37

RELATIVITY

37.1. **IDENTIFY** and **SET UP:** Consider the distance *A* to O' and *B* to O' as observed by an observer on the ground (Figure 37.1).

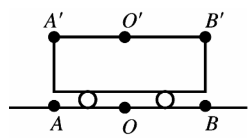


Figure 37.1

EXECUTE: Simultaneous to observer on train means light pulses from A' and B' arrive at O' at the same time. To observer at *O* light from A' has a longer distance to travel than light from B' so *O* will conclude that the pulse from $A(A')$ started before the pulse at $B(B')$. To observer at *O* bolt *A* appeared to strike first.

EVALUATE: Section 37.2 shows that if they are simultaneous to the observer on the ground then an observer on the train measures that the bolt at B' struck first.

37.2. **(a)** $\gamma = \dfrac{1}{\sqrt{1-(0.9)^2}} = 2.29.\quad t = \gamma\tau = (2.29)\,(2.20\times10^{-6}\text{s}) = 5.05\times10^{-6}\,\text{s}.$

(b) $d = vt = (0.900)\,(3.00\times10^8\text{ m/s})\,(5.05\times10^{-6}\text{s}) = 1.36\times10^3\text{ m} = 1.36\text{ km}.$

37.3. **IDENTIFY** and **SET UP:** The problem asks for *u* such that $\Delta t_0/\Delta t = \dfrac{1}{2}.$

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = (3.00\times10^8\text{ m/s})\sqrt{1-\left(\dfrac{1}{2}\right)^2} = 2.60\times10^8\text{ m/s}$; $\dfrac{u}{c} = 0.867$

Jet planes fly at less than ten times the speed of sound, less than about 3000 m/s. Jet planes fly at much lower speeds than we calculated for *u*.

37.4. **IDENTIFY:** Time dilation occurs because the rocket is moving relative to Mars.

SET UP: The time dilation equation is $\Delta t = \gamma\Delta t_0$, where t_0 is the proper time.

EXECUTE: **(a)** The two time measurements are made at the same place on Mars by an observer at rest there, so the observer on Mars measures the proper time.

(b) $\Delta t = \gamma\Delta t_0 = \dfrac{1}{\sqrt{1-(0.985)^2}}(75.0\ \mu\text{s}) = 435\ \mu\text{s}$

EVALUATE: The pulse lasts for a shorter time relative to the rocket than it does relative to the Mars observer.

37.5. **(a) IDENTIFY** and **SET UP:** $\Delta t_0 = 2.60\times10^{-8}$ s; $\Delta t = 4.20\times10^{-7}$ s. In the lab frame the pion is created and decays at different points, so this time is not the proper time.

EXECUTE: $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ says $1-\dfrac{u^2}{c^2} = \left(\dfrac{\Delta t_0}{\Delta t}\right)^2$

$$\frac{u}{c} = \sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1-\left(\frac{2.60\times10^{-8}\text{ s}}{4.20\times10^{-7}\text{ s}}\right)^2} = 0.998;\ u = 0.998c$$

EVALUATE: $u < c$, as it must be, but *u*/*c* is close to unity and the time dilation effects are large.

(b) IDENTIFY and **SET UP:** The speed in the laboratory frame is $u = 0.998c$; the time measured in this frame is Δt, so the distance as measured in this frame is $d = u\Delta t$

EXECUTE: $d = (0.998)(2.998\times10^8\text{ m/s})(4.20\times10^{-7}\text{ s}) = 126\text{ m}$

EVALUATE: The distance measured in the pion's frame will be different because the time measured in the pion's frame is different (shorter).

37.6. $\gamma = 1.667$

(a) $\Delta t_0 = \dfrac{\Delta t}{\gamma} = \dfrac{1.20\times 10^8 \text{ m}}{\gamma(0.800c)} = 0.300 \text{ s}.$

(b) $(0.300 \text{ s})(0.800c) = 7.20\times 10^7 \text{m}.$

(c) $\Delta t_0 = 0.300 \text{ s}/\gamma = 0.180 \text{ s}.$ (This is what the *racer* measures *your* clock to read at that instant.) At *your* origin you read the original $\dfrac{1.20\times 10^8 \text{m}}{(0.800)(3\times 10^8 \text{ m/s})} = 0.5 \text{ s}.$ Clearly the observers (you and the racer) will not agree on the order of events!

37.7. **IDENTIFY** and **SET UP:** A clock moving with respect to an observer appears to run more slowly than a clock at rest in the observer's frame. The clock in the spacecraft measurers the proper time Δt_0. $\Delta t = 365 \text{ days} = 8760 \text{ hours}.$

EXECUTE: The clock on the moving spacecraft runs slow and shows the smaller elapsed time.
$\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (8760 \text{ h})\sqrt{1-(4.80\times 10^6/3.00\times 10^8)^2} = 8758.88 \text{ h}$. The difference in elapsed times is $8760 \text{ h} - 8758.88 \text{ h} = 1.12 \text{ h}$.

37.8. **IDENTIFY** and **SET UP:** The proper time is measured in the frame where the two events occur at the same point.
EXECUTE: **(a)** The time of 12.0 ms measured by the first officer on the craft is the proper time.

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ gives $u = c\sqrt{1-(\Delta t_0/\Delta t)^2} = c\sqrt{1-(12.0\times 10^{-3}/0.190)^2} = 0.998c.$

EVALUATE: The observer at rest with respect to the searchlight measures a much shorter duration for the event.

37.9. **IDENTIFY** and **SET UP:** $l = l_0\sqrt{1-u^2/c^2}$. The length measured when the spacecraft is moving is $l = 74.0$ m; l_0 is the length measured in a frame at rest relative to the spacecraft.

EXECUTE: $l_0 = \dfrac{l}{\sqrt{1-u^2/c^2}} = \dfrac{74.0 \text{ m}}{\sqrt{1-(0.600c/c)^2}} = 92.5 \text{ m}.$

EVALUATE: $l_0 > l$. The moving spacecraft appears to an observer on the planet to be shortened along the direction of motion.

37.10. **IDENTIFY** and **SET UP:** When the meterstick is at rest with respect to you, you measure its length to be 1.000 m, and that is its proper length, l_0. $l = 0.3048$ m.

EXECUTE: $l = l_0\sqrt{1-u^2/c^2}$ gives $u = c\sqrt{1-(l/l_0)^2} = c\sqrt{1-(0.3048/1.00)^2} = 0.9524c = 2.86\times 10^8 \text{ m/s}.$

37.11. **IDENTIFY** and **SET UP:** The 2.2 μs lifetime is Δt_0 and the observer on earth measures Δt. The atmosphere is moving relative to the muon so in its frame the height of the atmosphere is l and l_0 is 10 km.

EXECUTE: **(a)** The greatest speed the muon can have is c, so the greatest distance it can travel in 2.2×10^{-6} s is $d = vt = (3.00\times 10^8 \text{ m/s})(2.2\times 10^{-6} \text{ s}) = 660 \text{ m} = 0.66 \text{ km}.$

(b) $\Delta t = \dfrac{\Delta t_0}{\sqrt{1-u^2/c^2}} = \dfrac{2.2\times 10^{-6} \text{ s}}{\sqrt{1-(0.999)^2}} = 4.9\times 10^{-5} \text{ s}$

$d = vt = (0.999)(3.00\times 10^8 \text{ m/s})(4.9\times 10^{-5} \text{ s}) = 15 \text{ km}$

In the frame of the earth the muon can travel 15 km in the atmosphere during its lifetime.

(c) $l = l_0\sqrt{1-u^2/c^2} = (10 \text{ km})\sqrt{1-(0.999)^2} = 0.45 \text{ km}$

In the frame of the muon the height of the atmosphere is less than the distance it moves during its lifetime.

37.12. **IDENTIFY** and **SET UP:** The scientist at rest on the earth's surface measures the proper length of the separation between the point where the particle is created and the surface of the earth, so $l_0 = 45.0$ km. The transit time measured in the particle's frame is the proper time, Δt_0.

EXECUTE: **(a)** $t = \dfrac{l_0}{v} = \dfrac{45.0\times 10^3 \text{ m}}{(0.99540)(3.00\times 10^8 \text{ m/s})} = 1.51\times 10^{-4} \text{ s}$

(b) $l = l_0\sqrt{1-u^2/c^2} = (45.0 \text{ km})\sqrt{1-(0.99540)^2} = 4.31 \text{ km}$

(c) time dilation formula: $\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = (1.51\times 10^{-4} \text{ s})\sqrt{1-(0.99540)^2} = 1.44\times 10^{-5} \text{ s}$

from Δl: $t = \dfrac{l}{v} = \dfrac{4.31\times 10^3 \text{ m}}{(0.99540)(3.00\times 10^8 \text{ m/s})} = 1.44\times 10^{-5} \text{ s}$

The two results agree.

37.13. **(a)** $l_0 = 3600$ m.

$$l = l_0\sqrt{1-\frac{u^2}{c^2}} = l_0(3600 \text{ m})\sqrt{1-\frac{(4.00\times 10^7 \text{ m/s})^2}{(3.00\times 10^8 \text{ m/s})^2}} = (3600 \text{ m})(0.991) = 3568 \text{ m}.$$

(b) $\Delta t_0 = \dfrac{l_0}{u} = \dfrac{3600\text{ m}}{4.00\times 10^7\text{ m/s}} = 9.00\times 10^{-5}\text{ s.}$

(c) $\Delta t = \dfrac{l}{u} = \dfrac{3568\text{ m}}{4.00\times 10^7\text{ m/s}} = 8.92\times 10^{-5}\text{ s.}$

37.14. Multiplying the last equation of (37.21) by u and adding to the first to eliminate t gives

$$x' + ut' = \gamma x\left(1-\frac{u^2}{c^2}\right) = \frac{1}{\gamma}x,$$

and multiplying the first by $\dfrac{u}{c^2}$ and adding to the last to eliminate x gives

$$t' + \frac{u}{c^2}x' = \gamma t\left(1-\frac{u^2}{c^2}\right) = \frac{1}{\gamma}t,$$

so $x = \gamma(x' + ut')$ and $t = \gamma(t' + ux'/c^2)$, which is indeed the same as Eq. (37.21) with the primed coordinates replacing the unprimed, and a change of sign of u.

37.15. **(a)** $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.400c + 0.600c}{1+(0.400)\,(0.600)} = 0.806c$

(b) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.900c + 0.600c}{1+(0.900)(0.600)} = 0.974c$

(c) $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.990c + 0.600c}{1+(0.990)(0.600)} = 0.997c.$

37.16. $\gamma = 1.667(\gamma = 5/3 \text{ if } u = (4/5)c).$

(a) In Mavis's frame the event "light on" has space-time coordinates $x' = 0$ and $t' = 5.00$ s, so from the result of Exercise 37.14 or Example 37.7, $x = \gamma(x' + ut')$ and $t = \gamma\left(t' + \dfrac{ux'}{c^2}\right) \Rightarrow x = \gamma ut' = 2.00\times 10^9\text{ m}, t = \gamma t' = 8.33\text{ s}$.

(b) The 5.00-s interval in Mavis's frame is the proper time Δt_0 in Eq.(37.6), so $\Delta t = \gamma\Delta t_0 = 8.33$ s, as in part (a).

(c) $(8.33\text{ s})\,(0.800c) = 2.00\times 10^9$ m, which is the distance x found in part (a).

37.17. **IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v_x' = \dfrac{v_x - u}{1-\dfrac{uv_x}{c^2}}$.

EXECUTE: **(a)** For the pursuit ship to catch the cruiser, the distance between them must be decreasing, so the velocity of the cruiser relative to the pursuit ship must be directed toward the pursuit ship.

(b) Let the unprimed frame be Tatooine and let the primed frame be the pursuit ship. We want the velocity v' of the cruiser knowing the velocity of the primed frame u and the velocity of the cruiser v in the unprimed frame (Tatooine).

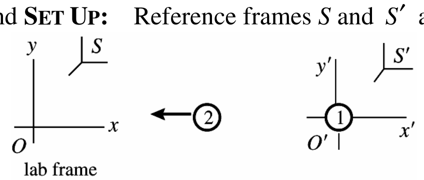
$$v_x' = \frac{v_x - u}{1-\dfrac{uv_x}{c^2}} = \frac{0.600c - 0.800c}{1-(0.600)\,(0.800)} = -0.385c$$

The result implies that the cruiser is moving toward the pursuit ship at $0.385c$.

EVALUATE: The nonrelativistic formula would have given $-0.200c$, which is considerably different from the correct result.

37.18. Let u_y be the y-component of the velocity of S' relative to S. Following the steps used in the derivation of Eq.(37.23) we get $v_y = \dfrac{v_y' + u_y}{1+u_y v_y'/c^2}$.

37.19. **IDENTIFY** and **SET UP:** Reference frames S and S' are shown in Figure 37.19.



Frame S is at rest in the laboratory. Frame S' is attached to particle 1.

Figure 37.19

u is the speed of S' relative to S; this is the speed of particle 1 as measured in the laboratory. Thus $u = +0.650c$. The speed of particle 2 in S' is $0.950c$. Also, since the two particles move in opposite directions, 2 moves in the $-x'$ direction and $v_x' = -0.950c$. We want to calculate v_x, the speed of particle 2 in frame S; use Eq.(37.23).

EXECUTE: $v_x = \dfrac{v'_x + u}{1 + uv'_x/c^2} = \dfrac{-0.950c + 0.650c}{1 + (0.950c)(-0.650c)/c^2} = \dfrac{-0.300c}{1 - 0.6175} = -0.784c$. The speed of the second particle, as measured in the laboratory, is $0.784c$.

EVALUATE: The incorrect Galilean expression for the relative velocity gives that the speed of the second particle in the lab frame is $0.300c$. The correct relativistic calculation gives a result more than twice this.

37.20. **IDENTIFY** and **SET UP:** Let S be the laboratory frame and let S' be the frame of one of the particles, as shown in Figure 37.20. Let the positive x direction for both frames be from particle 1 to particle 2. In the lab frame particle 1 is moving in the $+x$ direction and particle 2 is moving in the $-x$ direction. Then $u = 0.9520c$ and $v = -0.9520c$. v' is the velocity of particle 2 relative to particle 1.

EXECUTE: $v' = \dfrac{v - u}{1 - uv/c^2} = \dfrac{-0.9520c - 0.9520c}{1 - (0.9520c)(-0.9520c)/c^2} = -0.9988c$. The speed of particle 2 relative to particle 1 is $0.9988c$. $v' < 0$ shows particle 2 is moving toward particle 1.

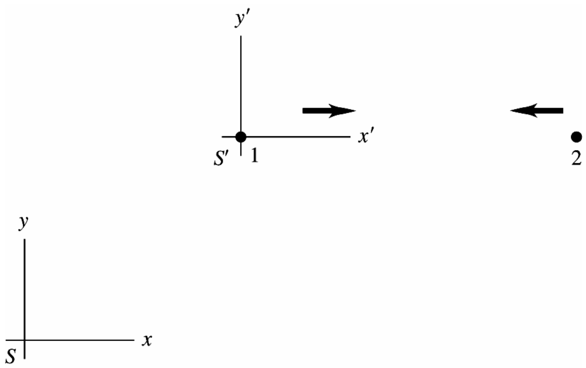


Figure 37.20

37.21. **IDENTIFY:** The relativistic velocity addition formulas apply since the speeds are close to that of light.

SET UP: The relativistic velocity addition formula is $v'_x = \dfrac{v_x - u}{1 - \dfrac{uv_x}{c^2}}$.

EXECUTE: In the relativistic velocity addition formula for this case, v'_x is the relative speed of particle 1 with respect to particle 2, v is the speed of particle 2 measured in the laboratory, and u is the speed of particle 1 measured in the laboratory, $u = -v$.

$v'_x = \dfrac{v - (-v)}{1 - (-v)v/c^2} = \dfrac{2v}{1 + v^2/c^2}$. $\dfrac{v'_x}{c^2}v^2 - 2v + v'_x = 0$ and $(0.890c)v^2 - 2c^2v + (0.890c^3) = 0$.

This is a quadratic equation with solution $v = 0.611c$ (v must be less than c).

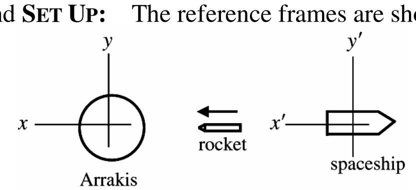
EVALUATE: The nonrelativistic result would be $0.445c$, which is considerably different from this result.

37.22. **IDENTIFY** and **SET UP:** Let the starfighter's frame be S and let the enemy spaceship's frame be S'. Let the positive x direction for both frames be from the enemy spaceship toward the starfighter. Then $u = +0.400c$. $v' = +0.700c$. v is the velocity of the missile relative to you.

EXECUTE: **(a)** $v = \dfrac{v' + u}{1 + uv'/c^2} = \dfrac{0.700c + 0.400c}{1 + (0.400)(0.700)} = 0.859c$

(b) Use the distance it moves as measured in your frame and the speed it has in your frame to calculate the time it takes in your frame. $t = \dfrac{8.00 \times 10^9 \text{ m}}{(0.859)(3.00 \times 10^8 \text{ m/s})} = 31.0 \text{ s}$.

37.23. **IDENTIFY** and **SET UP:** The reference frames are shown in Figure 37.23.



S = Arrakis frame
S' = spaceship frame
The object is the rocket.

Figure 37.23

u is the velocity of the spaceship relative to Arrakis.

$v_x = +0.360c$; $v'_x = +0.920c$

(In each frame the rocket is moving in the positive coordinate direction.)

Use the Lorentz velocity transformation equation, Eq.(37.22): $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$.

EXECUTE: $v'_x = \dfrac{v_x - u}{1 - uv_x/c^2}$ so $v'_x - u\left(\dfrac{v_x v'_x}{c^2}\right) = v_x - u$ and $u\left(1 - \dfrac{v_x v'_x}{c^2}\right) = v_x - v'_x$

$$u = \frac{v_x - v'_x}{1 - v_x v'_x/c^2} = \frac{0.360c - 0.920c}{1 - (0.360c)(0.920c)/c^2} = -\frac{0.560c}{0.6688} = -0.837c$$

The speed of the spacecraft relative to Arrakis is $0.837c = 2.51\times10^8$ m/s. The minus sign in our result for u means that the spacecraft is moving in the $-x$-direction, so it is moving away from Arrakis.

EVALUATE: The incorrect Galilean expression also says that the spacecraft is moving away from Arrakis, but with speed $0.920c - 0.360c = 0.560c$.

37.24. **IDENTIFY:** We need to use the relativistic Doppler shift formula.

SET UP: The relativistic Doppler shift formula, Eq.(37.25), is $f = \sqrt{\dfrac{c+u}{c-u}}f_0$.

EXECUTE: $f^2 = \dfrac{c+u}{c-u}f_0^2$. $(c-u)f^2 = (c+u)f_0^2$. $cf^2 - uf^2 = cf_0^2 + uf_0^2$. $cf^2 - cf_0^2 = uf^2 + uf_0^2$ and

$$u = \frac{c(f^2 - f_0^2)}{f^2 + f_0^2} = \frac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1}c.$$

(a) For $f/f_0 = 0.95$, $u = -0.051c$ moving away from the source.
(b) For $f/f_0 = 5.0$, $u = 0.923c$ moving towards the source.

EVALUATE: Note that the speed required to achieve a 10 times greater Doppler shift is not 10 times the original speed.

37.25. **IDENTIFY** and **SET UP:** Source and observer are approaching, so use Eq.(37.25): $f = \sqrt{\dfrac{c+u}{c-u}}f_0$. Solve for u, the speed of the light source relative to the observer.

(a) EXECUTE: $f^2 = \left(\dfrac{c+u}{c-u}\right)f_0^2$

$(c-u)f^2 = (c+u)f_0^2$ and $u = \dfrac{c(f^2 - f_0^2)}{f^2 + f_0^2} = c\left(\dfrac{(f/f_0)^2 - 1}{(f/f_0)^2 + 1}\right)$

$\lambda_0 = 675$ nm, $\lambda = 575$ nm

$u = \left(\dfrac{(675 \text{ nm}/575 \text{ nm})^2 - 1}{(675 \text{ nm}/575 \text{ nm})^2 + 1}\right)c = 0.159c = (0.159)(2.998\times10^8 \text{ m/s}) = 4.77\times10^7$ m/s; definitely speeding

(b) 4.77×10^7 m/s $= (4.77\times10^7$ m/s$)(1$ km/1000 m$)(3600$ s/1 h$) = 1.72\times10^8$ km/h. Your fine would be $\$1.72\times10^8$ (172 million dollars).

EVALUATE: The source and observer are approaching, so $f > f_0$ and $\lambda < \lambda_0$. Our result gives $u < c$, as it must.

37.26. Using $u = -0.600c = -(3/5)c$ in Eq.(37.25) gives

$$f = \sqrt{\frac{1-(3/5)}{1+(3/5)}}f_0 = \sqrt{\frac{2/5}{8/5}}f_0 = f_0/2.$$

37.27. **IDENTIFY** and **SET UP:** If $\vec{F}$ is parallel to $\vec{v}$ then $\vec{F}$ changes the magnitude of $\vec{v}$ and not its direction.

$$F = \frac{dp}{dt} = \frac{d}{dt}\left(\frac{mv}{\sqrt{1 - v^2/c^2}}\right)$$

Use the chain rule to evaluate the derivative: $\dfrac{d}{dt}f(v(t)) = \dfrac{df}{dv}\dfrac{dv}{dt}$.

EXECUTE: **(a)** $F = \dfrac{m}{(1 - v^2/c^2)^{1/2}}\left(\dfrac{dv}{dt}\right) + \dfrac{mv}{(1 - v^2/c^2)^{3/2}}\left(-\dfrac{1}{2}\right)\left(-\dfrac{2v}{c^2}\right)\left(\dfrac{dv}{dt}\right)$

$$F = \frac{dv}{dt}\frac{m}{(1 - v^2/c^2)^{3/2}}\left(1 - \frac{v^2}{c^2} + \frac{v^2}{c^2}\right) = \frac{dv}{dt}\frac{m}{(1 - v^2/c^2)^{3/2}}$$

But $\dfrac{dv}{dt} = a$, so $a = (F/m)(1 - v^2/c^2)^{3/2}$.

EVALUATE: Our result agrees with Eq.(37.30).

(b) IDENTIFY and **SET UP:** If $\vec{F}$ is perpendicular to $\vec{v}$ then $\vec{F}$ changes the direction of $\vec{v}$ and not its magnitude.

$\vec{F} = \dfrac{d}{dt}\left(\dfrac{m\vec{v}}{\sqrt{1-v^2/c^2}}\right)$.

$\vec{a} = d\vec{v}/dt$ but the magnitude of v in the denominator of Eq.(37.29) is constant.

EXECUTE: $F = \dfrac{ma}{\sqrt{1-v^2/c^2}}$ and $a = (F/m)(1-v^2/c^2)^{1/2}$.

EVALUATE: This result agrees with Eq.(37.33).

37.28. **IDENTIFY** and **SET UP:** $\gamma = \dfrac{1}{\sqrt{1-v^2/c^2}}$. If γ is 1.0% greater than 1 then $\gamma = 1.010$, if γ is 10% greater than 1 then $\gamma = 1.10$ and if γ is 100% greater than 1 then $\gamma = 2.00$.

EXECUTE: $v = c\sqrt{1-1/\gamma^2}$

(a) $v = c\sqrt{1-1/(1.010)^2} = 0.140c$

(b) $v = c\sqrt{1-1/(1.10)^2} = 0.417c$

(c) $v = c\sqrt{1-1/(2.00)^2} = 0.866c$

37.29. **(a)** $p = \dfrac{mv}{\sqrt{1-v^2/c^2}} = 2mv$.

$$\Rightarrow 1 = 2\sqrt{1-v^2/c^2} \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v^2 = \frac{3}{4}c^2 \Rightarrow v = \frac{\sqrt{3}}{2}c = 0.866c.$$

(b) $F = \gamma^3 ma = 2ma \Rightarrow \gamma^3 = 2 \Rightarrow \gamma = (2)^{1/3}$ so $\dfrac{1}{1-\dfrac{v^2}{c^2}} = 2^{2/3} \Rightarrow \dfrac{v}{c} = \sqrt{1-2^{-2/3}} = 0.608$

37.30. The force is found from Eq.(37.32) or Eq.(37.33).

(a) Indistinguishable from $F = ma = 0.145$ N.

(b) $\gamma^3 ma = 1.75$ N.

(c) $\gamma^3 ma = 51.7$ N.

(d) $\gamma ma = 0.145$ N, 0.333 N, 1.03 N.

37.31. **(a)** $K = \dfrac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = mc^2$

$$\Rightarrow \frac{1}{\sqrt{1-v^2/c^2}} = 2 \Rightarrow \frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow v = \sqrt{\frac{3}{4}}c = 0.866c.$$

(b) $K = 5mc^2 \Rightarrow \dfrac{1}{\sqrt{1-v^2/c^2}} = 6 \Rightarrow \dfrac{1}{36} = 1 - \dfrac{v^2}{c^2} \Rightarrow v = \sqrt{\dfrac{35}{36}}c = 0.986c.$

37.32. $E = 2mc^2 = 2(1.67\times10^{-27}\,\text{kg})(3.00\times10^8\,\text{m/s})^2 = 3.01\times10^{-10}\,\text{J} = 1.88\times10^9\,\text{eV}$.

37.33. **IDENTIFY** and **SET UP:** Use Eqs.(37.38) and (37.39).

EXECUTE: (a) $E = mc^2 + K$, so $E = 4.00mc^2$ means $K = 3.00mc^2 = 4.50\times10^{-10}$ J

(b) $E^2 = (mc^2)^2 + (pc)^2$; $E = 4.00mc^2$, so $15.0(mc^2)^2 = (pc)^2$

$p = \sqrt{15}mc = 1.94\times10^{-18}$ kg·m/s

(c) $E = mc^2/\sqrt{1-v^2/c^2}$

$E = 4.00mc^2$ gives $1 - v^2/c^2 = 1/16$ and $v = \sqrt{15/16}c = 0.968c$

EVALUATE: The speed is close to c since the kinetic energy is greater than the rest energy. Nonrelativistic expressions relating E, K, p and v will be very inaccurate.

37.34. **(a)** $W = \Delta K = (\gamma_f - 1)mc^2 = (4.07\times10^{-3})\,mc^2$.

(b) $(\gamma_f - \gamma_i)mc^2 = 4.79mc^2$.

(c) The result of part (b) is far larger than that of part (a).

37.35. **IDENTIFY:** Use $E = mc^2$ to relate the mass increase to the energy increase.

(a) SET UP: Your total energy E increases because your gravitational potential energy mgy increases.

EXECUTE: $\Delta E = mg\Delta y$

$\Delta E = (\Delta m)c^2$ so $\Delta m = \Delta E / c^2 = mg(\Delta y)/c^2$

$\Delta m / m = (g\Delta y)/c^2 = (9.80\text{ m/s}^2)(30\text{ m})/(2.998\times 10^8\text{ m/s})^2 = 3.3\times 10^{-13}\%$

This increase is much, much too small to be noticed.

(b) SET UP: The energy increases because potential energy is stored in the compressed spring.

EXECUTE: $\Delta E = \Delta U = \frac{1}{2}kx^2 = \frac{1}{2}(2.00\times 10^4\text{ N/m})(0.060\text{ m})^2 = 36.0\text{ J}$

$\Delta m = (\Delta E)/c^2 = 4.0\times 10^{-16}\text{ kg}$

Energy increases so mass increases. The mass increase is much, much too small to be noticed.

EVALUATE: In both cases the energy increase corresponds to a mass increase. But since c^2 is a very large number the mass increase is very small.

37.36. **(a)** $E_0 = m_0c^2$. $2E = mc^2 = 2m_0c^2$. Therefore, $m = 2m_0 \Rightarrow \dfrac{m_0}{\sqrt{1-v^2/c^2}} = 2m_0$.

$$\frac{1}{4} = 1 - \frac{v^2}{c^2} \Rightarrow \frac{v^2}{c^2} = \frac{3}{4} \Rightarrow v = c\sqrt{3/4} = 0.866c = 2.60\times 10^8\text{ m/s}$$

(b) $10\, m_0c^2 = mc^2 = \dfrac{m_0}{\sqrt{1-v^2/c^2}}c^2$.

$$1 - \frac{v^2}{c^2} = \frac{1}{100} \Rightarrow \frac{v^2}{c^2} = \frac{99}{100}.\quad v = c\sqrt{\frac{99}{100}} = 0.995c = 2.98\times 10^8\text{ m/s}.$$

37.37. **IDENTIFY** and **SET UP:** The energy equivalent of mass is $E = mc^2$. $\rho = 7.86\text{ g/cm}^3 = 7.86\times 10^3\text{ kg/m}^3$. For a cube, $V = L^3$.

EXECUTE: **(a)** $m = \dfrac{E}{c^2} = \dfrac{1.0\times 10^{20}\text{ J}}{(3.00\times 10^8\text{ m/s})^2} = 1.11\times 10^3\text{ kg}$

(b) $\rho = \dfrac{m}{V}$ so $V = \dfrac{m}{\rho} = \dfrac{1.11\times 10^3\text{ kg}}{7.86\times 10^3\text{ kg/m}^3} = 0.141\text{ m}^3$. $L = V^{1/3} = 0.521\text{ m} = 52.1\text{ cm}$

EVALUATE: Particle/antiparticle annihilation has been observed in the laboratory, but only with small quantities of antimatter.

37.38. $(5.52\times 10^{-27}\text{ kg})(3.00\times 10^8\text{ m/s})^2 = 4.97\times 10^{-10}\text{ J} = 3105\text{ MeV}$.

37.39. **IDENTIFY** and **SET UP:** The total energy is given in terms of the momentum by Eq.(37.39). In terms of the total energy *E*, the kinetic energy *K* is $K = E - mc^2$ (from Eq.37.38). The rest energy is mc^2.

EXECUTE: **(a)** $E = \sqrt{(mc^2)^2 + (pc)^2} =$

$\sqrt{[(6.64\times 10^{-27})(2.998\times 10^8)^2]^2 + [(2.10\times 10^{-18})(2.998\times 10^8)]^2}\text{ J}$

$E = 8.67\times 10^{-10}\text{ J}$

(b) $mc^2 = (6.64\times 10^{-27}\text{ kg})(2.998\times 10^8\text{ m/s})^2 = 5.97\times 10^{-10}\text{ J}$

$K = E - mc^2 = 8.67\times 10^{-10}\text{ J} - 5.97\times 10^{-10}\text{ J} = 2.70\times 10^{-10}\text{ J}$

(c) $\dfrac{K}{mc^2} = \dfrac{2.70\times 10^{-10}\text{ J}}{5.97\times 10^{-10}\text{ J}} = 0.452$

EVALUATE: The incorrect nonrelativistic expressions for *K* and *p* give $K = p^2/2m = 3.3\times 10^{-10}\text{ J}$; the correct relativistic value is less than this.

37.40. $E = (m^2c^4 + p^2c^2)^{1/2} = mc^2\left(1 + \left(\dfrac{p}{mc}\right)^2\right)^{1/2}$

$E \approx mc^2\left(1 + \dfrac{1}{2}\dfrac{p^2}{m^2c^2}\right) = mc^2 + \dfrac{p^2}{2m} = mc^2 + \dfrac{1}{2}mv^2$**,** the sum of the rest mass energy and the classical kinetic energy.

37.41. **(a)** $v = 8\times 10^7\text{ m/s} \Rightarrow \gamma = \dfrac{1}{\sqrt{1-v^2/c^2}} = 1.0376$. For $m = m_p$, $K_{\text{nonrel}} = \dfrac{1}{2}mv^2 = 5.34\times 10^{-12}\text{ J}$.

$K_{\text{rel}} = (\gamma - 1)mc^2 = 5.65\times 10^{-12}\text{ J}$. $\dfrac{K_{\text{rel}}}{K_{\text{nonrel}}} = 1.06$.

(b) $v = 2.85\times 10^8\text{ m/s}$; $\gamma = 3.203$.

$K_{\text{rel}} = \dfrac{1}{2}mv^2 = 6.78\times 10^{-11}\text{ J}$; $K_{\text{rel}} = (\gamma - 1)mc^2 = 3.31\times 10^{-10}\text{ J}$; $K_{\text{rel}}/K_{\text{nonrel}} = 4.88$.

37.42. **IDENTIFY:** Since the speeds involved are close to that of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic kinetic energy is $K=(\gamma-1)mc^2=\left(\dfrac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2$.

(a) $K=(\gamma-1)mc^2=\left(\dfrac{1}{\sqrt{1-v^2/c^2}}-1\right)mc^2=(1.67\times10^{-27}\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\dfrac{1}{\sqrt{1-(0.100c/c)^2}}-1\right)$

$K=(1.50\times10^{-10}\text{ J})\left(\dfrac{1}{\sqrt{1-0.0100}}-1\right)=7.56\times10^{-13}\text{ J}=4.73\text{ MeV}$

(b) $K=(1.50\times10^{-10}\text{J})\left(\dfrac{1}{\sqrt{1-(0.500)^2}}-1\right)=2.32\times10^{-11}\text{ J}=145\text{ MeV}$

(c) $K=(1.50\times10^{-10}\text{ J})\left(\dfrac{1}{\sqrt{1-(0.900)^2}}-1\right)=1.94\times10^{-10}\text{ J}=1210\text{ MeV}$

(d) $\Delta E=2.32\times10^{-11}\text{ J}-7.56\times10^{-13}\text{ J}=2.24\times10^{-11}\text{ J}=140\text{ MeV}$

(e) $\Delta E=1.94\times10^{-10}\text{ J}-2.32\times10^{-11}\text{ J}=1.71\times10^{-10}\text{ J}=1070\text{ MeV}$

(f) Without relativity, $K=\frac{1}{2}mv^2$. The work done in accelerating a proton from $0.100c$ to $0.500c$ in the nonrelativistic limit is $\Delta E=\frac{1}{2}m(0.500c)^2-\frac{1}{2}m(0.100c)^2=1.81\times10^{-11}\text{ J}=113\text{ MeV}$.

The work done in accelerating a proton from $0.500c$ to $0.900c$ in the nonrelativistic limit is

$$\Delta E=\frac{1}{2}m(0.900c)^2-\frac{1}{2}m(0.500c)^2=4.21\times10^{-11}\text{ J}=263\text{ MeV}.$$

EVALUATE: We see in the first case the nonrelativistic result is within 20% of the relativistic result. In the second case, the nonrelativistic result is very different from the relativistic result since the velocities are closer to c.

37.43. **IDENTIFY** and **SET UP:** Use Eq.(23.12) and conservation of energy to relate the potential difference to the kinetic energy gained by the electron. Use Eq.(37.36) to calculate the kinetic energy from the speed.

EXECUTE: **(a)** $K=q\Delta V=e\Delta V$

$K=mc^2\left(\dfrac{1}{\sqrt{1-v^2/c^2}}-1\right)=4.025mc^2=3.295\times10^{-13}\text{ J}=2.06\text{ MeV}$

$\Delta V=K/e=2.06\times10^6\text{ V}$

(b) From part (a), $K=3.30\times10^{-13}\text{ J}=2.06\text{ MeV}$

EVALUATE: The speed is close to c and the kinetic energy is four times the rest mass.

37.44. (a) According to Eq.(37.38) and conservation of mass-energy

$$2Mc^2+mc^2=\gamma 2Mc^2\Rightarrow\gamma=1+\frac{m}{2M}=1+\frac{9.75}{2(16.7)}=1.292.$$

Note that since $\gamma=\dfrac{1}{\sqrt{1-v^2/c^2}}$, we have that $\dfrac{v}{c}=\sqrt{1-\dfrac{1}{\gamma^2}}=\sqrt{1-\dfrac{1}{(1.292)^2}}=0.6331$.

(b) According to Eq.(37.36), the kinetic energy of each proton is

$K=(\gamma-1)Mc^2=(1.292-1)(1.67\times10^{-27}\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\dfrac{1.00\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)=274\text{ MeV}.$

(c) The rest energy of η^0 is $mc^2=(9.75\times10^{-28}\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\dfrac{1.00\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right)=548\text{ MeV}.$

(d) The kinetic energy lost by the protons is the energy that produces the η^0,

$$548\text{ MeV}=2(274\text{ MeV}).$$

37.45. **IDENTIFY:** The relativistic expression for the kinetic energy is $K=(\gamma-1)mc^2$, where $\gamma=\dfrac{1}{\sqrt{1-x}}$ and $x=v^2/c^2$.

The Newtonian expression for the kinetic energy is $K_{\text{N}}=\frac{1}{2}mv^2$.

SET UP: Solve for v such that $K=\frac{3}{2}K_{\text{N}}$.

EXECUTE: $(\gamma-1)mc^2=\frac{3}{4}mv^2$. $\frac{1}{\sqrt{1-x}}-1=\frac{3}{4}x$. $\frac{1}{1-x}=\left(1+\frac{3}{4}x\right)^2$. After a little algebra this becomes $9x^2+15x-8=0$. $x=\frac{1}{18}\left(-15\pm\sqrt{(15)^2+4(9)(8)}\right)$. The positive root is $x=0.425$. $x=v^2/c^2$, so $v=\sqrt{x}\ c=0.652c$.

EVALUATE: The fractional increase of the relativistic expression above the nonrelativistic one increases as v increases.

37.46. The fraction of the initial mass (a) that becomes energy is $1-\frac{(4.0015\text{ u})}{2(2.0136\text{ u})}=6.382\times10^{-3}$, and so the energy released per kilogram is $(6.382\times10^{-3})(1.00\text{ kg})(3.00\times10^{8}\text{ m/s})^2=5.74\times10^{14}\text{ J}$.

(b) $\frac{1.0\times10^{19}\text{ J}}{5.74\times10^{14}\text{ J/kg}}=1.7\times10^{4}\text{ kg}$.

37.47. **(a)** $E=mc^2$, $m=E/c^2=(3.8\times10^{26}\text{ J})/(2.998\times10^{8}\text{ m/s})^2=4.2\times10^{9}\text{ kg}$.

1 kg is equivalent to 2.2 lbs, so $m=4.6\times10^{6}$ tons

(b) The current mass of the sun is 1.99×10^{30} kg, so it would take it $(1.99\times10^{30}\text{ kg})/(4.2\times10^{9}\text{ kg/s})=4.7\times10^{20}\text{s}=1.5\times10^{13}$ years to use up all its mass.

37.48. **IDENTIFY:** Since the final speed is close to the speed of light, there will be a considerable difference between the relativistic and nonrelativistic results.

SET UP: The nonrelativistic work-energy theorem is $F\Delta x=\frac{1}{2}mv^2-\frac{1}{2}mv_0^2$, and the relativistic formula for a constant force is $F\Delta x=(\gamma-1)mc^2$.

(a) Using the classical work-energy theorem and solving for Δx, we obtain

$$\Delta x=\frac{m(v^2-v_0^2)}{2F}=\frac{(0.100\times10^{-9}\text{ kg})[(0.900)(3.00\times10^{8}\text{ m/s})]^2}{2(1.00\times10^{6}\text{ N})}=3.65\text{ m}.$$

(b) Using the relativistic work-energy theorem for a constant force, we obtain

$$\Delta x=\frac{(\gamma-1)mc^2}{F}.$$

For the given speed, $\gamma=\frac{1}{\sqrt{1-0.900^2}}=2.29$, thus

$$\Delta x=\frac{(2.29-1)(0.100\times10^{-9}\text{ kg})(3.00\times10^{8}\text{ m/s})^2}{(1.00\times10^{6}\text{N})}=11.6\text{ m}.$$

EVALUATE: **(c)** The distance obtained from the relativistic treatment is greater. As we have seen, more energy is required to accelerate an object to speeds close to c, so that force must act over a greater distance.

37.49. **(a) IDENTIFY** and **SET UP:** $\Delta t_0=2.60\times10^{-8}$ s is the proper time, measured in the pion's frame. The time measured in the lab must satisfy $d=c\Delta t$, where $u\approx c$. Calculate Δt and then use Eq.(37.6) to calculate u.

EXECUTE: $\Delta t=\frac{d}{c}=\frac{1.20\times10^{3}\text{ m}}{2.998\times10^{8}\text{ m/s}}=4.003\times10^{-6}\text{ s}$

$\Delta t=\frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ so $(1-u^2/c^2)^{1/2}=\frac{\Delta t_0}{\Delta t}$ and $(1-u^2/c^2)=\left(\frac{\Delta t_0}{\Delta t}\right)^2$

Write $u=(1-\Delta)c$ so that $(u/c)^2=(1-\Delta)^2=1-2\Delta+\Delta^2\approx1-2\Delta$ since Δ is small.

Using this in the above gives $1-(1-2\Delta)=\left(\frac{\Delta t_0}{\Delta t}\right)^2$

$\Delta=\frac{1}{2}\left(\frac{\Delta t_0}{\Delta t}\right)^2=\frac{1}{2}\left(\frac{2.60\times10^{-8}\text{ s}}{4.003\times10^{-6}\text{ s}}\right)^2=2.11\times10^{-5}$

EVALUATE: An alternative calculation is to say that the length of the tube must contract relative to the moving pion so that the pion travels that length before decaying. The contracted length must be $l=c\Delta t_0=(2.998\times10^{8}\text{ m/s})(2.60\times10^{-8}\text{ s})=7.79\text{ m}$.

$l=l_0\sqrt{1-u^2/c^2}$ so $1-u^2/c^2=\left(\frac{l}{l_0}\right)^2$

Then $u=(1-\Delta)c$ gives $\Delta=\frac{1}{2}\left(\frac{l}{l_0}\right)^2=\frac{1}{2}\left(\frac{7.79\text{ m}}{1.20\times10^3\text{ m}}\right)^2=2.11\times10^{-5}$, which checks.

(b) IDENTIFY and SET UP: $E=\gamma mc^2$ (Eq.(37.38).

EXECUTE: $\gamma=\frac{1}{\sqrt{1-u^2/c^2}}=\frac{1}{\sqrt{2\Delta}}=\frac{1}{\sqrt{2(2.11\times10^{-5})}}=154$

$E=154(139.6\text{ MeV})=2.15\times10^4\text{ MeV}=21.5\text{ GeV}$

EVALUATE: The total energy is 154 times the rest energy.

37.50. **IDENTIFY and SET UP:** The proper length of a side is $l_0=a$. The side along the direction of motion is shortened to $l=l_0\sqrt{1-v^2/c^2}$. The sides in the two directions perpendicular to the motion are unaffected by the motion and still have a length *a*.

EXECUTE: $V=a^2l=a^3\sqrt{1-v^2/c^2}$

37.51. **IDENTIFY and SET UP:** There must be a length contraction such that the length *a* becomes the same as *b*; $l_0=a,\ l=b.\ l_0$ is the distance measured by an observer at rest relative to the spacecraft. Use Eq.(37.16) and solve for *u*.

EXECUTE: $\frac{l}{l_0}=\sqrt{1-u^2/c^2}$ so $\frac{b}{a}=\sqrt{1-u^2/c^2}$;

$a=1.40b$ gives $b/1.40b=\sqrt{1-u^2/c^2}$ and thus $1-u^2/c^2=1/(1.40)^2$

$u=\sqrt{1-1/(1.40)^2}\,c=0.700c=2.10\times10^8\text{ m/s}$

EVALUATE: A length on the spacecraft in the direction of the motion is shortened. A length perpendicular to the motion is unchanged.

37.52. **IDENTIFY and SET UP:** The proper time $\Delta t_0$is the time that elapses in the frame of the space probe. Δt is the time that elapses in the frame of the earth. The distance traveled is 42.2 light years, as measured in the earth frame.

EXECUTE: **(a)** Light travels 42.2 light years in 42.2 yr, so $\Delta t=\left(\frac{c}{0.9910c}\right)(42.2\text{ yr})=42.6\text{ yr}$.

$\Delta t_0=\Delta t\sqrt{1-u^2/c^2}=(42.6\text{ yr})\sqrt{1-(0.9910)^2}=5.7\text{ yr}$. She measures her biological age to be $19\text{ yr}+5.7\text{ yr}=24.7\text{ yr}$.

(b) Her age measured by someone on earth is $19\text{ yr}+42.6\text{ yr}=61.6\text{ yr}$.

37.53. **(a)** $E=\gamma mc^2$ and $\gamma=10=\frac{1}{\sqrt{1-(v/c)^2}}\Rightarrow\frac{v}{c}=\sqrt{\frac{\gamma^2-1}{\gamma^2}}\Rightarrow\frac{v}{c}=\sqrt{\frac{99}{100}}=0.995.$

(b) $(pc)^2=m^2v^2\gamma^2c^2,\ E^2=m^2c^4\left(\left(\frac{v}{c}\right)^2\gamma^2+1\right)$

$$\Rightarrow\frac{E^2-(pc)^2}{E^2}=\frac{1}{1+\gamma^2\left(\frac{v}{c}\right)^2}=\frac{1}{1+(10/(0.995))^2}=0.01=1\%.$$

37.54. **IDENTIFY and SET UP:** The clock on the plane measures the proper time Δt_0.

$\Delta t=4.00\text{ h}=4.00\text{ h }(3600\text{ s/1 h})=1.44\times10^4\text{ s}.$

$\Delta t=\frac{\Delta t_0}{\sqrt{1-u^2/c^2}}$ and $\Delta t_0=\Delta t\sqrt{1-u^2/c^2}$

EXECUTE: $\frac{u}{c}$ small so $\sqrt{1-u^2/c^2}=(1-u^2/c^2)^{1/2}\approx1-\frac{1}{2}\frac{u^2}{c^2}$; thus $\Delta t_0=\Delta t\left(1-\frac{1}{2}\frac{u^2}{c^2}\right)$

The difference in the clock readings is $\Delta t-\Delta t_0=\frac{1}{2}\frac{u^2}{c^2}\Delta t=\frac{1}{2}\left(\frac{250\text{ m/s}}{2.998\times10^8\text{ m/s}}\right)^2(1.44\times10^4\text{ s})=5.01\times10^{-9}\text{ s}$. The clock on the plane has the shorter elapsed time.

EVALUATE: Δt_0 is always less than Δt; our results agree with this. The speed of the plane is much less than the speed of light, so the difference in the reading of the two clocks is very small.

37.55. **IDENTIFY:** Since the speed is very close to the speed of light, we must use the relativistic formula for kinetic energy.

SET UP: The relativistic formula for kinetic energy is $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$ and the relativistic mass is $m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}}$.

EXECUTE: (a) $K = 7\times10^{12}\ \text{eV} = 1.12\times10^{-6}\ \text{J}$. Using this value in the relativistic kinetic energy formula and substituting the mass of the proton for *m*, we get $K = mc^2\left(\frac{1}{\sqrt{1-v^2/c^2}} - 1\right)$

which gives $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$ and $1-\frac{v^2}{c^2} = \frac{1}{(7.45\times10^3)^2}$. Solving for *v* gives

$1-\frac{v^2}{c^2} = \frac{(c+v)(c-v)}{c^2} = \frac{2(c-v)}{c}$, since $c + v \approx 2c$. Substituting $v = (1-\Delta)c$, we have.

$1-\frac{v^2}{c^2} = \frac{2(c-v)}{c} = \frac{2\left[c-(1-\Delta)c\right]}{c} = 2\Delta$. Solving for Δ gives $\Delta = \frac{1-v^2/c^2}{2} = \frac{\frac{1}{\left(7.45\times10^3\right)^2}}{2} = 9\times10^{-9}$, to one significant digit.

(b) Using the relativistic mass formula and the result that $\frac{1}{\sqrt{1-v^2/c^2}} = 7.45\times10^3$, we have

$m_{\text{rel}} = \frac{m}{\sqrt{1-v^2/c^2}} = m\left(\frac{1}{\sqrt{1-v^2/c^2}}\right) = (7\times10^3)m$, to one significant digit.

EVALUATE: At such high speeds, the proton's mass is over 7000 times as great as its rest mass.

37.56. **IDENTIFY** and **SET UP:** The energy released is $E = (\Delta m)c^2$. $\Delta m = \left(\frac{1}{10^4}\right)(8.00\ \text{kg})$. $P_{\text{av}} = \frac{E}{t}$. The change in gravitational potential energy is $mg\Delta y$.

EXECUTE: **(a)** $E = (\Delta m)c^2 = \left(\frac{1}{10^4}\right)(8.00\ \text{kg})(3.00\times10^8\ \text{m/s})^2 = 7.20\times10^{13}\ \text{J}$

(b) $P_{\text{av}} = \frac{E}{t} = \frac{7.20\times10^{13}\ \text{J}}{4.00\times10^{-6}\ \text{s}} = 1.80\times10^{19}\ \text{W}$

(c) $E = \Delta U = mg\Delta y$. $m = \frac{E}{g\Delta y} = \frac{7.20\times10^{13}\ \text{J}}{(9.80\ \text{m/s}^2)(1.00\times10^3\ \text{m})} = 7.35\times10^9\ \text{kg}$

37.57. **IDENTIFY** and **SET UP:** In crown glass the speed of light is $v = \frac{c}{n}$. Calculate the kinetic energy of an electron that has this speed.

EXECUTE: $v = \frac{2.998\times10^8\ \text{m/s}}{1.52} = 1.972\times10^8\ \text{m/s}$.

$K = mc^2(\gamma - 1)$

$mc^2 = (9.109\times10^{-31}\ \text{kg})(2.998\times10^8\ \text{m/s})^2 = 8.187\times10^{-14}\ \text{J}(1\ \text{eV}/1.602\times10^{-19}\ \text{J}) = 0.5111\ \text{MeV}$

$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-((1.972\times10^8\ \text{m/s})/(2.998\times10^8\ \text{m/s}))^2}} = 1.328$

$K = mc^2(\gamma-1) = (0.5111\ \text{MeV})(1.328-1) = 0.168\ \text{MeV}$

EVALUATE: No object can travel faster than the speed of light in vacuum but there is nothing that prohibits an object from traveling faster than the speed of light in some material.

37.58. **(a)** $v = \frac{p}{m} = \frac{(E/c)}{m} = \frac{E}{mc}$, where the atom and the photon have the same magnitude of momentum, E/c.

(b) $v = \frac{E}{mc} \ll c$, so $E \ll mc^2$.

37.59. **IDENTIFY** and **SET UP:** Let *S* be the lab frame and S' be the frame of the proton that is moving in the +*x* direction, so $u = +c/2$. The reference frames and moving particles are shown in Figure 37.59. The other proton moves in the $-x$ direction in the lab frame, so $v=-c/2$. A proton has rest mass $m_p=1.67\times10^{-27}$ kg and rest energy $m_pc^2=938$ MeV.

EXECUTE: (a) $v'=\dfrac{v-u}{1-uv/c^2}=\dfrac{-c/2-c/2}{1-(c/2)(-c/2)/c^2}=-\dfrac{4c}{5}$

The speed of each proton relative to the other is $\dfrac{4}{5}c$.

(b) In nonrelativistic mechanics the speeds just add and the speed of each relative to the other is c.

(c) $K=\dfrac{mc^2}{\sqrt{1-v^2/c^2}}-mc^2$

(i) Relative to the lab frame each proton has speed $v=c/2$. The total kinetic energy of each proton is

$$K=\frac{938\text{ MeV}}{\sqrt{1-\left(\frac{1}{2}\right)^2}}-(938\text{ MeV})=145\text{ MeV}.$$

(ii) In its rest frame one proton has zero speed and zero kinetic energy and the other has speed $\dfrac{4}{5}c$. In this frame the kinetic energy of the moving proton is $K=\dfrac{938\text{ MeV}}{\sqrt{1-\left(\frac{4}{5}\right)^2}}-(938\text{ MeV})=625\text{ MeV}$

(d) (i) Each proton has speed $v=c/2$ and kinetic energy

$$K=\frac{1}{2}mv^2=\left(\frac{1}{2}m\right)(c/2)^2=\frac{mc^2}{8}=\frac{938\text{ MeV}}{8}=117\text{ MeV}$$

(ii) One proton has speed $v=0$ and the other has speed c. The kinetic energy of the moving proton is $K=\dfrac{1}{2}mc^2=\dfrac{938\text{ MeV}}{2}=469\text{ MeV}$

EVALUATE: The relativistic expression for K gives a larger value than the nonrelativistic expression. The kinetic energy of the system is different in different frames.

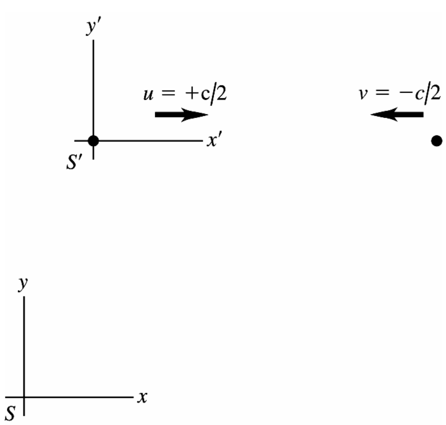


Figure 37.59

37.60. IDENTIFY and **SET UP:** Let S be the lab frame and let S' the frame of the proton that is moving in the $+x$ direction in the lab frame, as shown in Figure 37.60. In S' the other proton moves in the $-x'$ direction with speed $c/2$, so $v'=-c/2$. In the lab frame each proton has speed αc, where α is a constant that we need to solve for.

EXECUTE: (a) $v=\dfrac{v'+u}{1+uv'/c^2}$ with $v=-\alpha c$, $u=+\alpha c$ and $v'=-0.50c$ gives $-\alpha c=\dfrac{-0.50c+\alpha c}{1+(\alpha c)(-0.50c)/c^2}$ and $-\alpha=\dfrac{-0.50+\alpha}{1-0.50\alpha}$. $\alpha^2-4\alpha+1=0$ and $\alpha=0.268$ or $\alpha=3.73$. Can't have $v>c$, so only $\alpha=0.268$ is physically allowed. The speed measured by the observer in the lab is $0.268c$.

(b) (i) $v=0.269c$. $\gamma=1.0380$. $K=(\gamma-1)mc^2=35.6$ MeV.

(ii) $v = 0.500c$. $\gamma = 1.1547$. $K = (\gamma - 1)mc^2 = 145\text{ MeV}$.

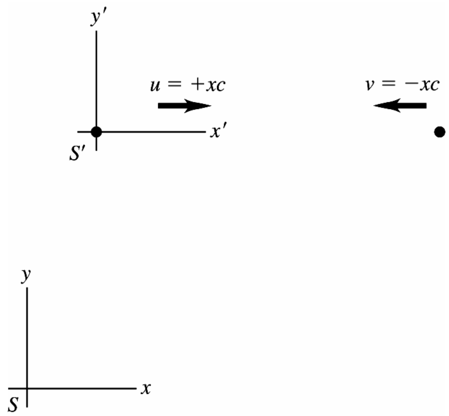


Figure 37.60

37.61. $x'^2 = c^2t'^2 \Rightarrow (x-ut)^2\gamma^2 = c^2\gamma^2\left(t - ux/c^2\right)^2$

$\Rightarrow x - ut = c(t - ux/c^2) \;\Rightarrow x\left(1+\dfrac{u}{c}\right) = \dfrac{1}{c}x(u+c) = t(u+c) \;\Rightarrow x = ct \;\Rightarrow x^2 = c^2t^2.$

37.62. **IDENTIFY** and **SET UP:** Let S be the lab frame and let S' be the frame of the nucleus. Let the $+x$ direction be the direction the nucleus is moving. $u = 0.7500c$.

EXECUTE: **(a)** $v' = +0.9995c$. $v = \dfrac{v'+u}{1+uv'/c^2} = \dfrac{0.9995c + 0.7500c}{1+(0.7500)(0.9995)} = 0.999929c$

(b) $v' = -0.9995c$. $v = \dfrac{-0.9995c + 0.7500c}{1+(0.7500)(-0.9995)} = -0.9965c$

(c) emitted in same direction:

(i) $K = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.999929)^2}} - 1\right) = 42.4\text{ MeV}$

(ii) $K' = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.9995)^2}} - 1\right) = 15.7\text{ MeV}$

(d) emitted in opposite direction:

(i) $K = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.9965)^2}} - 1\right) = 5.60\text{ MeV}$

(ii) $K' = \left(\dfrac{1}{\sqrt{1-v^2/c^2}} - 1\right)mc^2 = (0.511\text{ MeV})\left(\dfrac{1}{\sqrt{1-(0.9995)^2}} - 1\right) = 15.7\text{ MeV}$

37.63. **IDENTIFY** and **SET UP:** Use Eq.(37.30), with $a = dv/dt$, to obtain an expression for dv/dt. Separate the variables v and t and integrate to obtain an expression for $v(t)$. In this expression, let $t \to \infty$.

EXECUTE: $a = \dfrac{dv}{dt} = \dfrac{F}{m}(1 - v^2/c^2)^{3/2}$. (One-dimensional motion is assumed, and all the F, v, and a refer to x-components.)

$$\frac{dv}{(1-v^2/c^2)^{3/2}} = \left(\frac{F}{m}\right)dt$$

Integrate from $t = 0$, when $v = 0$, to time t, when the velocity is v.

$$\int_0^v \frac{dv}{(1-v^2/c^2)^{3/2}} = \int_0^t \left(\frac{F}{m}\right)dt$$

Since F is constant, $\displaystyle\int_0^t \left(\frac{F}{m}\right)dt = \frac{Ft}{m}$. In the velocity integral make the change of variable $y = v/c$; then $dy = dv/c$.

$$\int_0^v \frac{dv}{(1-v^2/c^2)^{3/2}} = c\int_0^{v/c} \frac{dy}{(1-y^2)^{3/2}} = c\left[\frac{y}{(1-y^2)^{1/2}}\right]_0^{v/c} = \frac{v}{\sqrt{1-v^2/c^2}}$$

Thus $\dfrac{v}{\sqrt{1-v^2/c^2}} = \dfrac{Ft}{m}$.

Solve this equation for v:

$$\frac{v^2}{1-v^2/c^2} = \left(\frac{Ft}{m}\right)^2 \text{ and } v^2 = \left(\frac{Ft}{m}\right)^2 (1-v^2/c^2)$$

$$v^2\left(1+\left(\frac{Ft}{mc}\right)^2\right) = \left(\frac{Ft}{m}\right)^2 \text{ so } v = \frac{(Ft/m)}{\sqrt{1+(Ft/mc)^2}} = c\frac{Ft}{\sqrt{m^2c^2+F^2t^2}}$$

As $t \to \infty$, $\dfrac{Ft}{\sqrt{m^2c^2+F^2t^2}} \to \dfrac{Ft}{\sqrt{F^2t^2}} \to 1$, so $v \to c$.

EVALUATE: Note that $\dfrac{Ft}{\sqrt{m^2c^2+F^2t^2}}$ is always less than 1, so $v<c$ always and v approaches c only when $t \to \infty$.

37.64. Setting $x=0$ in Eq.(37.21), the first equation becomes $x' = -\gamma ut$ and the last, upon multiplication by c, becomes $ct' = \gamma ct$.Squaring and subtracting gives $c^2t'^2 - x'^2 = \gamma^2(c^2t^2 - u^2t^2) = c^2t^2$, or $x' = c\sqrt{t'^2 - t^2} = 4.53\times 10^8$ m.

37.65. **(a) IDENTIFY** and **SET UP:** Use the Lorentz coordinate transformation (Eq.37.21) for (x_1, t_1) and (x_2, t_2):

$$x_1' = \frac{x_1 - ut_1}{\sqrt{1-u^2/c^2}}, \quad x_2' = \frac{x_2 - ut_2}{\sqrt{1-u^2/c^2}}$$

$$t_1' = \frac{t_1 - ux_1/c^2}{\sqrt{1-u^2/c^2}}, \quad t_2' = \frac{t_2 - ux_2/c^2}{\sqrt{1-u^2/c^2}}$$

Same point in S' implies $x_1' = x_2'$. What then is $\Delta t' = t_2' - t_1'$?

EXECUTE: $x_1' = x_2'$ implies $x_1 - ut_1 = x_2 - ut_2$

$$u(t_2 - t_1) = x_2 - x_1 \text{ and } u = \frac{x_2 - x_1}{t_2 - t_1} = \frac{\Delta x}{\Delta t}$$

From the time transformation equations,

$$\Delta t' = t_2' - t_1' = \frac{1}{\sqrt{1-u^2/c^2}}(\Delta t - u\Delta x/c^2)$$

Using the result that $u = \dfrac{\Delta x}{\Delta t}$ gives

$$\Delta t' = \frac{1}{\sqrt{1-(\Delta x)^2/((\Delta t)^2c^2)}}(\Delta t - (\Delta x)^2/((\Delta t)c^2))$$

$$\Delta t' = \frac{\Delta t}{\sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}}(\Delta t - (\Delta x)^2/((\Delta t)c^2))$$

$$\Delta t' = \frac{(\Delta t)^2 - (\Delta x)^2/c^2}{\sqrt{(\Delta t)^2 - (\Delta x)^2/c^2}} = \sqrt{(\Delta t)^2 - (\Delta x/c)^2}, \text{ as was to be shown.}$$

This equation doesn't have a physical solution (because of a negative square root) if $(\Delta x/c)^2 > (\Delta t)^2$ or $\Delta x \ge c\Delta t$.

(b) IDENTIFY and **SET UP:** Now require that $t_2' = t_1'$ (the two events are simultaneous in S') and use the Lorentz coordinate transformation equations.

EXECUTE: $t_2' = t_1'$ implies $t_1 - ux_1/c^2 = t_2 - ux_2/c^2$

$$t_2 - t_1 = \left(\frac{x_2 - x_1}{c^2}\right)u \text{ so } \Delta t = \left(\frac{\Delta x}{c^2}\right)u \text{ and } u = \frac{c^2\Delta t}{\Delta x}$$

From the Lorentz transformation equations,

$$\Delta x' = x_2' - x_1' = \left(\frac{1}{\sqrt{1-u^2/c^2}}\right)(\Delta x - u\Delta t).$$

Using the result that $u = c^2\Delta t/\Delta x$ gives

$$\Delta x' = \frac{1}{\sqrt{1-c^2(\Delta t)^2/(\Delta x)^2}}(\Delta x - c^2(\Delta t)^2/\Delta x)$$

$$\Delta x' = \frac{\Delta x}{\sqrt{(\Delta x)^2 - c^2(\Delta t)^2}}(\Delta x - c^2(\Delta t)^2/\Delta x)$$

$$\Delta x' = \frac{(\Delta x)^2 - c^2(\Delta t)^2}{\sqrt{(\Delta x)^2 - c^2(\Delta t)^2}} = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$$

(c) IDENTIFY and **SET UP:** The result from part (b) is $\Delta x' = \sqrt{(\Delta x)^2 - c^2(\Delta t)^2}$

Solve for Δt: $(\Delta x')^2 = (\Delta x)^2 - c^2(\Delta t)^2$

EXECUTE: $\Delta t = \dfrac{\sqrt{(\Delta x)^2 - (\Delta x')^2}}{c} = \dfrac{\sqrt{(5.00\text{ m})^2 - (2.50\text{ m})^2}}{2.998\times10^8\text{ m/s}} = 1.44\times10^{-8}\text{ s}$

EVALUATE: This provides another illustration of the concept of simultaneity (Section 37.2): events observed to be simultaneous in one frame are not simultaneous in another frame that is moving relative to the first.

37.66. **(a)** 80.0 m/s is non-relativistic, and $K = \frac{1}{2}mv^2 = 186\text{ J}$.

(b) $(\gamma - 1)mc^2 = 1.31\times10^{15}\text{ J}$.

(c) In Eq. (37.23), c) $v' = 2.20\times10^8\text{ m/s}$, $u = -1.80\times10^8\text{ m/s}$, and so $v = 7.14\times10^7\text{m/s}$.

(d) $\dfrac{20.0\text{ m}}{\gamma} = 13.6\text{ m}$.

(e) $\dfrac{20.0\text{ m}}{2.20\times10^8\text{ m/s}} = 9.09\times10^{-8}\text{ s}$.

(f) $t' = \dfrac{t}{\gamma} = 6.18\times10^{-8}\text{ s}$, or $t' = \dfrac{13.6\text{ m}}{2.20\times10^8\text{ m/s}} = 6.18\times10^{-8}\text{ s}$.

37.67. **IDENTIFY** and **SET UP:** An increase in wavelength corresponds to a decrease in frequency $(f = c/\lambda)$, so the atoms are moving away from the earth. Receding, so use Eq.(37.26): $f = \sqrt{\dfrac{c-u}{c+u}}f_0$

EXECUTE: Solve for u: $(f/f_0)^2(c+u) = c-u$ and $u = c\left(\dfrac{1-(f/f_0)^2}{1+(f/f_0)^2}\right)$

$f = c/\lambda,\ f_0 = c/\lambda_0$ so $f/f_0 = \lambda_0/\lambda$

$$u = c\left(\frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2}\right) = c\left(\frac{1-(656.3/953.4)^2}{1+(656.3/953.4)^2}\right) = 0.357c = 1.07\times10^8\text{ m/s}$$

EVALUATE: The relative speed is large, 36% of c. The cosmological implication of such observations will be discussed in Section 44.6.

37.68. The baseball had better be moving non-relativistically, so the Doppler shift formula (Eq.(37.25)) becomes $f \cong f_0(1-(u/c))$. In the baseball's frame, this is the frequency with which the radar waves strike the baseball, and the baseball reradiates at f. But in the coach's frame, the reflected waves are Doppler shifted again, so the detected frequency is $f(1-(u/c)) = f_0(1-(u/c))^2 \approx f_0(1-2(u/c))$, so $\Delta f = 2f_0(u/c)$ and the fractional frequency shift is $\dfrac{\Delta f}{f_0} = 2(u/c)$. In this case,

$$u = \frac{\Delta f}{2f_0}c = \frac{(2.86\times10^{-7})}{2}(3.00\times10^8\text{ m}) = 42.9\text{ m/s} = 154\text{ km/h} = 92.5\text{ mi/h}.$$

37.69. **IDENTIFY** and **SET UP:** 500 light years $= 4.73\times10^{18}$ m. The proper distance l_0 to the star is 500 light years. The energy needed is the kinetic energy of the rocket at its final speed.

EXECUTE: **(a)** $u = 0.50c$. $\Delta t = \dfrac{d}{u} = \dfrac{4.73\times10^{18}\text{ m}}{(0.50)(3.00\times10^8\text{ m/s})} = 3.2\times10^{10}\text{ s} = 1000\text{ yr}$

The proper time is measured by the astronauts. $\Delta t_0 = \Delta t\sqrt{1-u^2/c^2} = 866\text{ yr}$

$$K = \frac{mc^2}{\sqrt{1-v^2/c^2}} - mc^2 = (1000\text{ kg})(3.00\times10^8\text{ m/s})^2\left(\frac{1}{\sqrt{1-(0.500)^2}} - 1\right) = 1.4\times10^{19}\text{ J}$$

This is 140% of the U.S. yearly use of energy.

(b) $u = 0.99c$. $\Delta t = \dfrac{d}{u} = \dfrac{4.73\times10^{18}\text{ m}}{(0.99)(3.00\times10^8\text{ m/s})} = 1.6\times10^{10}\text{ s} = 505\text{ yr}$, $\Delta t_0 = 71\text{ yr}$

$$K = (9.00\times10^{19}\text{ J})\left(\frac{1}{\sqrt{1-(0.99)^2}} - 1\right) = 5.5\times10^{20}\text{ J}$$

This is 55 times the U.S. yearly use.

(c) $u = 0.9999c$. $\Delta t = \dfrac{d}{u} = \dfrac{4.73\times10^{18}\text{ m}}{(0.9999)(3.00\times10^{8}\text{ m/s})} = 1.58\times10^{10}\text{ s} = 501\text{ yr}$, $\Delta t_0 = 7.1\text{ yr}$

$$K = (9.00\times10^{19}\text{ J})\left(\frac{1}{\sqrt{1-(0.9999)^2}} - 1\right) = 6.3\times10^{21}\text{ J}$$

This is 630 times the U.S. yearly use.
The energy cost of accelerating a rocket to these speeds is immense.

37.70. **(a)** As in the hint, both the sender and the receiver measure the same distance. However, in our frame, the ship has moved between emission of successive wavefronts, and we can use the time $T = 1/f$ as the proper time, with the result that $f = \gamma f_0 > f_0$.

(b) Toward: $f = f_0\sqrt{\dfrac{c+u}{c-u}} = 345\text{ MHz}\left(\dfrac{1+0.758}{1-0.758}\right)^{1/2} = 930\text{ MHz}$

$$f - f_0 = 930\text{ MHz} - 345\text{ MHz} = 585\text{ MHz}.$$

Away: $f = f_0\sqrt{\dfrac{c-u}{c+u}} = 345\text{ MHz}\left(\dfrac{1-0.758}{1+0.758}\right)^{1/2} = 128\text{ MHz}$ and $f - f_0 = -217\text{ MHz}$.

(c) $\gamma f_0 = 1.53 f_0 = 528\text{ MHz}$, $f - f_0 = 183\text{ MHz}$. The shift is still bigger than f_0, but not as large as the approaching frequency.

37.71. The crux of this problem is the question of simultaneity. To be "in the barn at one time" for the runner is different than for a stationary observer in the barn. The diagram in Figure 37.71a shows the rod fitting into the barn at time $t = 0$, according to the stationary observer. The diagram in Figure 37.71b is in the runner's frame of reference. The front of the rod enters the barn at time t_1 and leaves the back of the barn at time t_2. However, the back of the rod does not enter the front of the barn until the later time t_3.

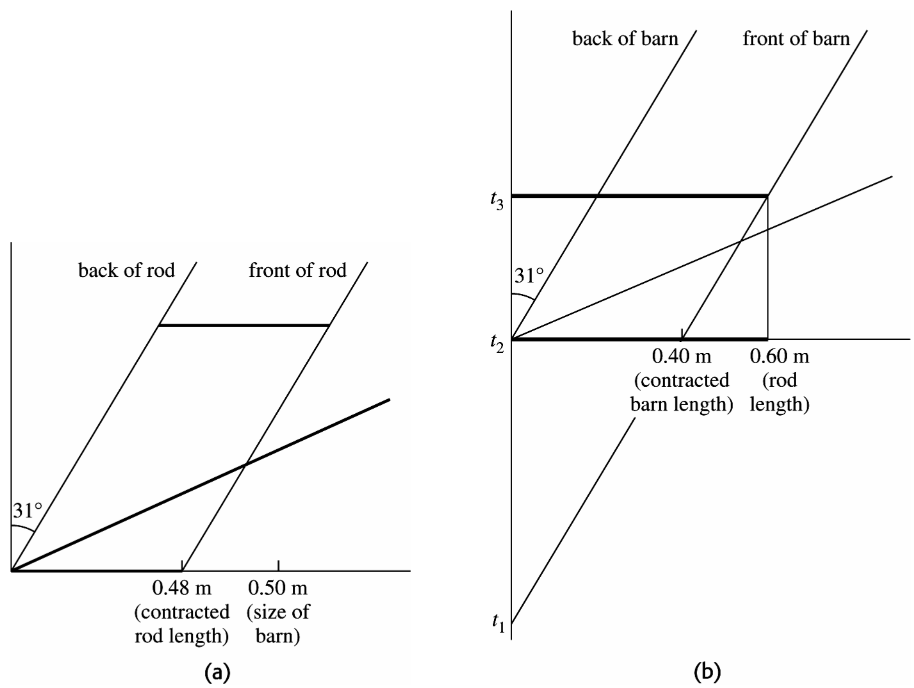


Figure 37.71

37.72. In Eq.(37.23), $u = V$, $v' = (c/n)$, and so $v = \dfrac{(c/n)+V}{1+\dfrac{cV}{nc^2}} = \dfrac{(c/n)+V}{1+(V/nc)}$. For V non-relativistic, this is

$v \approx ((cn)+V)(1-(V/nc)) = (nc/n) + V - (V/n^2) - (V^2/nc) \approx \dfrac{c}{n} + \left(1 - \dfrac{1}{n^2}\right)V$, so $k = \left(1 - \dfrac{1}{n^2}\right)$. For water, $n = 1.333$ and $k = 0.437$.

37.73. **(a)** $a' = \dfrac{dv}{dt'}$. $dt' = \gamma(dt - u\,dx/c^2)$. $dv' = \dfrac{dv}{(1-uv/c^2)} + \dfrac{v-u}{(1-uv/c^2)^2}\dfrac{u}{c^2}dv$

$\dfrac{dv'}{dv} = \dfrac{1}{1-uv/c^2} + \dfrac{v-u}{(1-uv/c^2)^2}\left(\dfrac{u}{c^2}\right)$.

$$dv' = dv\left(\frac{1}{1-uv/c^2} + \frac{(v-u)u/c^2}{(1-uv/c^2)^2}\right) = dv\left(\frac{1-u^2/c^2}{(1-uv/c^2)^2}\right)$$

$$a' = \frac{dv\dfrac{(1-u^2/c^2)}{(1-uv/c^2)^2}}{\gamma dt - u\gamma dx/c^2} = \frac{dv}{dt}\frac{(1-u^2/c^2)}{(1-uv/c^2)^2}\frac{1}{\gamma(1-uv/c^2)}$$

$$= a(1-u^2/c^2)^{3/2}\,(1-uv/c^2)^{-3}.$$

(b) Changing frames from $S' \to S$ just involves changing $a \to a'$, $v \to -v' \Rightarrow a = a'(1-u^2/c^2)^{3/2}\left(1+\dfrac{uv'}{c^2}\right)^{-3}$.

37.74. **(a)** The speed v' is measured relative to the rocket, and so for the rocket and its occupant, $v' = 0$. The acceleration as seen in the rocket is given to be $a' = g$, and so the acceleration as measured on the earth is

$a = \dfrac{du}{dt} = g\left(1-\dfrac{u^2}{c^2}\right)^{3/2}$.

(b) With $v_1 = 0$ when $t = 0$,

$dt = \dfrac{1}{g}\dfrac{du}{(1-u^2/c^2)^{3/2}}$. $\displaystyle\int_0^{t_1} dt = \frac{1}{g}\int_0^{v_1}\frac{du}{(1-u^2/c^2)^{3/2}}$. $t_1 = \dfrac{v_1}{g\sqrt{1-v_1^2/c^2}}$.

(c) $dt' = \gamma dt = dt/\sqrt{1-u^2/c^2}$, so the relation in part (b) between *dt* and *du,* expressed in terms of dt' and *du,* is

$dt' = \gamma dt = \dfrac{1}{\sqrt{1-u^2/c^2}}\,\dfrac{du}{g(1-u^2/c^2)^{3/2}} = \dfrac{1}{g}\,\dfrac{du}{(1-u^2/c^2)^2}$.

Integrating as above (perhaps using the substitution $z = u/c$) gives $t_1' = \dfrac{c}{g}\,\text{arctanh}\left(\dfrac{v_1}{c}\right)$. For those who wish to avoid inverse hyperbolic functions, the above integral may be done by the method of partial fractions;

$g\,dt' = \dfrac{du}{(1+u/c)(1-u/c)} = \dfrac{1}{2}\left[\dfrac{du}{1+u/c} + \dfrac{du}{1-uc}\right]$, which integrates to $t_1' = \dfrac{c}{2g}\ln\left(\dfrac{c+v_1}{c-v_1}\right)$.

(d) Solving the expression from part (c) for v_1 in terms of t_1, $(v_1/c) = \tanh(gt_1'/c)$, so that

$\sqrt{1-(v_1/c)^2} = 1/\cosh(gt_1'/c)$, using the appropriate indentities for hyperbolic functions. Using this in the expression found in part (b), $t_1 = \dfrac{c}{g}\dfrac{\tanh(gt_1'/c)}{1/\cosh(gt_1'/c)} = \dfrac{c}{g}\sinh(gt_1'/c)$, which may be rearranged slightly as $\dfrac{gt_1}{c} = \sinh\left(\dfrac{gt_1'}{c}\right)$. If hyperbolic functions are not used, v_1 in terms of t_1' is found to be $\dfrac{v_1}{c} = \dfrac{e^{gt_1'/c} - e^{-gt_1'/c}}{e^{gt_1'/c} + e^{-gt_1'/c}}$ which is the same as $\tanh(gt_1'/c)$. Inserting this expression into the result of part (b) gives, after much algebra, $t_1 = \dfrac{c}{2g}(e^{gt_1'/c} - e^{-gt_1'/c})$, which is equivalent to the expression found using hyperbolic functions.

(e) After the first acceleration period (of 5 years by Stella's clock), the elapsed time on earth is

$$t_1' = \frac{c}{g}\sinh(gt_1'/c) = 2.65\times10^9\ \text{s} = 84.0\ \text{yr}.$$

The elapsed time will be the same for each of the four parts of the voyage, so when Stella has returned, Terra has aged 336 yr and the year is 2436. (Keeping more precision than is given in the problem gives February 7 of that year.)

37.75. **(a)** $f_0 = 4.568110\times10^{14}$ Hz; $f_+ = 4.568910\times10^{14}$ Hz; $f_- = 4.567710\times10^{14}$ Hz

$$\left.\begin{array}{l} f_+ = \sqrt{\dfrac{c+(u+v)}{c-(u+v)}}\,f_0 \\[2ex] f_- = \sqrt{\dfrac{c+(u-v)}{c-(u-v)}}\,f_0 \end{array}\right\} \Rightarrow \begin{array}{l} f_+^2(c-(u+v)) = f_0^2\,(c+(u+v)) \\ f_-^2(c-(u-v)) = f_0^2(c+(u-v)) \end{array}$$

where u is the velocity of the center of mass and v is the orbital velocity.

$$\Rightarrow (u+v) = \frac{(f_+/f_0)^2 - 1}{(f_+/f_0)^2 + 1}c \text{ and } (u-v) = \frac{(f_-^2/f_0^2) - 1}{(f_-^2/f_0^2) + 1}c$$

$$\Rightarrow u+v = 5.25\times10^4 \text{ m/s and } u-v = -2.63\times10^4 \text{ m/s}.$$

This gives $u = +1.31\times10^4$ m/s (moving toward at 13.1 km/s) and $v = 3.94\times10^4$ m/s.

(b) $v = 3.94\times10^4$ m/s; $T = 11.0$ days. $2\pi R = vt \Rightarrow$

$R = \dfrac{(3.94\times10^4 \text{ m/s})(11.0 \text{ days})(24 \text{ hrs/day})(3600 \text{ sec/hr})}{2\pi} = 5.96\times10^9$ m. This is about 0.040 times the earth-sun distance.

Also the gravitational force between them (a distance of $2R$) must equal the centripetal force from the center of mass:

$$\frac{(Gm^2)}{(2R)^2} = \frac{mv^2}{R} \Rightarrow m = \frac{4Rv^2}{G} = \frac{4(5.96\times10^9 \text{ m})(3.94\times10^4 \text{ m/s})^2}{6.672\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2} = 5.55\times10^{29} \text{ kg} = 0.279 \text{ m}_{\text{sun}}.$$

37.76. For any function $f = f(x,t)$ and $x = x(x',t'), t = t(x',t')$, let $F(x',t') = f(x(x',t'), t(x',t'))$ and use the standard (but mathematically improper) notation $F(x',t') = f(x',t')$. The chain rule is then

$$\frac{\partial f(x',t')}{\partial x} = \frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial x} + \frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial x},$$
$$\frac{\partial f(x',t')}{\partial t} = \frac{\partial f(x,t)}{\partial x'}\frac{\partial x'}{\partial t} + \frac{\partial f(x',t')}{\partial t'}\frac{\partial t'}{\partial t}.$$

In this solution, the explicit dependence of the functions on the sets of dependent variables is suppressed, and the above relations are then $\dfrac{\partial f}{\partial x} = \dfrac{\partial f}{\partial x'}\dfrac{\partial x'}{\partial x} + \dfrac{\partial f}{\partial t'}\dfrac{\partial t'}{\partial x}$, $\dfrac{\partial f}{\partial t} = \dfrac{\partial f}{\partial x'}\dfrac{\partial x'}{\partial t} + \dfrac{\partial f}{\partial t'}\dfrac{\partial t'}{\partial t}$.

(a) $\dfrac{\partial x'}{\partial x} = 1, \dfrac{\partial x'}{\partial t} = -v, \dfrac{\partial t'}{\partial x} = 0$ and $\dfrac{\partial t'}{\partial t} = 1$. Then, $\dfrac{\partial E}{\partial x} = \dfrac{\partial E}{\partial x'}$, and $\dfrac{\partial^2 E}{\partial x^2} = \dfrac{\partial^2 E}{\partial x'^2}$. For the time derivative, $\dfrac{\partial E}{\partial t} = -v\dfrac{\partial E}{\partial x'} + \dfrac{\partial E}{\partial t'}$. To find the second time derivative, the chain rule must be applied to both terms; that is,

$$\frac{\partial}{\partial t}\frac{\partial E}{\partial x'} = -v\frac{\partial^2 E}{\partial x'^2} + \frac{\partial^2 E}{\partial t' \partial x'},$$
$$\frac{\partial}{\partial t}\frac{\partial E}{\partial t'} = -v\frac{\partial^2 E}{\partial x' \partial t'} + \frac{\partial^2 E}{\partial t'^2}.$$

Using these in $\dfrac{\partial^2 E}{\partial t^2}$, collecting terms and equating the mixed partial derivatives gives

$\dfrac{\partial^2 E}{\partial t^2} = v^2\dfrac{\partial^2 E}{\partial x'^2} - 2v\dfrac{\partial^2 E}{\partial x' \partial t'} + \dfrac{\partial^2 E}{\partial t'^2}$, and using this and the above expression for $\dfrac{\partial^2 E}{\partial x'^2}$ gives the result.

(b) For the Lorentz transformation, $\dfrac{\partial x'}{\partial x} = \gamma, \dfrac{\partial x'}{\partial t} = \gamma v, \dfrac{\partial t'}{\partial x} = \gamma v/c^2$ and $\dfrac{\partial t'}{\partial t} = \gamma$.

The first partials are then

$$\frac{\partial E}{\partial x} = \gamma\frac{\partial E}{\partial x'} - \gamma\frac{v}{c^2}\frac{\partial E}{\partial t'}, \frac{\partial E}{\partial t} = -\gamma v\frac{\partial E}{\partial x'} + \gamma\frac{\partial E}{\partial t'}$$

and the second partials are (again equating the mixed partials)

$$\frac{\partial^2 E}{\partial x^2} = \gamma^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{v^2}{c^4}\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2\frac{v}{c^2}\frac{\partial^2 E}{\partial x' \partial t'}$$
$$\frac{\partial^2 E}{\partial t^2} = \gamma^2 v^2\frac{\partial^2 E}{\partial x'^2} + \gamma^2\frac{\partial^2 E}{\partial t'^2} - 2\gamma^2 v\frac{\partial^2 E}{\partial x' \partial t'}.$$

Substituting into the wave equation and combining terms (note that the mixed partials cancel),

$$\frac{\partial^2 E}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = \gamma^2\left(1 - \frac{v^2}{c^2}\right)\frac{\partial^2 E}{\partial x'^2} + \gamma^2\left(\frac{v^2}{c^4} - \frac{1}{c^2}\right)\frac{\partial^2 E}{\partial t'^2} = \frac{\partial^2 E}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t'^2} = 0.$$

37.77. **(a)** In the center of momentum frame, the two protons approach each other with equal velocities (since the protons have the same mass). After the collision, the two protons are at rest—but now there are kaons as well. In this situation the kinetic energy of the protons must equal the total rest energy of the two kaons $\Rightarrow 2(\gamma_{\text{cm}}-1)m_{\text{p}}c^2 =$

$2m_{\text{k}}c^2 \Rightarrow \gamma_{\text{cm}} = 1+\dfrac{m_{\text{k}}}{m_{\text{p}}} = 1.526.$ The velocity of a proton in the center of momentum frame is then

$$v_{\text{cm}} = c\sqrt{\frac{\gamma_{\text{cm}}^2-1}{\gamma_{\text{cm}}^2}} = 0.7554c.$$

To get the velocity of this proton in the lab frame, we must use the Lorentz velocity transformations. This is the same as "hopping" into the proton that will be our target and asking what the velocity of the projectile proton is. Taking the lab frame to be the unprimed frame moving to the left, $u = v_{\text{cm}}$ and $v' = v_{\text{cm}}$ (the velocity of the projectile proton in the center of momentum frame).

$$v_{\text{lab}} = \frac{v'+u}{1+\dfrac{uv'}{c^2}} = \frac{2v_{\text{cm}}}{1+\dfrac{v_{\text{cm}}^2}{c^2}} = 0.9619c \Rightarrow \gamma_{\text{lab}} = \frac{1}{\sqrt{1-\dfrac{v_{\text{lab}}^2}{c^2}}} = 3.658 \Rightarrow K_{\text{lab}} = (\gamma_{\text{lab}}-1)m_{\text{p}}c^2 = 2494\ \text{MeV}.$$

(b) $\dfrac{K_{\text{lab}}}{2m_{\text{k}}} = \dfrac{2494\ \text{MeV}}{2(493.7\ \text{MeV})} = 2.526.$

(c) The center of momentum case considered in part (a) is the same as this situation. Thus, the kinetic energy required *is* just twice the rest mass energy of the kaons. $K_{\text{cm}} = 2(493.7\ \text{MeV}) = 987.4\ \text{MeV}.$ This offers a substantial advantage over the fixed target experiment in part (b). It takes less energy to create two kaons in the proton center of momentum frame.