving for the separation radius r,

$$r = \frac{1}{4\pi\epsilon_0}\frac{(92e)\,(2e)}{K} = \frac{1}{4\pi\epsilon_0}\frac{(184)\,(1.60\times10^{-19}\ \text{C})}{(4.78\times10^{6}\ \text{J/C})} = 5.54\times10^{-14}\ \text{m}.$$

(b) The above result may be substituted into Coulomb's law, or, the relation between the magnitude of the force and the magnitude of the potential energy in a Coulombic field is

$$F = \frac{K}{r} = \frac{(4.78\times10^{6}\ \text{eV})\,(1.6\times10^{-19}\ \text{J/ev})}{(5.54\times10^{-14}\ \text{m})} = 13.8\ \text{N}.$$

38.21. **(a) IDENTIFY:** If the particles are treated as point charges, $U = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{r}$.

SET UP: $q_1 = 2e$ (alpha particle); $q_2 = 82e$ (gold nucleus); r is given so we can solve for U.

EXECUTE: $U = (8.987\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\frac{(2)(82)(1.602\times10^{-19}\ \text{C})^2}{6.50\times10^{-14}\ \text{m}} = 5.82\times10^{-13}\ \text{J}$

$U = 5.82\times10^{-13}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 3.63\times10^{6}\ \text{eV} = 3.63\ \text{MeV}$

(b) IDENTIFY: Apply conservation of energy: $K_1 + U_1 = K_2 + U_2$.

SET UP: Let point 1 be the initial position of the alpha particle and point 2 be where the alpha particle momentarily comes to rest. Alpha particle is initially far from the lead nucleus implies $r_1 \approx \infty$ and $U_1 = 0$. Alpha particle stops implies $K_2 = 0$.

EXECUTE: Conservation of energy thus says $K_1 = U_2 = 5.82\times10^{-13}\ \text{J} = 3.63\ \text{MeV}$.

(c) $K = \frac{1}{2}mv^2$ so $v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(5.82\times10^{-13}\ \text{J})}{6.64\times10^{-27}\ \text{kg}}} = 1.32\times10^{7}\ \text{m/s}$

EVALUATE: $v/c = 0.044$, so it is ok to use the nonrelativistic expression to relate K and v. When the alpha particle stops, all its initial kinetic energy has been converted to electrostatic potential energy.

38.22. **(a)**, **(b)** For either atom, the magnitude of the angular momentum is $\frac{h}{2\pi} = 1.05\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$.

38.23. **IDENTIFY** and **SET UP:** Use the energy to calculate n for this state. Then use the Bohr equation, Eq.(38.10), to calculate L.

EXECUTE: $E_n = -(13.6\ \text{eV})/n^2$, so this state has $n = \sqrt{13.6/1.51} = 3$. In the Bohr model. $L = n\hbar$ so for this state $L = 3\hbar = 3.16\times10^{-34}\ \text{kg}\cdot\text{m}^2/\text{s}$.

EVALUATE: We will find in Section 41.1 that the modern quantum mechanical description gives a different result.

38.24. **IDENTIFY** and **SET UP:** For a hydrogen atom $E_n = -\frac{13.6\ \text{eV}}{n^2}$. $\Delta E = \frac{hc}{\lambda}$, where ΔE is the magnitude of the energy change for the atom and λ is the wavelength of the photon that is absorbed or emitted.

EXECUTE: $\Delta E = E_4 - E_1 = -(13.6\ \text{eV})\left(\frac{1}{4^2} - \frac{1}{1^2}\right) = +12.75\ \text{eV}$.

$\lambda = \frac{hc}{\Delta E} = \frac{(4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{12.75\ \text{eV}} = 97.3\ \text{nm}$. $f = \frac{c}{\lambda} = 3.08\times10^{15}\ \text{Hz}$.

38.25. **IDENTIFY:** The force between the electron and the nucleus in Be^{3+} is $F = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2}$, where $Z = 4$ is the nuclear charge. All the equations for the hydrogen atom apply to Be^{3+} if we replace e^2 by Ze^2.

(a) SET UP: Modify Eq.(38.18).

EXECUTE: $E_n = -\frac{1}{\epsilon_0}\frac{me^4}{8n^2h^2}$ (hydrogen) becomes

$$E_n = -\frac{1}{\epsilon_0}\frac{m(Ze^2)^2}{8n^2h^2} = Z^2\left(-\frac{1}{\epsilon_0}\frac{me^4}{8n^2h^2}\right) = Z^2\left(-\frac{13.60\ \text{eV}}{n^2}\right)\ (\text{for Be}^{3+})$$

The ground-level energy of Be^{3+} is $E_1 = 16\left(-\frac{13.60\ \text{eV}}{1^2}\right) = -218\ \text{eV}$.

EVALUATE: The ground-level energy of Be^{3+} is $Z^2 = 16$ times the ground-level energy of H.

(b) SET UP: The ionization energy is the energy difference between the $n \to \infty$ level energy and the $n = 1$ level energy.

EXECUTE: The $n \to \infty$ level energy is zero, so the ionization energy of Be^{3+} is 218 eV.
EVALUATE: This is 16 times the ionization energy of hydrogen.

(c) SET UP: $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$ just as for hydrogen but now *R* has a different value.

EXECUTE: $R_\text{H} = \frac{me^4}{8\epsilon_0 h^3 c} = 1.097\times 10^7\ \text{m}^{-1}$ for hydrogen becomes

$R_\text{Be} = Z^2 \frac{me^4}{8\epsilon_0 h^3 c} = 16(1.097\times 10^7\ \text{m}^{-1}) = 1.755\times 10^8\ \text{m}^{-1}$ for Be^{3+}.

For $n = 2$ to $n = 1$, $\frac{1}{\lambda} = R_\text{Be}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 3R/4.$

$\lambda = 4/(3R) = 4/(3(1.755\times 10^8\ \text{m}^{-1})) = 7.60\times 10^{-9}\ \text{m} = 7.60\ \text{nm}.$

EVALUATE: This wavelength is smaller by a factor of 16 compared to the wavelength for the corresponding transition in the hydrogen atom.

(d) SET UP: Modify Eq.(38.12): $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m e^2}$ (hydrogen).

EXECUTE: $r_n = \epsilon_0 \frac{n^2 h^2}{\pi m (Ze^2)}$ (Be^{3+}).

EVALUATE: For a given *n* the orbit radius for Be^{3+} is smaller by a factor of $Z = 4$ compared to the corresponding radius for hydrogen.

38.26. **(a)** We can find the photon's energy from Eq. 38.8

$E = hcR\left(\frac{1}{2^2} - \frac{1}{n^2}\right) = (6.63\times 10^{-34}\ \text{J}\cdot\text{s})\,(3.00\times 10^8\ \text{m/s})\,(1.097\times 10^7 \text{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 4.58\times 10^{-19}\ \text{J}.$ The

corresponding wavelength is $\lambda = \frac{E}{hc} = 434\ \text{nm}.$

(b) In the Bohr model, the angular momentum of an electron with principal quantum number *n* is given by Eq. 38.10: $L = n\frac{h}{2\pi}.$ Thus, when an electron makes a transition from *n* = 5 to *n* = 2 orbital, there is the following loss in angular momentum (which we would assume is transferred to the photon):

$$\Delta L = (2-5)\frac{h}{2\pi} = -\frac{3(6.63\times 10^{-34}\ \text{J}\cdot\text{s})}{2\pi} = -3.17\times 10^{-34}\ \text{J}\cdot\text{s}.$$

However, this prediction of the Bohr model is wrong (as shown in Chapter 41).

38.27. **(a)** $v_n = \frac{1}{\epsilon_0}\frac{e^2}{2nh} : n = 1 \Rightarrow v_1 = \frac{(1.60\times 10^{-19}\ \text{C})^2}{\epsilon_0 2\,(6.63\times 10^{-34}\ \text{J}\cdot\text{s})} = 2.18\times 10^6\ \text{m/s}$

$$h = 2 \Rightarrow v_2 = \frac{v_1}{2} = 1.09\times 10^6\ \text{m/s}.\ \ h = 3 \Rightarrow v_3 = \frac{v_1}{3} = 7.27\times 10^5\ \text{m/s}.$$

(b) Orbital period $= \frac{2\pi r_n}{v_n} = \frac{2\epsilon_0 n^2 h^2 / me^2}{1/\epsilon_0 \cdot e^2/2nh} = \frac{4\epsilon_0^2 n^3 h^3}{me^4}$

$$n = 1 \Rightarrow T_1 = \frac{4\epsilon_0^2 (6.63\times 10^{-34}\ \text{J}\cdot\text{s})^3}{(9.11\times 10^{-31}\ \text{kg})\,(1.60\times 10^{-19}\ \text{C})^4} = 1.53\times 10^{-16}\ \text{s}$$

$$n = 2:\ T_2 = T_1(2)^3 = 1.22\times 10^{-15}\ \text{s}.\ \ n = 3:\ T_3 = T_1(3)^3 = 4.13\times 10^{-15}\ \text{s}.$$

(c) number of orbits $= \frac{1.0\times 10^{-8}\text{s}}{1.22\times 10^{-15}\ \text{s}} = 8.2\times 10^6.$

38.28. **IDENTIFY** and **SET UP:** $E_n = -\frac{13.6\ \text{eV}}{n^2}$

EXECUTE: **(a)** $E_n = -\frac{13.6\ \text{eV}}{n^2}$ and $E_{n+1} = -\frac{13.6\ \text{eV}}{(n+1)^2}$

$\Delta E = E_{n+1} - E_n = (-13.6\ \text{eV})\left[\frac{1}{(n+1)^2} - \frac{1}{n^2}\right] = -(13.6\ \text{eV})\frac{n^2 - (n+1)^2}{(n^2)(n+1)^2}$

$\Delta E = (13.6\ \text{eV})\frac{2n+1}{(n^2)(n+1)^2}$ As *n* becomes large, $\Delta E \to (13.6\ \text{eV})\frac{2n}{n^4} = (13.6\ \text{eV})\frac{2}{n^3}$

Thus ΔE becomes small as n becomes large.

(b) $r_n = n^2 r_1$ so the orbits get farther apart in space as n increases.

38.29. **IDENTIFY** and **SET UP:** The number of photons emitted each second is the total energy emitted divided by the energy of one photon. The energy of one photon is given by Eq.(38.2). $E = Pt$ gives the energy emitted by the laser in time t.

EXECUTE: In 1.00 s the energy emitted by the laser is $(7.50\times10^{-3}\text{ W})(1.00\text{ s}) = 7.50\times10^{-3}\text{ J}$.

The energy of each photon is $E = \dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{10.6\times10^{-6}\text{ m}} = 1.874\times10^{-20}\text{ J}$.

Therefore $\dfrac{7.50\times10^{-3}\text{ J/s}}{1.874\times10^{-20}\text{ J/photon}} = 4.00\times10^{17}\text{ photons/s}$

EVALUATE: The number of photons emitted per second is extremely large.

38.30. **IDENTIFY** and **SET UP:** Visible light has wavelengths from about 400 nm to about 700 nm. The energy of each photon is $E = hf = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\text{ J}\cdot\text{m}}{\lambda}$. The power is the total energy per second and the total energy E_{tot} is the number of photons N times the energy E of each photon.

EXECUTE: **(a)** 193 nm is shorter than visible light so is in the ultraviolet.

(b) $E = \dfrac{hc}{\lambda} = 1.03\times10^{-18}\text{ J} = 6.44\text{ eV}$

(c) $P = \dfrac{E_{\text{tot}}}{t} = \dfrac{NE}{t}$ so $N = \dfrac{Pt}{E} = \dfrac{(1.50\times10^{-3}\text{ W})(12.0\times10^{-9}\text{ s})}{1.03\times10^{-18}\text{ J}} = 1.75\times10^{7}$ photons

EVALUATE: A very small amount of energy is delivered to the lens in each pulse, but this still corresponds to a large number of photons.

38.31. **IDENTIFY:** Apply Eq.(38.21): $\dfrac{n_{5s}}{n_{3p}} = e^{-(E_{5s}-E_{3p})/kT}$

SET UP: From Fig.38.24a in the textbook, $E_{5s} = 20.66\text{ eV}$ and $E_{3p} = 18.70\text{ eV}$

EXECUTE: $E_{5s} - E_{3p} = 20.66\text{ eV} - 18.70\text{ eV} = 1.96\text{ eV}(1.602\times10^{-19}\text{ J/1 eV}) = 3.140\times10^{-19}\text{ J}$

(a) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\text{ J})/[(1.38\times10^{-23}\text{ J/K})(300\text{ K})]} = e^{-75.79} = 1.2\times10^{-33}$

(b) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\text{ J})/[(1.38\times10^{-23}\text{ J/K})(600\text{ K})]} = e^{-37.90} = 3.5\times10^{-17}$

(c) $\dfrac{n_{5s}}{n_{3p}} = e^{-(3.140\times10^{-19}\text{ J})/[(1.38\times10^{-23}\text{ J/K})(1200\text{ K})]} = e^{-18.95} = 5.9\times10^{-9}$

(d) EVALUATE: At each of these temperatures the number of atoms in the 5s excited state, the initial state for the transition that emits 632.8 nm radiation, is quite small. The ratio increases as the temperature increases.

38.32. $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(E_{2P3/2} - E_{2P1/2})/KT}$.

From the diagram $\Delta E_{3/2-\text{g}} = \dfrac{hc}{\lambda_1} = \dfrac{(6.626\times10^{-34}\text{ J})(3.000\times10^{8}\text{ m/s})}{5.890\times10^{-7}\text{ m}} = 3.375\times10^{-19}\text{ J}$.

$\Delta E_{1/2-\text{g}} = \dfrac{hc}{\lambda_2} = \dfrac{(6.626\times10^{-34}\text{ J})(3.000\times10^{8}\text{ m/s})}{5.896\times10^{-7}\text{ m}} = 3.371\times10^{-19}\text{ J}$. so $\Delta E_{3/2-1/2} = 3.375\times10^{-19}\text{ J} - 3.371\times10^{-19}\text{ J} =$

$4.00\times10^{-22}\text{ J}$. $\dfrac{n_{2P_{3/2}}}{n_{2P_{1/2}}} = e^{-(4.00\times10^{-22}\text{ J})/(1.38\times10^{-23}\text{ J/K}\cdot 500\text{ K}).} = 0.944$. So more atoms are in the $2p_{1/2}$ state.

38.33. $eV_{\text{AC}} = hf_{\max} = \dfrac{hc}{\lambda_{\min}}$

$$\Rightarrow \lambda_{\min} = \frac{hc}{eV_{\text{AC}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(4000\text{ V})} = 3.11\times10^{-10}\text{ m}$$

This is the same answer as would be obtained if electrons of this energy were used. Electron beams are much more easily produced and accelerated than proton beams.

38.34. **IDENTIFY** and **SET UP:** $\frac{hc}{\lambda} = eV$, where λ is the wavelength of the x ray and V is the accelerating voltage.

EXECUTE: **(a)** $V = \frac{hc}{e\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(0.150\times10^{-9}\text{ m})} = 8.29\text{ kV}$

(b) $\lambda = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(30.0\times10^{3}\text{ V})} = 4.14\times10^{-11}\text{ m} = 0.0414\text{ nm}$

(c) No. A proton has the same magnitude of charge as an electron and therefore gains the same amount of kinetic energy when accelerated by the same magnitude of potential difference.

38.35. **IDENTIFY:** The initial electrical potential energy of the accelerated electrons is converted to kinetic energy which is then given to a photon.

SET UP: The electrical potential energy of an electron is eV_{AC}, where V_{AC} is the accelerating potential, and the energy of a photon is hf. Since the energy of the electron is all given to a photon, we have $eV_{AC} = hf$. For any wave, $f\lambda = v$.

EXECUTE: **(a)** $eV_{AC} = hf_{min}$ gives

$$f_{min} = eV_{AC}/h = (1.60\times10^{-19}\text{ C})(25{,}000\text{ V})/(6.626\times10^{-34}\text{ J}\cdot\text{s}) = 6.037\times10^{18}\text{ Hz}$$

$$= 6.04\times10^{18}\text{ Hz, rounded to three digits}$$

(b) $\lambda_{min} = c/f_{max} = (3.00\times10^{8}\text{ m/s})/(6.037\times10^{18}\text{ Hz}) = 4.97\times10^{-11}\text{ m} = 0.0497\text{ nm}$

(c) We assume that all the energy of the electron produces only one photon on impact with the screen.

EVALUATE: These photons are in the x-ray and γ-ray part of the electromagnetic spectrum (see Figure 32.4 in the textbook) and would be harmful to the eyes without protective glass on the screen to absorb them.

38.36. **IDENTIFY** and **SET UP:** The wavelength of the x rays produced by the tube is give by $\frac{hc}{\lambda} = eV$.

$\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi)$. $\frac{h}{mc} = 2.426\times10^{-12}\text{ m}$. The energy of the scattered x ray is $\frac{hc}{\lambda'}$.

EXECUTE: **(a)** $\lambda = \frac{hc}{eV} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(1.60\times10^{-19}\text{ C})(18.0\times10^{3}\text{ V})} = 6.91\times10^{-11}\text{ m} = 0.0691\text{ nm}$

(b) $\lambda' = \lambda + \frac{h}{mc}(1-\cos\phi) = 6.91\times10^{-11}\text{ m} + (2.426\times10^{-12}\text{ m})(1-\cos45.0°)$.

$\lambda' = 6.98\times10^{-11}\text{ m} = 0.0698\text{ nm}$.

(c) $E = \frac{hc}{\lambda'} = \frac{(4.136\times10^{-15}\text{ eV}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{6.98\times10^{-11}\text{ m}} = 17.8\text{ keV}$

EVALUATE: The incident x ray has energy 18.0 keV. In the scattering event, the photon loses energy and its wavelength increases.

38.37. **IDENTIFY:** Apply Eq.(38.23): $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi) = \lambda_C(1-\cos\phi)$

SET UP: Solve for λ': $\lambda' = \lambda + \lambda_C(1-\cos\phi)$

The largest λ' corresponds to $\phi = 180°$, so $\cos\phi = -1$.

EXECUTE: $\lambda' = \lambda + 2\lambda_C = 0.0665\times10^{-9}\text{ m} + 2(2.426\times10^{-12}\text{ m}) = 7.135\times10^{-11}\text{ m} = 0.0714\text{ nm}$. This wavelength occurs at a scattering angle of $\phi = 180°$.

EVALUATE: The incident photon transfers some of its energy and momentum to the electron from which it scatters. Since the photon loses energy its wavelength increases, $\lambda' > \lambda$.

38.38. **(a)** From Eq. (38.23), $\cos\phi = 1 - \frac{\Delta\lambda}{(h/mc)}$, and so $\Delta\lambda = 0.0542\text{ nm} - 0.0500\text{ nm}$,

$\cos\phi = 1 - \frac{0.0042\text{ nm}}{0.002426\text{ nm}} = -0.731$, and $\phi = 137°$.

(b) $\Delta\lambda = 0.0521\text{ nm} - 0.0500\text{ nm}$. $\cos\phi = 1 - \frac{0.0021\text{ nm}}{0.002426\text{ nm}} = 0.134$. $\phi = 82.3°$.

(c) $\Delta\lambda = 0$, the photon is undeflected, $\cos\phi = 1$ and $\phi = 0$.

38.39. **IDENTIFY** and **SET UP:** The shift in wavelength of the photon is $\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi)$ where λ' is the wavelength after the scattering and $\frac{h}{mc} = \lambda_c = 2.426\times10^{-12}\text{ m}$. The energy of a photon of wavelength λ is

$E=\dfrac{hc}{\lambda}=\dfrac{1.24\times10^{-6}\ \text{eV}\cdot\text{m}}{\lambda}$. Conservation of energy applies to the collision, so the energy lost by the photon equals the energy gained by the electron.

EXECUTE: **(a)** $\lambda'-\lambda=\lambda_c(1-\cos\phi)=(2.426\times10^{-12}\ \text{m})(1-\cos35.0°)=4.39\times10^{-13}\ \text{m}=4.39\times10^{-4}\ \text{nm}$

(b) $\lambda'=\lambda+4.39\times10^{-4}\ \text{nm}\ =\ 0.04250\ \text{nm}+4.39\times10^{-4}\ \text{nm}=0.04294\ \text{nm}$

(c) $E_\lambda=\dfrac{hc}{\lambda}=2.918\times10^4\ \text{eV}$ and $E_{\lambda'}=\dfrac{hc}{\lambda'}=2.888\times10^4\ \text{eV}$ so the photon loses 300 eV of energy.

(d) Energy conservation says the electron gains 300 eV of energy.

38.40. The change in wavelength of the scattered photon is given by Eq. 38.23

$$\frac{\Delta\lambda}{\lambda}=\frac{h}{mc\lambda}(1-\cos\phi)\Rightarrow\lambda=\frac{h}{mc\left(\dfrac{\Delta\lambda}{\lambda}\right)}(1-\cos\phi).$$

Thus, $\lambda=\dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(1.67\times10^{-27}\ \text{kg})(3.00\times10^8\ \text{m/s})(0.100)}(1+1)=2.65\times10^{-14}\ \text{m}.$

38.41. The derivation of Eq.(38.23) is explicitly shown in Equations (38.24) through (38.27) with the final substitution of $p'=h/\lambda'$ and $p=h/\lambda$ yielding $\lambda'-\lambda=\dfrac{h}{mc}(1-\cos\phi)$.

38.42. From Eq. (38.30), (a) $\lambda_m=\dfrac{2.898\times10^{-3}\ \text{m}\cdot\text{K}}{3.00\ \text{K}}=0.966\ \text{mm}$, and $f=\dfrac{c}{\lambda_m}=3.10\times10^{11}$ Hz. Note that a more precise value of the Wien displacement law constant has been used.

(b) A factor of 100 increase in the temperature lowers λ_m by a factor of 100 to 9.66 μm and raises the frequency by the same factor, to 3.10×10^{13} Hz.

(c) Similarly, $\lambda_m=966\ \text{nm}$ and $f=3.10\times10^{14}$ Hz.

38.43. **(a)** $H=Ae\sigma T^4;\ A=\pi r^2 l$

$$T=\left(\frac{H}{Ae\sigma}\right)^{1/4}=\left(\frac{100\ \text{W}}{2\pi(0.20\times10^{-3}\ \text{m})(0.30\ \text{m})(0.26)(5.671\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)}\right)^{1/4}$$

$T=2.06\times10^3$ K

(b) $\lambda_m T=2.90\times10^{-3}\ \text{m}\cdot\text{K};\quad \lambda_m=1410\ \text{nm}$

Much of the emitted radiation is in the infrared.

38.44. $T=\dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{\lambda_m}=\dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{400\times10^{-9}\ \text{m}}=7.25\times10^3\ \text{K}.$

38.45. **IDENTIFY** and **SET UP:** The wavelength λ_m where the Planck distribution peaks is given by Eq.(38.30).

EXECUTE: $\lambda_m=\dfrac{2.90\times10^{-3}\ \text{m}\cdot\text{K}}{2.728\ \text{K}}=1.06\times10^{-3}\ \text{m}=1.06\ \text{mm}.$

EVALUATE: This wavelength is in the microwave portion of the electromagnetic spectrum. This radiation is often referred to as the "microwave background" (Section 44.7). Note that in Eq.(38.30), T must be in kelvins.

38.46. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law and Wien's displacement law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I=\sigma T^4$, so the total radiated power is $P=\sigma AT^4$. Wien's displacement law tells us that the peak-intensity wavelength is λ_m = (constant)/T.

EXECUTE: **(a)** The hot and cool stars radiate the same total power, so the Stefan-Boltzmann law gives $\sigma A_h T_h^4=\sigma A_c T_c^4\ \Rightarrow\ 4\pi R_h^2 T_h^4=4\pi R_c^2 T_c^4=4\pi(3R_h)^2 T_c^4\Rightarrow T_h^4=9T^4\ \Rightarrow T_h=T\sqrt{3}\ =1.7T$, rounded to two significant digits.

(b) Using Wien's law, we take the ratio of the wavelengths, giving

$$\frac{\lambda_m(\text{hot})}{\lambda_m(\text{cool})}=\frac{T_c}{T_h}=\frac{T}{T\sqrt{3}}=\frac{1}{\sqrt{3}}=0.58,$$ rounded to two significant digits.

EVALUATE: Although the hot star has only 1/9 the surface area of the cool star, its absolute temperature has to be only 1.7 times as great to radiate the same amount of energy.

38.47. **(a)** Let $\alpha=hc/kT$. To find the maximum in the Planck distribution:

$$\frac{dI}{d\lambda}=\frac{d}{d\lambda}\left(\frac{2\pi hc^2}{\lambda^5(e^{\alpha/\lambda}-1)}\right)=0=-5\frac{(2\pi hc^2)}{\lambda^5(e^{\alpha/\lambda}-1)}-\frac{2\pi hc^2(-\alpha/\lambda^2)}{\lambda^5(e^{\alpha/\lambda}-1)^2}$$

$\Rightarrow-5(e^{\alpha/\lambda}-1)\lambda=\alpha\ \Rightarrow\ -5e^{\alpha/\lambda}+5=\alpha/\lambda\ \Rightarrow$ Solve $5-x=5e^x$ where $x=\dfrac{\alpha}{\lambda}=\dfrac{hc}{\lambda kT}$.

Its root is 4.965, so $\frac{\alpha}{\lambda} = 4.965 \Rightarrow \lambda = \frac{hc}{(4.965)kT}$.

(b) $\lambda_m T = \frac{hc}{(4.965)k} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(4.965)(1.38\times10^{-23}\ \text{J/K})} = 2.90\times10^{-3}\ \text{m}\cdot\text{K}.$

38.48. **IDENTIFY:** Since the stars radiate as blackbodies, they obey the Stefan-Boltzmann law.

SET UP: The Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** $I = \sigma T^4 = (5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(24{,}000\ \text{K})^4 = 1.9\times10^{10}\ \text{W/m}^2$

(b) Wien's law gives $\lambda_m = (0.00290\ \text{m}\cdot\text{K})/(24{,}000\ \text{K}) = 1.2\times10^{-7}\ \text{m} = 120\ \text{nm}$

This is not visible since the wavelength is less than 400 nm.

(c) $P = AI \Rightarrow 4\pi R^2 = P/I = (1.00\times10^{25}\ \text{W})/(1.9\times10^{10}\ \text{W/m}^2)$

which gives $R_{\text{Sirius}} = 6.51\times10^6\ \text{m} = 6510\ \text{km}$.

$R_{\text{Sirius}}/R_{\text{sun}} = (6.51\times10^6\ \text{m})/(6.96\times10^9\ \text{m}) = 0.0093$, which gives

$$R_{\text{Sirius}} = 0.0093\ R_{\text{sun}} \approx 1\%\ R_{\text{sun}}$$

(d) Using the Stefan-Boltzmann law, we have

$$\frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \frac{\sigma A_{\text{sun}} T_{\text{sun}}^4}{\sigma A_{\text{Sirius}} T_{\text{Sirius}}^4} = \frac{4\pi R_{\text{sun}}^2 T_{\text{sun}}^4}{4\pi R_{\text{Sirius}}^2 T_{\text{Sirius}}^4} = \left(\frac{R_{\text{sun}}}{R_{\text{Sirius}}}\right)^2 \left(\frac{T_{\text{sun}}}{T_{\text{Sirius}}}\right)^4 \cdot \frac{P_{\text{sun}}}{P_{\text{Sirius}}} = \left(\frac{R_{\text{sun}}}{0.00935 R_{\text{sun}}}\right)^2 \left(\frac{5800\ \text{K}}{24{,}000\ \text{K}}\right)^4 = 39$$

EVALUATE: Even though the absolute surface temperature of Sirius B is about 4 times that of our sun, it radiates only 1/39 times as much energy per second as our sun because it is so small.

38.49. Eq. (38.32): $I(\lambda) = \frac{2\pi hc^2}{\lambda^5 (e^{hc/\lambda kT} - 1)}$ but $e^x = 1 + x + \frac{x^2}{2} + \cdots \approx 1 + x$ for

$x \ll 1 \Rightarrow I(\lambda) \approx \frac{2\pi hc^2}{\lambda^5 (hc/\lambda kT)} = \frac{2\pi ckT}{\lambda^4} =$ Eq. (38.31), which is Rayleigh's distribution.

38.50. **(a)** Wien's law: $\lambda_m = \frac{k}{T}$. $\lambda_m = \frac{2.90\times10^{-3}\ \text{K}\cdot\text{m}}{30{,}000\ \text{K}} = 9.7\times10^{-8}\text{m} = 97\ \text{nm}$

This peak is in the ultraviolet region, which is *not* visible. The star is blue because the largest part of the visible light radiated is in the blue/violet part of the visible spectrum

(b) $P = \sigma A T^4$ (Stefan-Boltzmann law)

$$(100{,}000)(3.86\times10^{26}\ \text{W}) = \left(5.67\times10^{-8}\ \frac{\text{W}}{\text{m}^2\text{K}^4}\right)(4\pi R^2)(30{,}000\ \text{K})^4$$

$$R = 8.2\times10^9\ \text{m}$$

$$R_{\text{star}}/R_{\text{sun}} = \frac{8.2\times10^9\ \text{m}}{6.96\times10^8\ \text{m}} = 12$$

(c) The visual luminosity is proportional to the power radiated at visible wavelengths. Much of the power is radiated nonvisible wavelengths, which does not contribute to the visible luminosity.

38.51. **IDENTIFY** and **SET UP:** Use $c = f\lambda$ to relate frequency and wavelength and use $E = hf$ to relate photon energy and frequency.

EXECUTE: **(a)** One photon dissociates one AgBr molecule, so we need to find the energy required to dissociate a single molecule. The problem states that it requires $1.00\times10^5\ \text{J}$ to dissociate one mole of AgBr, and one mole contains Avogadro's number (6.02×10^{23}) of molecules, so the energy required to dissociate one AgBr is

$$\frac{1.00\times10^5\ \text{J/mol}}{6.02\times10^{23}\ \text{molecules/mol}} = 1.66\times10^{-19}\ \text{J/molecule}.$$

The photon is to have this energy, so $E = 1.66\times10^{-19}\ \text{J}(1\text{eV}/1.602\times10^{-19}\ \text{J}) = 1.04\ \text{eV}$.

(b) $E = \frac{hc}{\lambda}$ so $\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^8\ \text{m/s})}{1.66\times10^{-19}\ \text{J}} = 1.20\times10^{-6}\ \text{m} = 1200\ \text{nm}$

(c) $c = f\lambda$ so $f = \frac{c}{\lambda} = \frac{2.998\times10^8\ \text{m/s}}{1.20\times10^{-6}\ \text{m}} = 2.50\times10^{14}\ \text{Hz}$

(d) $E = hf = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(100\times10^6\ \text{Hz}) = 6.63\times10^{-26}\ \text{J}$

$E = 6.63\times10^{-26}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 4.14\times10^{-7}\ \text{eV}$

(e) EVALUATE: A photon with frequency $f = 100$ MHz has too little energy, by a large factor, to dissociate a AgBr molecule. The photons in the visible light from a firefly do individually have enough energy to dissociate AgBr. The huge number of 100 MHz photons can't compensate for the fact that individually they have too little energy.

38.52. **(a)** Assume a non-relativistic velocity and conserve momentum $\Rightarrow mv = \dfrac{h}{\lambda} \Rightarrow v = \dfrac{h}{m\lambda}$.

(b) $K = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\left(\dfrac{h}{m\lambda}\right)^2 = \dfrac{h^2}{2m\lambda^2}$.

(c) $\dfrac{K}{E} = \dfrac{h^2}{2m\lambda^2}\cdot\dfrac{\lambda}{hc} = \dfrac{h}{2mc\lambda}$. Recoil becomes an important concern for small m and small λ since this ratio becomes large in those limits.

(d) $E = 10.2\text{ eV} \Rightarrow \lambda = \dfrac{hc}{E} = \dfrac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(10.2\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 1.22\times10^{-7}\text{ m} = 122\text{ nm}$.

$$K = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})^2}{2(1.67\times10^{-27}\text{ kg})(1.22\times10^{-7}\text{ m})^2} = 8.84\times10^{-27}\text{ J} = 5.53\times10^{-8}\text{ eV}.$$

$\dfrac{K}{E} = \dfrac{5.53\times10^{-8}\text{ eV}}{10.2\text{ eV}} = 5.42\times10^{-9}$. This is quite small so recoil can be neglected.

38.53. **IDENTIFY** and **SET UP:** $f = \dfrac{c}{\lambda}$. The (f, V_0) values are: $(8.20\times10^{14}\text{ Hz, 1.48 V})$, $(7.41\times10^{14}\text{ Hz, 1.15 V})$, $(6.88\times10^{14}\text{ Hz, 0.93 V})$, $(6.10\times10^{14}\text{ Hz, 0.62 V})$, $(5.49\times10^{14}\text{ Hz, 0.36 V})$, $(5.18\times10^{14}\text{ Hz, 0.24 V})$. The graph of V_0 versus f is given in Figure 38.53.

EXECUTE: **(a)** The threshold frequency, f_{th}, is f where $V_0 = 0$. From the graph this is $f_{\text{th}} = 4.56\times10^{14}$ Hz.

(b) $\lambda_{\text{th}} = \dfrac{c}{f_{\text{th}}} = \dfrac{3.00\times10^{8}\text{ m/s}}{4.56\times10^{14}\text{ Hz}} = 658\text{ nm}$

(c) $\phi = hf_{\text{th}} = (4.136\times10^{-15}\text{ eV}\cdot\text{s})(4.56\times10^{14}\text{ Hz}) = 1.89\text{ eV}$

(d) $eV_0 = hf - \phi$ so $V_0 = \left(\dfrac{h}{e}\right)f - \phi$. The slope of the graph is $\dfrac{h}{e}$.

$\dfrac{h}{e} = \left(\dfrac{1.48\text{ V} - 0.24\text{ V}}{8.20\times10^{14}\text{ Hz} - 5.18\times10^{14}\text{ Hz}}\right) = 4.11\times10^{-15}$ V/Hz and

$h = (4.11\times10^{-15}\text{ V/Hz})(1.60\times10^{-19}\text{ C}) = 6.58\times10^{-34}\text{ J}\cdot\text{s}$.

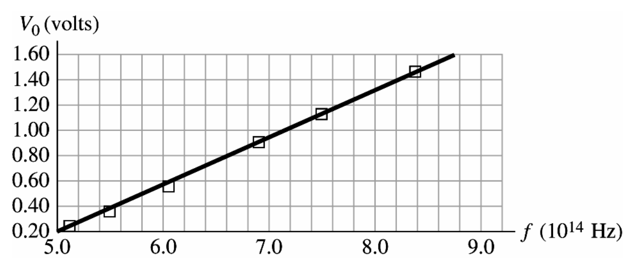

Figure 38.53

38.54. **(a)** $\dfrac{dN}{dt} = \dfrac{(dE/dt)}{(dE/dN)} = \dfrac{P}{hf} = \dfrac{(200\text{ W})(0.10)}{h(5.00\times10^{14}\text{ Hz})} = 6.03\times10^{19}$ photons/sec.

(b) Demand $\dfrac{(dN/dt)}{4\pi r^2} = 1.00\times10^{11}$ photons/sec$\cdot$cm^2.

Therefore, $r = \left(\dfrac{6.03\times10^{19}\text{ photons/sec}}{4\pi(1.00\times10^{11}\text{ photons/sec}\cdot\text{cm}^2)}\right)^{1/2} = 6930\text{ cm} = 69.3\text{ m}$.

38.55. **(a) IDENTIFY:** Apply the photoelectric effect equation, Eq.(38.4).

SET UP: $eV_0 = hf - \phi = (hc/\lambda) - \phi$. Call the stopping potential V_{01} for λ_1 and V_{02} for λ_2. Thus $eV_{01} = (hc/\lambda_1) - \phi$ and $eV_{02} = (hc/\lambda_2) - \phi$. Note that the work function ϕ is a property of the material and is independent of the wavelength of the light.

EXECUTE: Subtracting one equation from the other gives $e(V_{02} - V_{01}) = hc\left(\dfrac{\lambda_1 - \lambda_2}{\lambda_1\lambda_2}\right)$.

(b) $\Delta V_0 = \dfrac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{1.602\times10^{-19}\ \text{C}}\left(\dfrac{295\times10^{-9}\ \text{m}-265\times10^{-9}\ \text{m}}{(295\times10^{-9}\ \text{m})(265\times10^{-9}\ \text{m})}\right)=0.476\ \text{V}.$

EVALUATE: $e\Delta V_0$, which is 0.476 eV, is the increase in photon energy from 295 nm to 265 nm. The stopping potential increases when λ deceases because the photon energy increases when the wavelength decreases.

38.56. **IDENTIFY:** The photoelectric effect occurs, so the energy of the photon is used to eject an electron, with any excess energy going into kinetic energy of the electron.

SET UP: Conservation of energy gives $hf = hc/\lambda = K_{max} + \phi$.

EXECUTE: **(a)** Using $hc/\lambda = K_{max} + \phi$, we solve for the work function:

$$\phi = hc/\lambda - K_{max} = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(124\ \text{nm}) - 4.16\ \text{eV} = 5.85\ \text{eV}$$

(b) The number N of photoelectrons per second is equal to the number of photons per second that strike the metal per second. $N\times$ (energy of a photon) = 2.50 W. $N(hc/\lambda)$ = 2.50 W.

$N = (2.50\ \text{W})(124\ \text{nm})/[(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})] = 1.56\times10^{18}$ electrons/s

(c) N is proportional to the power, so if the power is cut in half, so is N, which gives

$$N = (1.56\times10^{18}\ \text{el/s})/2 = 7.80\times10^{17}\ \text{el/s}$$

(d) If we cut the wavelength by half, the energy of each photon is doubled since $E = hc/\lambda$. To maintain the same power, the number of photons must be half of what they were in part (b), so N is cut in half to 7.80×10^{17} el/s. We could also see this from part (b), where N is proportional to λ. So if the wavelength is cut in half, so is N.

EVALUATE: In part (c), reducing the power does not reduce the maximum kinetic energy of the photons; it only reduces the number of ejected electrons. In part (d), reducing the wavelength *does* change the maximum kinetic energy of the photoelectrons because we have increased the energy of each photon.

38.57. **IDENTIFY** and **SET UP:** The energy added to mass m of the blood to heat it to $T_f = 100\ ^\circ\text{C}$ and to vaporize it is $Q = mc(T_f - T_i) + mL_v$, with $c = 4190\ \text{J/kg}\cdot\text{K}$ and $L_v = 2.256\times10^{6}\ \text{J/kg}$. The energy of one photon is $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{\lambda}$.

EXECUTE: **(a)** $Q = (2.0\times10^{-9}\ \text{kg})(4190\ \text{J/kg}\cdot\text{K})(100^\circ\text{C}-33^\circ\text{C}) + (2.0\times10^{-9}\ \text{kg})(2.256\times10^{6}\ \text{J/kg}) = 5.07\times10^{-3}\ \text{J}$

The pulse must deliver 5.07 mJ of energy.

(b) $P = \dfrac{\text{energy}}{t} = \dfrac{5.07\times10^{-3}\ \text{J}}{450\times10^{-6}\ \text{s}} = 11.3\ \text{W}$

(c) One photon has energy $E = \dfrac{hc}{\lambda} = \dfrac{1.99\times10^{-25}\ \text{J}\cdot\text{m}}{585\times10^{-9}\ \text{m}} = 3.40\times10^{-19}\ \text{J}$. The number N of photons per pulse is the energy per pulse divided by the energy of one photon: $N = \dfrac{5.07\times10^{-3}\ \text{J}}{3.40\times10^{-19}\ \text{J/photon}} = 1.49\times10^{16}$ photons

38.58. **(a)** $\lambda_0 = \dfrac{hc}{E}$, and the wavelengths are: cesium: 590 nm, copper: 264 nm, potassium: 539 nm, zinc: 288 nm.

b) The wavelengths of copper and zinc are in the ultraviolet, and visible light is not energetic enough to overcome the threshold energy of these metals.

38.59. **(a) IDENTIFY** and **SET UP:** Apply Eq.(38.20): $m_r = \dfrac{m_1 m_2}{m_1 + m_2} = \dfrac{207 m_e m_p}{207 m_e + m_p}$

EXECUTE: $m_r = \dfrac{207(9.109\times10^{-31}\ \text{kg})(1.673\times10^{-27}\ \text{kg})}{207(9.109\times10^{-31}\ \text{kg}) + 1.673\times10^{-27}\ \text{kg}} = 1.69\times10^{-28}\ \text{kg}$

We have used m_e to denote the electron mass.

(b) IDENTIFY: In Eq.(38.18) replace $m = m_e$ by $m_r : E_n = -\dfrac{1}{\epsilon_0^2}\dfrac{m_r e^4}{8n^2h^2}$.

SET UP: Write as $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8n^2h^2}\right)$, since we know that $\dfrac{1}{\epsilon_0^2}\dfrac{m_H e^4}{8h^2} = 13.60$ eV. Here m_H denotes the reduced mass for the hydrogen atom; $m_H = 0.99946(9.109\times10^{-31}\ \text{kg}) = 9.104\times10^{-31}\ \text{kg}$.

EXECUTE: $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{13.60\ \text{eV}}{n^2}\right)$

$E_1 = \dfrac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(-13.60\ \text{eV}) = 186(-13.60\ \text{eV}) = -2.53\ \text{keV}$

(c) SET UP: From part (b), $E_n = \left(\dfrac{m_r}{m_H}\right)\left(-\dfrac{R_H ch}{n^2}\right)$, where $R_H = 1.097\times10^7\ \text{m}^{-1}$ is the Rydberg constant for the hydrogen atom. Use this result in $\dfrac{hc}{\lambda} = E_i - E_f$ to find an expression for $1/\lambda$. The initial level for the transition is the $n_i = 2$ level and the final level is the $n_f = 1$ level.

EXECUTE: $\dfrac{hc}{\lambda} = \dfrac{m_r}{m_H}\left(-\dfrac{R_H ch}{n_i^2} - \left(-\dfrac{R_H ch}{n_f^2}\right)\right)$

$\dfrac{1}{\lambda} = \dfrac{m_r}{m_H}R_H\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$

$\dfrac{1}{\lambda} = \dfrac{1.69\times10^{-28}\ \text{kg}}{9.104\times10^{-31}\ \text{kg}}(1.097\times10^7\ \text{m}^{-1})\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) = 1.527\times10^9\ \text{m}^{-1}$

$\lambda = 0.655\ \text{nm}$

EVALUATE: From Example 38.6 the wavelength of the radiation emitted in this transition in hydrogen is 122 nm. The wavelength for muonium is $\dfrac{m_H}{m_r} = 5.39\times10^{-3}$ times this. The reduced mass for hydrogen is very close to the electron mass because the electron mass is much less then the proton mass: $m_p/m_e = 1836$. The muon mass is $207m_e = 1.886\times10^{-28}$ kg. The proton is only about 10 times more massive than the muon, so the reduced mass is somewhat smaller than the muon mass. The muon-proton atom has much more strongly bound energy levels and much shorter wavelengths in its spectrum than for hydrogen.

38.60. **(a)** The change in wavelength of the scattered photon is given by Eq. 38.23

$$\lambda' - \lambda = \frac{h}{mc}(1-\cos\phi) \Rightarrow \lambda = \lambda' - \frac{h}{mc}(1-\cos\phi) =$$

$$(0.0830\times10^{-9}\ \text{m}) - \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^8\ \text{m/s})}(1+1) = 0.0781\ \text{nm}.$$

(b) Since the collision is one-dimensional, the magnitude of the electron's momentum must be equal to the magnitude of the change in the photon's momentum. Thus,

$$p_e = h\left(\frac{1}{\lambda} - \frac{-1}{\lambda'}\right) = (6.63\times10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{0.0781} + \frac{1}{0.0830}\right)(10^9\ \text{m}^{-1})$$
$$= 1.65\times10^{-23}\ \text{kg}\cdot\text{m/s} \approx 2\times10^{-23}\ \text{kg}\cdot\text{m/s}.$$

(c) Since the electron is non relativistic $(\beta = 0.06)$, $K_e = \dfrac{p_e^2}{2m} = 1.49\times10^{-16}\ \text{J} \approx 10^{-16}$ J.

38.61. **IDENTIFY** and **SET UP:** $\lambda' = \lambda + \dfrac{h}{mc}(1-\cos\phi)$

$\phi = 180°$ so $\lambda' = \lambda + \dfrac{2h}{mc} = 0.09485$ m. Use Eq.(38.5) to calculate the momentum of the scattered photon. Apply conservation of energy to the collision to calculate the kinetic energy of the electron after the scattering. The energy of the photon is given by Eq.(38.2),

EXECUTE: **(a)** $p' = h/\lambda' = 6.99\times10^{-24}\ \text{kg}\cdot\text{m/s}$.

(b) $E = E' + E_e$; $hc/\lambda = hc/\lambda' + E_e$

$E_e = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) = (hc)\dfrac{\lambda' - \lambda}{\lambda\lambda'} = 1.129\times10^{-16}\ \text{J} = 705\ \text{eV}$

EVALUATE: The energy of the incident photon is 13.8 keV, so only about 5% of its energy is transferred to the electron. This corresponds to a fractional shift in the photon's wavelength that is also 5%.

38.62. **(a)** $\phi = 180°$ so $(1-\cos\phi) = 2 \Rightarrow \Delta\lambda = \dfrac{2h}{mc} = 0.0049$ nm, so $\lambda' = 0.1849$ nm.

(b) $\Delta E = hc\left(\dfrac{1}{\lambda} - \dfrac{1}{\lambda'}\right) = 2.93\times10^{-17}\ \text{J} = 183\ \text{eV}$. This will be the kinetic energy of the electron.

(c) The kinetic energy is far less than the rest mass energy, so a non-relativistic calculation is adequate; $v = \sqrt{2K/m} = 8.02\times10^6$ m/s.

38.63. **IDENTIFY** and **SET UP:** The H_α line in the Balmer series corresponds to the $n=3$ to $n=2$ transition.

$E_n = -\dfrac{13.6\text{ eV}}{n^2}$. $\dfrac{hc}{\lambda} = \Delta E$.

EXECUTE: **(a)** The atom must be given an amount of energy $E_3 - E_1 = -(13.6\text{ eV})\left(\dfrac{1}{3^2} - \dfrac{1}{1^2}\right) = 12.1\text{ eV}$.

(b) There are three possible transitions. $n=3 \to n=1$: $\Delta E = 12.1\text{ eV}$ and $\lambda = \dfrac{hc}{\Delta E} = 103\text{ nm}$;

$n=3 \to n=2$: $\Delta E = -(13.6\text{ eV})\left(\dfrac{1}{3^2} - \dfrac{1}{2^2}\right) = 1.89\text{ eV}$ and $\lambda = 657\text{ nm}$; $n=2 \to n=1$:

$\Delta E = -(13.6\text{ eV})\left(\dfrac{1}{2^2} - \dfrac{1}{1^2}\right) = 10.2\text{ eV}$ and $\lambda = 122\text{ nm}$.

38.64. $\dfrac{n_2}{n_1} = e^{-(E_{\text{ex}} - E_{\text{g}})/kT} \Rightarrow T = \dfrac{-(E_{\text{ex}} - E_{\text{g}})}{k\ln(n_2/n_1)}$.

$$E_{\text{ex}} = E_2 = \frac{-13.6\text{ eV}}{4} = -3.4\text{ eV}.\ E_{\text{g}} = -13.6\text{ eV}.\ E_{\text{ex}} - E_{\text{g}} = 10.2\text{ eV} = 1.63\times10^{-18}\text{ J}.$$

(a) $\dfrac{n_2}{n_1} = 10^{-12}$. $T = \dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-12})} = 4275\text{ K}$.

(b) $\dfrac{n_2}{n_1} = 10^{-8}$. $T = \dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-8})} = 6412\text{ K}$.

(c) $\dfrac{n_2}{n_1} = 10^{-4}$. $T = \dfrac{-(1.63\times10^{-18}\text{ J})}{(1.38\times10^{-23}\text{ J/K})\ln(10^{-4})} = 12824\text{ K}$.

(d) For absorption to take place in the Balmer series, hydrogen must *start* in the $n=2$ state. From part (a), colder stars have fewer atoms in this state leading to weaker absorption lines.

38.65. **(a) IDENTIFY** and **SET UP:** The photon energy is given to the electron in the atom. Some of this energy overcomes the binding energy of the atom and what is left appears as kinetic energy of the free electron. Apply $hf = E_{\text{f}} - E_{\text{i}}$, the energy given to the electron in the atom when a photon is absorbed.

EXECUTE: The energy of one photon is $\dfrac{hc}{\lambda} = \dfrac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(2.998\times10^{8}\text{ m/s})}{85.5\times10^{-9}\text{ m}}$

$\dfrac{hc}{\lambda} = 2.323\times10^{-18}\text{ J}(1\text{ eV}/1.602\times10^{-19}\text{ J}) = 14.50\text{ eV}$.

The final energy of the electron is $E_{\text{f}} = E_{\text{i}} + hf$. In the ground state of the hydrogen atom the energy of the electron is $E_{\text{i}} = -13.60\text{ eV}$. Thus $E_{\text{f}} = -13.60\text{ eV} + 14.50\text{ eV} = 0.90\text{ eV}$.

(b) EVALUATE: At thermal equilibrium a few atoms will be in the $n=2$ excited levels, which have an energy of $-13.6\text{ eV}/4 = -3.40\text{ eV}$, 10.2 eV greater than the energy of the ground state. If an electron with $E = -3.40\text{ eV}$ gains 14.5 eV from the absorbed photon, it will end up with $14.5\text{ eV} - 3.4\text{ eV} = 11.1\text{ eV}$ of kinetic energy.

38.66. **IDENTIFY:** The diffraction grating allows us to determine the peak-intensity wavelength of the light. Then Wien's displacement law allows us to calculate the temperature of the blackbody, and the Stefan-Boltzmann law allows us to calculate the rate at which it radiates energy.

SET UP: The bright spots for a diffraction grating occur when $d\sin\theta = m\lambda$. Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times10^{-3}\text{ m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First find the wavelength of the light:

$$\lambda = d\sin\theta = [1/(385{,}000\text{ lines/m})]\sin(11.6°) = 5.22\times10^{-7}\text{ m}$$

Now use Wien's law to find the temperature: $T = (2.90\times10^{-3}\text{ m}\cdot\text{K})/(5.22\times10^{-7}\text{ m}) = 5550\text{ K}$.

(b) The energy radiated by the blackbody is equal to the power times the time, giving $U = Pt = IAt = \sigma A T^4 t$, which gives

$t = U/(\sigma A T^4) = (12.0\times10^6\text{ J})/[(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(4\pi)(0.0750\text{ m})^2(5550\text{ K})^4] = 3.16\text{ s}$.

EVALUATE: By ordinary standards, this blackbody is very hot, so it does not take long to radiate 12.0 MJ of energy.

38.67. IDENTIFY: Assuming that Betelgeuse radiates like a perfect blackbody, Wien's displacement and the Stefan-Boltzmann law apply to its radiation.

SET UP: Wien's displacement law is $\lambda_{\text{peak}} = \dfrac{2.90\times10^{-3}\,\text{m}\cdot\text{K}}{T}$, and the Stefan-Boltzmann law says that the intensity of the radiation is $I = \sigma T^4$, so the total radiated power is $P = \sigma A T^4$.

EXECUTE: **(a)** First use Wien's law to find the peak wavelength:

$$\lambda_{\text{m}} = (2.90\times10^{-3}\ \text{m}\cdot\text{K})/(3000\ \text{K}) = 9.667\times10^{-7}\ \text{m}$$

Call N the number osf photons/second radiated. $N \times$ (energy per photon) $= IA = \sigma A T^4$.

$N(hc/\lambda_{\text{m}}) = \sigma A T^4$. $N = \dfrac{\lambda_{\text{m}}\sigma A T^4}{hc}$.

$$N = \frac{(9.667\times10^{-7}\ \text{m})(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(4\pi)(600\times6.96\times10^8\ \text{m})^2(3000\,\text{K})^4}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^8\ \text{m/s})}.$$

$N = 5\times10^{49}$ photons/s.

(b) $\dfrac{I_{\text{B}}A_{\text{B}}}{I_{\text{S}}A_{\text{S}}} = \dfrac{\sigma A_{\text{B}}T_{\text{B}}^4}{\sigma A_{\text{S}}T_{\text{S}}^4} = \dfrac{4\pi R_{\text{B}}^2T_{\text{B}}^4}{4\pi R_{\text{S}}^2T_{\text{S}}^4} = \left(\dfrac{600R_{\text{S}}}{R_{\text{S}}}\right)^2\left(\dfrac{3000\ \text{K}}{5800\ \text{K}}\right)^4 = 3\times10^4$

EVALUATE: Betelgeuse radiates 30,000 times as much energy per second as does our sun!

38.68. IDENTIFY: The blackbody radiates heat into the water, but the water also radiates heat back into the blackbody. The net heat entering the water causes evaporation. Wien's law tells us the peak wavelength radiated, but a thermophile in the water measures the wavelength and frequency of the light in the water.

SET UP: By the Stefan-Boltzman law, the net power radiated by the blackbody is $\dfrac{dQ}{dt} = \sigma A\left(T_{\text{sphere}}^4 - T_{\text{water}}^4\right)$. Since this heat evaporates water, the rate at which water evaporates is $\dfrac{dQ}{dt} = L_v\dfrac{dm}{dt}$. Wien's displacement law is $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\,\text{m}\cdot\text{K}}{T}$, and the wavelength in the water is $\lambda_{\text{w}} = \lambda_0/n$.

EXECUTE: (a) The net radiated heat is $\dfrac{dQ}{dt} = \sigma A\left(T_{\text{sphere}}^4 - T_{\text{water}}^4\right)$ and the evaporation rate is $\dfrac{dQ}{dt} = L\text{v}\dfrac{dm}{dt}$, where dm is the mass of water that evaporates in time dt. Equating these two rates gives $L_{\text{v}}\dfrac{dm}{dt} = \sigma A\left(T_{\text{sphere}}^4 - T_{\text{water}}^4\right)$.

$$\frac{dm}{dt} = \frac{\sigma\left(4\pi R^2\right)\left(T_{\text{sphere}}^4 - T_{\text{water}}^4\right)}{L_{\text{v}}}.$$

$$\frac{dm}{dt} = \frac{\left(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)(4\pi)(0.120\,\text{m})^2\left[(498\,\text{K})^4 - (373\,\text{K})^4\right]}{2256\times10^3\ \text{J/Kg}} = 1.92\times10^{-4}\ \text{kg/s} = 0.193\ \text{g/s}$$

(b) (i) Wien's law gives $\lambda_{\text{m}} = (0.00290\ \text{m}\cdot\text{K})/(498\ \text{K}) = 5.82\times10^{-6}$ m

But this would be the wavelength in vacuum. In the water the thermophile organism would measure $\lambda_{\text{w}} = \lambda_0/n = (5.82\times10^{-6}\ \text{m})/1.333 = 4.37\times10^{-6}\ \text{m} = 4.37\ \mu\text{m}$

(ii) The frequency is the same as if the wave were in air, so

$$f = c/\lambda_0 = (3.00\times10^8\ \text{m/s})/(5.82\times10^{-6}\ \text{m}) = 5.15\times10^{13}\ \text{Hz}$$

EVALUATE: An alternative way is to use the quantities in the water: $f = \dfrac{c/n}{\lambda_0/n} = c/\lambda_0$, which gives the same answer for the frequency. An organism in the water would measure the light coming to it through the water, so the wavelength it would measure would be reduced by a factor of $1/n$.

38.69. IDENTIFY: The energy of the peak-intensity photons must be equal to the energy difference between the $n = 1$ and the $n = 4$ states. Wien's law allows us to calculate what the temperature of the blackbody must be for it to radiate with its peak intensity at this wavelength.

SET UP: In the Bohr model, the energy of an electron in shell n is $E_n = -\dfrac{13.6\ \text{eV}}{n^2}$, and Wien's displacement law is $\lambda_{\text{m}} = \dfrac{2.90\times10^{-3}\,\text{m}\cdot\text{K}}{T}$. The energy of a photon is $E = hf = hc/\lambda$.

EXECUTE: First find the energy (ΔE) that a photon would need to excite the atom. The ground state of the atom is $n = 1$ and the third excited state is $n = 4$. This energy is the *difference* between the two energy levels. Therefore

$\Delta E = (-13.6\ \text{eV})\left(\frac{1}{4^2}-\frac{1}{1^2}\right)$ = 12.8 eV. Now find the wavelength of the photon having this amount of energy.

hc/λ = 12.8 eV and

$$\lambda = (4.136\times10^{-15}\ \text{eV}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})/(12.8\ \text{eV}) = 9.73\times10^{-8}\ \text{m}$$

Now use Wien's law to find the temperature. $T = (0.00290\ \text{m}\cdot\text{K})/(9.73\times10^{-8}\ \text{m}) = 2.98\times10^{4}$ K.

EVALUATE: This temperature is well above ordinary room temperatures, which is why hydrogen atoms are not in excited states during everyday conditions.

38.70. **IDENTIFY** and **SET UP:** Electrical power is *VI*. $Q = mc\Delta T$.

EXECUTE: **(a)** $(0.010)VI = (0.010)(18.0\times10^{3}\ \text{V})(60.0\times10^{-3}\ \text{A}) = 10.8\ \text{W} = 10.8\ \text{J/s}$

(b) The energy in the electron beam that isn't converted to x rays stays in the target and appears as thermal energy. For $t = 1.00$ s, $Q = (0.990)VI(1.00\ \text{s}) = 1.07\times10^{3}$ J and $\Delta T = \frac{Q}{mc} = \frac{1.07\times10^{3}\ \text{J}}{(0.250\ \text{kg})(130\ \text{J/kg}\cdot\text{K})} = 32.9\ \text{K}$. The temperature rises at a rate of 32.9 K/s.

EVALUATE: The target must be made of a material that has a high melting point.

38.71. **IDENTIFY:** Apply conservation of energy and conservation of linear momentum to the system of atom plus photon.

(a) SET UP: Let E_{tr} be the transition energy, E_{ph} be the energy of the photon with wavelength λ', and E_{r} be the kinetic energy of the recoiling atom. Conservation of energy gives $E_{\text{ph}} + E_{\text{r}} = E_{\text{tr}}$.

$E_{\text{ph}} = \frac{hc}{\lambda'}$ so $\frac{hc}{\lambda'} = E_{\text{tr}} - E_{\text{r}}$ and $\lambda' = \frac{hc}{E_{\text{tr}} - E_{\text{r}}}$.

EXECUTE: If the recoil energy is neglected then the photon wavelength is $\lambda = hc/E_{\text{tr}}$.

$$\Delta\lambda = \lambda' - \lambda = hc\left(\frac{1}{E_{\text{tr}}-E_{\text{r}}} - \frac{1}{E_{\text{tr}}}\right) = \left(\frac{hc}{E_{\text{tr}}}\right)\left(\frac{1}{1-E_{\text{r}}/E_{\text{tr}}} - 1\right)$$

$$\frac{1}{1-E_{\text{r}}/E_{\text{tr}}} = \left(1-\frac{E_{\text{r}}}{\text{E}_{\text{tr}}}\right)^{-1} \approx 1 + \frac{E_{\text{r}}}{E_{\text{tr}}} \text{ since } \frac{E_{\text{r}}}{E_{\text{tr}}} \ll 1$$

(We have used the binomial theorem, Appendix B.)

Thus $\Delta\lambda = \frac{hc}{E_{\text{tr}}}\left(\frac{E_{\text{r}}}{E_{\text{tr}}}\right)$, or since $E_{\text{tr}} = hc/\lambda$, $\Delta\lambda = \left(\frac{E_{\text{r}}}{hc}\right)\lambda^2$.

SET UP: Use conservation of linear momentum to find E_{r}: Assuming that the atom is initially at rest, the momentum p_{r} of the recoiling atom must be equal in magnitude and opposite in direction to the momentum $p_{\text{ph}} = h/\lambda$ of the emitted photon: $h/\lambda = p_{\text{r}}$.

EXECUTE: $E_{\text{r}} = \frac{p_{\text{r}}^2}{2m}$, where *m* is the mass of the atom, so $E_{\text{r}} = \frac{h^2}{2m\lambda^2}$.

Use this result in the above equation: $\Delta\lambda = \left(\frac{E_{\text{r}}}{hc}\right)\lambda^2 = \left(\frac{h^2}{2m\lambda^2}\right)\left(\frac{\lambda^2}{hc}\right) = \frac{h}{2mc}$;

note that this result for $\Delta\lambda$ is independent of the atomic transition energy.

(b) For a hydrogen atom $m = m_{\text{p}}$ and $\Delta\lambda = \frac{h}{2m_{\text{p}}c} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{2(1.673\times10^{-27}\ \text{kg})(2.998\times10^{8}\ \text{m/s})} = 6.61\times10^{-16}\ \text{m}$

EVALUATE: The correction is independent of *n*. The wavelengths of photons emitted in hydrogen atom transitions are on the order of $100\ \text{nm} = 10^{-7}$ m, so the recoil correction is exceedingly small.

38.72. **(a)** $\Delta\lambda_1 = (h/mc)(1-\cos\theta_1)$, $\Delta\lambda_2 = (h/mc)(1-\cos\theta_2)$, and so the overall wavelength shift is $\Delta\lambda = (h/mc)(2-\cos\theta_1-\cos\theta_2)$.

(b) For a single scattering through angle θ, $\Delta\lambda_{\text{s}} = (h/mc)(1-\cos\theta)$. For two successive scatterings through an angle of $\theta/2$ for each scattering,

$$\Delta\lambda_{\text{t}} = 2(h/mc)(1-\cos\theta/2).$$

$$1-\cos\theta = 2(1-\cos^2(\theta/2)) \text{ and } \Delta\lambda_{\text{s}} = (h/mc)2(1-\cos^2(\theta/2))$$

$$\cos(\theta/2) \le 1 \text{ so } 1-\cos^2(\theta/2) \ge (1-\cos(\theta/2)) \text{ and } \Delta\lambda_{\text{s}} \ge \Delta\lambda_{\text{t}}$$

Equality holds only when $\theta = 180°$.

(c) $(h/mc)2(1-\cos 30.0°) = 0.268(h/mc)$.

(d) $(h/mc)(1-\cos 60°) = 0.500(h/mc)$, which is indeed greater than the shift found in part (c).

38.73. **IDENTIFY** and **SET UP:** Find the average change in wavelength for one scattering and use that in $\Delta\lambda$ in Eq.(38.23) to calculate the average scattering angle ϕ.

EXECUTE: **(a)** The wavelength of a 1 MeV photon is

$$\lambda = \frac{hc}{E} = \frac{(4.136\times 10^{-15}\ \text{eV}\cdot\text{s})(2.998\times 10^{8}\ \text{m/s})}{1\times 10^{6}\ \text{eV}} = 1\times 10^{-12}\ \text{m}$$

The total change in wavelength therefore is $500\times 10^{-9}\ \text{m} - 1\times 10^{-12}\ \text{m} = 500\times 10^{-9}\ \text{m}$.

If this shift is produced in 10^{26} Compton scattering events, the wavelength shift in each scattering event is

$$\Delta\lambda = \frac{500\times 10^{-9}\ \text{m}}{1\times 10^{26}} = 5\times 10^{-33}\ \text{m}.$$

(b) Use this $\Delta\lambda$ in $\Delta\lambda = \frac{h}{mc}(1-\cos\phi)$ and solve for ϕ. We anticipate that ϕ will be very small, since $\Delta\lambda$ is much less than h/mc, so we can use $\cos\phi \approx 1-\phi^2/2$.

$$\Delta\lambda = \frac{h}{mc}(1-(1-\phi^2/2)) = \frac{h}{2mc}\phi^2$$

$$\phi = \sqrt{\frac{2\Delta\lambda}{(h/mc)}} = \sqrt{\frac{2(5\times 10^{-33}\ \text{m})}{2.426\times 10^{-12}\ \text{m}}} = 6.4\times 10^{-11}\ \text{rad} = (4\times 10^{-9})°$$

ϕ in radians is much less than 1 so the approximation we used is valid.

(c) IDENTIFY and **SET UP:** We know the total transit time and the total number of scatterings, so we can calculate the average time between scatterings.

EXECUTE: The total time to travel from the core to the surface is $(10^6\ \text{y})(3.156\times 10^7\ \text{s/y}) = 3.2\times 10^{13}\ \text{s}$. There are 10^{26} scatterings during this time, so the average time between scatterings is $t = \frac{3.2\times 10^{13}\ \text{s}}{10^{26}} = 3.2\times 10^{-13}\ \text{s}$.

The distance light travels in this time is $d = ct = (3.0\times 10^8\ \text{m/s})(3.2\times 10^{-13}\ \text{s}) = 0.1\ \text{mm}$

EVALUATE: The photons are on the average scattered through a very small angle in each scattering event. The average distance a photon travels between scatterings is very small.

38.74. **(a)** The final energy of the photon is $E' = \frac{hc}{\lambda'}$, and $E = E' + K$, where K is the kinetic energy of the electron after the collision. Then,

$$\lambda = \frac{hc}{E'+K} = \frac{hc}{(hc/\lambda')+K} = \frac{hc}{(hc/\lambda')+(\gamma-1)mc^2} = \frac{\lambda'}{1+\frac{\lambda' mc}{h}\left[\frac{1}{(1-v^2/c^2)^{1/2}}-1\right]}.$$

$(K = mc^2(\gamma-1)$ since the relativistic expression must be used for three-figure accuracy).

(b) $\phi = \arccos(1-\Delta\lambda/(h/mc))$.

(c) $\gamma - 1 = \frac{1}{\left(1-\left(\frac{1.80}{3.00}\right)^2\right)^{1/2}} - 1 = 1.25 - 1 = 0.250,\ \frac{h}{mc} = 2.43\times 10^{-12}\ \text{m}$

$$\Rightarrow \lambda = \frac{5.10\times 10^{-3}\ \text{mm}}{1+\frac{(5.10\times 10^{-12}\ \text{m})(9.11\times 10^{-31}\ \text{kg})(3.00\times 10^{8}\ \text{m/s})(0.250)}{(6.63\times 10^{-34}\ \text{J}\cdot\text{s})}} = 3.34\times 10^{-3}\ \text{nm}.$$

$$\phi = \arccos\left(1-\frac{(5.10\times 10^{-12}\ \text{m} - 3.34\times 10^{-12}\ \text{m})}{2.43\times 10^{-12}\ \text{m}}\right) = 74.0°.$$

38.75. **(a) IDENTIFY** and **SET UP:** Conservation of energy applied to the collision gives $E_\lambda = E_{\lambda'} + E_e$, where E_e is the kinetic energy of the electron after the collision and E_λ and $E_{\lambda'}$ are the energies of the photon before and after the collision. The energy of a photon is related to its wavelength according to Eq.(38.2).

EXECUTE: $E_e = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right) = hc\left(\frac{\lambda' - \lambda}{\lambda\lambda'}\right)$

$$E_e = (6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})\left(\frac{0.0032\times10^{-9}\ \text{m}}{(0.1100\times10^{-9}\ \text{m})(0.1132\times10^{-9}\ \text{m})}\right)$$

$$E_e = 5.105\times10^{-17}\ \text{J} = 319\ \text{eV}$$

$$E_e = \frac{1}{2}mv^2 \text{ so } v = \sqrt{\frac{2E_e}{m}} = \sqrt{\frac{2(5.105\times10^{-17}\ \text{J})}{9.109\times10^{-31}\ \text{kg}}} = 1.06\times10^{7}\ \text{m/s}$$

(b) The wavelength λ of a photon with energy E_e is given by $E_e = hc/\lambda$ so

$$\lambda = \frac{hc}{E_e} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{5.105\times10^{-17}\ \text{J}} = 3.89\ \text{nm}$$

EVALUATE: Only a small portion of the incident photon's energy is transferred to the struck electron; this is why the wavelength calculated in part (b) is much larger than the wavelength of the incident photon in the Compton scattering.

38.76. **IDENTIFY:** Apply the Compton scattering formula $\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1-\cos\phi) = \lambda_c(1-\cos\phi)$

(a) SET UP: Largest $\Delta\lambda$ is for $\phi = 180°$.

EXECUTE: For $\phi = 180°$, $\Delta\lambda = 2\lambda_c = 2(2.426\ \text{pm}) = 4.85\ \text{pm}$.

(b) SET UP: $\lambda' - \lambda = \lambda_c(1-\cos\phi)$

Wavelength doubles implies $\lambda' = 2\lambda$ so $\lambda' - \lambda = \lambda$. Thus $\lambda = \lambda_C(1-\cos\phi)$. λ is related to E by Eq.(38.2).

EXECUTE: $E = hc/\lambda$, so smallest energy photon means largest wavelength photon, so $\phi = 180°$ and

$\lambda = 2\lambda_c = 4.85$ pm. Then $E = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\ \text{J}\cdot\text{s})(2.998\times10^{8}\ \text{m/s})}{4.85\times10^{-12}\ \text{m}} = 4.096\times10^{-14}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) =$ 0.256 MeV.

EVALUATE: Any photon Compton scattered at $\phi = 180°$ has a wavelength increase of $2\lambda_c = 4.85$ pm. 4.85 pm is near the short-wavelength end of the range of x-ray wavelengths.

38.77. **(a)** $I(\lambda) = \frac{2\pi hc^2}{\lambda^5(e^{hc/\lambda kT}-1)}$ but $\lambda = \frac{c}{f}$

$$\Rightarrow I(f) = \frac{2\pi hc^2}{(c/f)^5(e^{hf/kT}-1)} = \frac{2\pi hf^5}{c^3(e^{hf/kT}-1)}$$

(b) $\int_0^\infty I(\lambda)\,d\lambda = \int_\infty^0 I(f)\,df\left(\frac{-c}{f^2}\right)$

$$= \int_0^\infty \frac{2\pi hf^3\,df}{c^2(e^{hf/kT}-1)} = \frac{2\pi(kT)^4}{c^2h^3}\int_0^\infty \frac{x^3}{e^x-1}\,dx = \frac{2\pi(kT)^4}{c^2h^3}\frac{1}{240}(2\pi)^4 = \frac{(2\pi)^5(kT)^4}{240h^3c^2} = \frac{2\pi^5k^4T^4}{15c^2h^3}$$

(c) The expression $\frac{2\pi^5k^4T^4}{15h^3c^2} = \sigma$ as shown in Eq. (38.36). Plugging in the values for the constants we get $\sigma = 5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$.

38.78. $I = \sigma T^4, P = IA$, and $\Delta E = Pt$; combining,

$$t = \frac{\Delta E}{A\sigma T^4} = \frac{(100\ \text{J})}{(4.00\times10^{-6}\ \text{m}^2)(5.67\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4)(473\ \text{K})^4} = 8.81\times10^{3}\ \text{s} = 2.45\ \text{hrs}.$$

38.79. **(a)** The period was found in Exercise 38.27b: $T = \frac{4\epsilon_0^2n^3h^3}{me^4}$ and frequency is just $f = \frac{1}{T} = \frac{me^4}{4\epsilon_0^2n^3h^3}$.

(b) Eq. (38.6) tells us that $f = \frac{1}{h}(E_2 - E_1)$. So $f = \frac{me^4}{8\epsilon_0^2h^3}\left(\frac{1}{n_2^2} - \frac{1}{n_1^2}\right)$ (from Eq. (38.18)).

If $n_2 = n$ and $n_1 = n+1$, then $\frac{1}{n_2^2} - \frac{1}{n_1^2} = \frac{1}{n^2} - \frac{1}{(n+1)^2}$

$$= \frac{1}{n^2}\left(1 - \frac{1}{(1+1/n)^2}\right) \approx \frac{1}{n^2}\left(1 - \left(1 - \frac{2}{n} + \cdots\right)\right) = \frac{2}{n^3} \text{ for large } n \Rightarrow f \approx \frac{me^4}{4\epsilon_0^2n^3h^3}.$$

38.80. Each photon has momentum $p = \dfrac{h}{\lambda}$, and if the rate at which the photons strike the surface is (dN/dt), the force on the surface is $(h/\lambda)(dN/dt)$, and the pressure is $(h/\lambda)(dN/dt)/A$. The intensity is $I = (dN/dt)(E)/A = (dN/dt)(hc/\lambda)/A$, and comparison of the two expressions gives the pressure as (I/c).

38.81. Momentum: $\vec{\boldsymbol{p}} + \vec{\boldsymbol{P}} = \vec{\boldsymbol{p}}' + \vec{\boldsymbol{P}}' \Rightarrow p - P = -p' - P' \;\Rightarrow\; p' = P - (p + P')$

energy: $pc + E = p'c + E' = p'c + \sqrt{(P'c)^2 + (mc^2)^2}$

$$\Rightarrow (pc - p'c + E)^2 = (P'c)^2 + (mc^2)^2 = (Pc)^2 + ((p + p')c)^2 - 2P(p + p')c^2 + (mc^2)^2$$

$$(pc - p'c)^2 + E^2 = E^2 + (pc + p'c)^2 - 2(Pc^2)(p + p') + 2Ec(p - p') - 4pp'c^2 + 2Ec(p - p')$$

$$+2(Pc^2)(p + p') = 0$$

$$\Rightarrow p'(Pc^2 - 2pc^2 - Ec) = p(-Ec - Pc^2)$$

$$\Rightarrow p' = p\frac{Ec + Pc^2}{2pc^2 + Ec - Pc^2} = p\frac{E + Pc}{2pc + (E - Pc)}$$

$$\Rightarrow \lambda' = \lambda\left(\frac{2hc/\lambda + (E - Pc)}{E + Pc}\right) = \lambda\left(\frac{E - Pc}{E + Pc}\right) + \frac{2hc}{E + Pc}$$

$$\Rightarrow \lambda' = \frac{(\lambda(E - Pc) + 2hc)}{E + Pc}$$

If $E \gg mc^2, Pc = \sqrt{E^2 - (mc^2)^2} = E\sqrt{1 - \left(\dfrac{mc^2}{E}\right)^2} \approx E\left(1 - \dfrac{1}{2}\left(\dfrac{mc^2}{E}\right)^2 + \cdots\right)$

$$\Rightarrow E - Pc \approx \frac{1}{2}\frac{(mc^2)^2}{E} \;\Rightarrow \lambda_1 \approx \frac{\lambda(mc^2)^2}{2E(2E)} + \frac{hc}{E} = \frac{hc}{E}\left(1 + \frac{m^2c^4\lambda}{4hcE}\right)$$

(b) If $\lambda = 10.6 \times 10^{-6}$ m, $E = 1.00 \times 10^{10}$ eV $= 1.60 \times 10^{-9}$ J

$$\Rightarrow \lambda' \approx \frac{hc}{1.60 \times 10^{-9}\ \text{J}}\left(1 + \frac{(9.11 \times 10^{-31}\ \text{kg})^2 c^4 (10.6 \times 10^{-6}\ \text{m})}{4hc\,(1.6 \times 10^{-9}\ \text{J})}\right)$$

$$= (1.24 \times 10^{-16}\ \text{m})(1 + 56.0) = 7.08 \times 10^{-15}\ \text{m}.$$

(c) These photons are gamma rays. We have taken infrared radiation and converted it into gamma rays! Perhaps useful in nuclear medicine, nuclear spectroscopy, or high energy physics: wherever controlled gamma ray sources might be useful.