35

INTERFERENCE

35.1. **IDENTIFY:** Compare the path difference to the wavelength.
SET UP: The separation between sources is 5.00 m, so for points between the sources the largest possible path difference is 5.00 m.
EXECUTE: **(a)** For constructive interference the path difference is $m\lambda, m = 0, \pm 1, \pm 2, \ldots$ Thus only the path difference of zero is possible. This occurs midway between the two sources, 2.50 m from A.
(b) For destructive interference the path difference is $(m + \frac{1}{2})\lambda,\ m = 0, \pm 1, \pm 2, \ldots$
A path difference of $\pm\lambda/2 = 3.00$ m is possible but a path difference as large as $3\lambda/2 = 9.00$ m is not possible. For a point a distance x from A and $5.00 - x$ from B the path difference is
$x - (5.00\text{ m} - x)$. $x - (5.00\text{ m} - x) = +3.00$ m gives $x = 4.00$ m. $x - (5.00\text{ m} - x) = -3.00$ m gives $x = 1.00$ m .
EVALUATE: The point of constructive interference is midway between the points of destructive interference.

35.2. **IDENTIFY:** For destructive interference the path difference is $(m + \frac{1}{2})\lambda, m = 0, \pm 1, \pm 2, \ldots$. The longest wavelength is for $m = 0$. For constructive interference the path difference is $m\lambda,\ m = 0, \pm 1, \pm 2, \ldots$ The longest wavelength is for $m = 1$.
SET UP: The path difference is 120 m.
EXECUTE: **(a)** For destructive interference $\frac{\lambda}{2} = 120\text{ m} \Rightarrow \lambda = 240$ m.
(b) The longest wavelength for constructive interference is $\lambda = 120$ m.
EVALUATE: The path difference doesn't depend on the distance of point Q from B.

35.3. **IDENTIFY:** Use $c = f\lambda$ to calculate the wavelength of the transmitted waves. Compare the difference in the distance from A to P and from B to P. For constructive interference this path difference is an integer multiple of the wavelength.
SET UP: Consider Figure 35.3

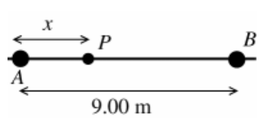

Figure 35.3

The distance of point P from each coherent source is $r_A = x$ and $r_B = 9.00\text{ m} - x$.

EXECUTE: The path difference is $r_B - r_A = 9.00\text{ m} - 2x$.

$r_B - r_A = m\lambda,\ m = 0,\ \pm 1,\ \pm 2,\ \ldots$

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8\text{ m/s}}{120 \times 10^6\text{ Hz}} = 2.50\text{ m}$$

Thus $9.00\text{ m} - 2x = m(2.50\text{ m})$ and $x = \frac{9.00\text{ m} - m(2.50\text{ m})}{2} = 4.50\text{ m} - (1.25\text{ m})m$. x must lie in the range 0 to 9.00 m since P is said to be between the two antennas.

$m = 0$ gives $x = 4.50$ m
$m = +1$ gives $x = 4.50\text{ m} - 1.25\text{ m} = 3.25$ m
$m = +2$ gives $x = 4.50\text{ m} - 2.50\text{ m} = 2.00$ m
$m = +3$ gives $x = 4.50\text{ m} - 3.75\text{ m} = 0.75$ m
$m = -1$ gives $x = 4.50\text{ m} + 1.25\text{ m} = 5.75$ m
$m = -2$ gives $x = 4.50\text{ m} + 2.50\text{ m} = 7.00$ m
$m = -3$ gives $x = 4.50\text{ m} + 3.75\text{ m} = 8.25$ m

All other values of m give values of x out of the allowed range. Constructive interference will occur for $x = 0.75$ m, 2.00 m, 3.25 m, 4.50 m, 5.75 m, 7.00 m, and 8.25 m.

EVALUATE: Constructive interference occurs at the midpoint between the two sources since that point is the same distance from each source. The other points of constructive interference are symmetrically placed relative to this point.

35.4. **IDENTIFY:** For constructive interference the path difference d is related to λ by $d = m\lambda,\ m = 0,1,2,\ldots$ For destructive interference $d = (m+\frac{1}{2})\lambda,\ m = 0,1,2,\ldots$

SET UP: $d = 2040$ nm

EXECUTE: **(a)** The brightest wavelengths are when constructive interference occurs:

$d = m\lambda_m \Rightarrow \lambda_m = \dfrac{d}{m} \Rightarrow \lambda_3 = \dfrac{2040\text{ nm}}{3} = 680\text{ nm},\ \lambda_4 = \dfrac{2040\text{ nm}}{4} = 510\text{ nm}$ and

$\lambda_5 = \dfrac{2040\text{ nm}}{5} = 408\text{ nm}.$

(b) The path-length difference is the same, so the wavelengths are the same as part (a).

(c) $d = (m+\frac{1}{2})\lambda_m$ so $\lambda_m = \dfrac{d}{m+\frac{1}{2}} = \dfrac{2040\text{ nm}}{m+\frac{1}{2}}$. The visible wavelengths are $\lambda_3 = 583\text{ nm}$ and $\lambda_4 = 453\text{ nm}$.

EVALUATE: The wavelengths for constructive interference are between those for destructive interference.

35.5. **IDENTIFY:** If the path difference between the two waves is equal to a whole number of wavelengths, constructive interference occurs, but if it is an odd number of half-wavelengths, destructive interference occurs.

SET UP: We calculate the distance traveled by both waves and subtract them to find the path difference.

EXECUTE: Call P_1 the distance from the right speaker to the observer and P_2 the distance from the left speaker to the observer.

(a) $P_1 = 8.0$ m and $P_2 = \sqrt{(6.0\text{ m})^2 + (8.0\text{ m})^2} = 10.0\text{ m}$. The path distance is

$$\Delta P = P_2 - P_1 = 10.0\text{ m} - 8.0\text{ m} = 2.0\text{ m}$$

(b) The path distance is one wavelength, so constructive interference occurs.

(c) $P_1 = 17.0$ m and $P_2 = \sqrt{(6.0\,\text{m})^2 + (17.0\,\text{m})^2} = 18.0\,\text{m}$. The path difference is 18.0 m – 17.0 m = 1.0 m, which is one-half wavelength, so destructive interference occurs.

EVALUATE: Constructive interference also occurs if the path difference 2λ, 3λ, 4λ, etc., and destructive interference occurs if it is $\lambda/2$, $3\lambda/2$, $5\lambda/2$, etc.

35.6. **IDENTIFY:** At an antinode the interference is constructive and the path difference is an integer number of wavelengths; path difference $= m\lambda,\ m = 0,\pm1,\pm2,\ldots$ at an antinode.

SET UP: The maximum magnitude of the path difference is the separation d between the two sources.

EXECUTE: **(a)** At $S_1, r_2 - r_1 = 4\lambda$, and this path difference stays the same all along the y-axis, so $m = +4$. At $S_2, r_2 - r_1 = -4\lambda,$ and the path difference below this point, along the negative y-axis, stays the same, so $m = -4$.

(b) The wave pattern is sketched in Figure 35.6.

(c) The maximum and minimum m-values are determined by the largest integer less than or equal to $\dfrac{d}{\lambda}$.

(d) If $d = 7\frac{1}{2}\lambda \Rightarrow -7 \le m \le +7$, so there will be a total of 15 antinodes between the sources.

EVALUATE: We are considering points close to the two sources and the antinodal curves are not straight lines.

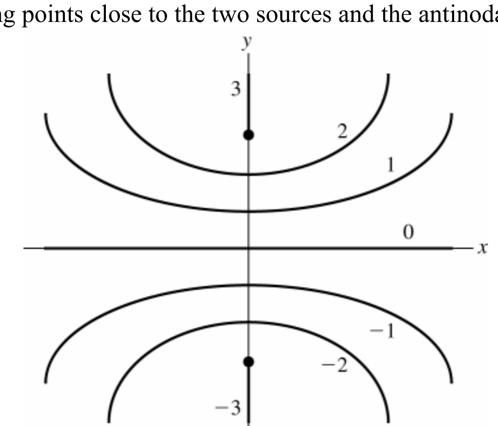

Figure 35.6

35.7. **IDENTIFY:** At an antinodal point the path difference is equal to an integer number of wavelengths.
SET UP: For $m=3$, the path difference is 3λ.
EXECUTE: Measuring with a ruler from both S_1 and S_2 to the different points in the antinodal line labeled $m=3$, we find that the difference in path length is three times the wavelength of the wave, as measured from one crest to the next on the diagram.
EVALUATE: There is a whole curve of points where the path difference is 3λ.

35.8. **IDENTIFY:** The value of y_{20} is much smaller than *R* and the approximate expression $y_m = R\dfrac{m\lambda}{d}$ is accurate.

SET UP: $y_{20} = 10.6\times10^{-3}$ m.

EXECUTE: $d = \dfrac{20R\lambda}{y_{20}} = \dfrac{(20)(1.20 \text{ m})(502\times10^{-9} \text{ m})}{10.6\times10^{-3} \text{ m}} = 1.14\times10^{-3} \text{ m} = 1.14 \text{ mm}$

EVALUATE: $\tan\theta_{20} = \dfrac{y_{20}}{R}$ so $\theta_{20} = 0.51^\circ$ and the approximation $\sin\theta_{20} \approx \tan\theta_{20}$ is very accurate.

35.9. **IDENTIFY** and **SET UP:** The dark lines correspond to destructive interference and hence are located by Eq.(35.5):

$$d\sin\theta = \left(m+\frac{1}{2}\right)\lambda \text{ so } \sin\theta = \frac{\left(m+\frac{1}{2}\right)\lambda}{d},\quad m = 0, \pm1, \pm2, \ldots$$

Solve for θ that locates the second and third dark lines. Use $y = R\tan\theta$ to find the distance of each of the dark lines from the center of the screen.
EXECUTE: 1st dark line is for $m=0$

2nd dark line is for $m=1$ and $\sin\theta_1 = \dfrac{3\lambda}{2d} = \dfrac{3(500\times10^{-9} \text{ m})}{2(0.450\times10^{-3}\text{m})} = 1.667\times10^{-3}$ and $\theta_1 = 1.667\times10^{-3}$ rad

3rd dark line is for $m=2$ and $\sin\theta_2 = \dfrac{5\lambda}{2d} = \dfrac{5(500\times10^{-9} \text{ m})}{2(0.450\times10^{-3}\text{m})} = 2.778\times10^{-3}$ and $\theta_2 = 2.778\times10^{-3}$ rad

(Note that θ_1 and θ_2 are small so that the approximation $\theta \approx \sin\theta \approx \tan\theta$ is valid.) The distance of each dark line from the center of the central bright band is given by $y_m = R\tan\theta$, where $R = 0.850$ m is the distance to the screen.

$\tan\theta \approx \theta$ so $y_m = R\theta_m$

$y_1 = R\theta_1 = (0.750 \text{ m})(1.667\times10^{-3} \text{ rad}) = 1.25\times10^{-3} \text{ m}$

$y_2 = R\theta_2 = (0.750 \text{ m})(2.778\times10^{-3} \text{ rad}) = 2.08\times10^{-3} \text{ m}$

$\Delta y = y_2 - y_1 = 2.08\times10^{-3} \text{ m} - 1.25\times10^{-3} \text{ m} = 0.83 \text{ mm}$

EVALUATE: Since θ_1 and θ_2 are very small we could have used Eq.(35.6), generalized to destructive interference: $y_m = R\left(m+\dfrac{1}{2}\right)\lambda/d.$

35.10. **IDENTIFY:** Since the dark fringes are eqully spaced, $R \gg y_m$, the angles are small and the dark bands are located by $y_{m+\frac{1}{2}} = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: The separation between adjacent dark bands is $\Delta y = \dfrac{R\lambda}{d}$.

EXECUTE: $\Delta y = \dfrac{R\lambda}{d} \Rightarrow d = \dfrac{R\lambda}{\Delta y} = \dfrac{(1.80 \text{ m})\,(4.50\times10^{-7} \text{ m})}{4.20\times10^{-3} \text{ m}} = 1.93\times10^{-4} \text{ m} = 0.193 \text{ m}.$

EVALUATE: When the separation between the slits decreases, the separation between dark fringes increases.

35.11. **IDENTIFY** and **SET UP:** The positions of the bright fringes are given by Eq.(35.6): $y_m = R(m\lambda/d)$. For each fringe the adjacent fringe is located at $y_{m+1} = R(m+1)\lambda/d$. Solve for λ.
EXECUTE: The separation between adjacent fringes is $\Delta y = y_{m+1} - y_m = R\lambda/d$.

$$\lambda = \frac{d\Delta y}{R} = \frac{(0.460\times10^{-3} \text{ m})(2.82\times10^{-3} \text{ m})}{2.20 \text{ m}} = 5.90\times10^{-7} \text{ m} = 590 \text{ nm}$$

EVALUATE: Eq.(35.6) requires that the angular position on the screen be small. The angular position of bright fringes is given by $\sin\theta = m\lambda/d$. The slit separation is much larger than the wavelength $(\lambda/d = 1.3\times10^{-3})$, so θ is small so long as *m* is not extremely large.

35.12. **IDENTIFY:** The width of a bright fringe can be defined to be the distance between its two adjacent destructive minima. Assuming the small angle formula for destructive interference $y_m = R\dfrac{(m+\frac{1}{2})\lambda}{d}$.

SET UP: $d = 0.200\times10^{-3}$ m. $R = 4.00$ m.

EXECUTE: The distance between any two successive minima is $y_{m+1} - y_m = R\dfrac{\lambda}{d} = (4.00\text{ m})\dfrac{(400\times10^{-9}\text{m})}{(0.200\times10^{-3}\text{ m})} = 8.00$ mm. Thus, the answer to both part (a) and part (b) is that the width is 8.00 mm.

EVALUATE: For small angles, when $y_m \ll R$, the interference minima are equally spaced.

35.13. **IDENTIFY** and **SET UP:** The dark lines are located by $d\sin\theta = \left(m+\dfrac{1}{2}\right)\lambda.$ The distance of each line from the center of the screen is given by $y = R\tan\theta.$

EXECUTE: First dark line is for $m = 0$ and $d\sin\theta_1 = \lambda/2.$

$\sin\theta_1 = \dfrac{\lambda}{2d} = \dfrac{550\times10^{-9}\text{ m}}{2(1.80\times10^{-6}\text{ m})} = 0.1528$ and $\theta_1 = 8.789°.$ Second dark line is for $m = 1$ and $d\sin\theta_2 = 3\lambda/2.$

$\sin\theta_2 = \dfrac{3\lambda}{2d} = 3\left(\dfrac{550\times10^{-9}\text{ m}}{2(1.80\times10^{-6}\text{ m})}\right) = 0.4583$ and $\theta_2 = 27.28°.$

$y_1 = R\tan\theta_1 = (0.350\text{ m})\tan 8.789° = 0.0541\text{ m}$

$y_2 = R\tan\theta_2 = (0.350\text{ m})\tan 27.28° = 0.1805\text{ m}$

The distance between the lines is $\Delta y = y_2 - y_1 = 0.1805\text{ m} - 0.0541\text{ m} = 0.126\text{ m} = 12.6\text{ cm}.$

EVALUATE: $\sin\theta_1 = 0.1528$ and $\tan\theta_1 = 0.1546.$ $\sin\theta_2 = 0.4583$ and $\tan\theta_2 = 0.5157.$ As the angle increases, $\sin\theta \approx \tan\theta$ becomes a poorer approximation.

35.14. **IDENTIFY:** Using Eq.(35.6) for small angles: $y_m = R\dfrac{m\lambda}{d}$.

SET UP: First-order means $m = 1$.

EXECUTE: The distance between corresponding bright fringes is

$$\Delta y = \frac{Rm}{d}\Delta\lambda = \frac{(5.00\text{ m})(1)}{(0.300\times10^{-3}\text{ m})}(660-470)\times(10^{-9}\text{ m}) = 3.17\text{ mm}.$$

EVALUATE: The separation between these fringes for different wavelengths increases when the slit separation decreases.

35.15. **IDENTIFY** and **SET UP:** Use the information given about the bright fringe to find the distance d between the two slits. Then use Eq.(35.5) and $y = R\tan\theta$ to calculate λ for which there is a first-order dark fringe at this same place on the screen.

EXECUTE: $y_1 = \dfrac{R\lambda_1}{d}$, so $d = \dfrac{R\lambda_1}{y_1} = \dfrac{(3.00\text{ m})(600\times10^{-9}\text{ m})}{4.84\times10^{-3}\text{ m}} = 3.72\times10^{-4}$ m. (*R* is much greater than *d,* so Eq.35.6 is valid.) The dark fringes are located by $d\sin\theta = \left(m+\dfrac{1}{2}\right)\lambda,\ m = 0, \pm1, \pm2, \ldots$ The first order dark fringe is located by $\sin\theta = \lambda_2/2d,$ where λ_2 is the wavelength we are seeking.

$y = R\tan\theta \approx R\sin\theta = \dfrac{\lambda_2 R}{2d}$

We want λ_2 such that $y = y_1.$ This gives $\dfrac{R\lambda_1}{d} = \dfrac{R\lambda_2}{2d}$ and $\lambda_2 = 2\lambda_1 = 1200$ nm.

EVALUATE: For $\lambda = 600$ nm the path difference from the two slits to this point on the screen is 600 nm. For this same path difference (point on the screen) the path difference is $\lambda/2$ when $\lambda = 1200$ nm.

35.16. **IDENTIFY:** Bright fringes are located at $y_m = R\dfrac{m\lambda}{d}$, when $y_m \ll R$. Dark fringes are at $d\sin\theta = (m+\frac{1}{2})\lambda$ and $y = R\tan\theta$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{6.32\times10^{14}\text{ Hz}} = 4.75\times10^{-7}$ m. For the third bright fringe (not counting the central bright spot), $m = 3$. For the third dark fringe, $m = 2$.

EXECUTE: **(a)** $d=\dfrac{m\lambda R}{y_m}=\dfrac{3(4.75\times10^{-7}\text{ m})(0.850\text{ m})}{0.0311\text{ m}}=3.89\times10^{-5}\text{ m}=0.0389\text{ mm}$

(b) $\sin\theta=(2+\tfrac{1}{2})\dfrac{\lambda}{d}=(2.5)\left(\dfrac{4.75\times10^{-7}\text{ m}}{3.89\times10^{-5}\text{ m}}\right)=0.0305$ and $\theta=1.75°$. $y=R\tan\theta=(85.0\text{ cm})\tan 1.75°=2.60\text{ cm}$.

EVALUATE: The third dark fringe is closer to the center of the screen than the third bright fringe on one side of the central bright fringe.

35.17. **IDENTIFY:** Bright fringes are located at angles θ given by $d\sin\theta=m\lambda$.

SET UP: The largest value $\sin\theta$ can have is 1.00.

EXECUTE: **(a)** $m=\dfrac{d\sin\theta}{\lambda}$. For $\sin\theta=1$, $m=\dfrac{d}{\lambda}=\dfrac{0.0116\times10^{-3}\text{ m}}{5.85\times10^{-7}\text{ m}}=19.8$. Therefore, the largest *m* for fringes on the screen is $m=19$. There are $2(19)+1=39$ bright fringes, the central one and 19 above and 19 below it.

(b) The most distant fringe has $m=\pm19$. $\sin\theta=m\dfrac{\lambda}{d}=\pm19\left(\dfrac{5.85\times10^{-7}\text{ m}}{0.0116\times10^{-3}\text{ m}}\right)=\pm0.958$ and $\theta=\pm73.3°$.

EVALUATE: For small θ the spacing Δy between adjacent fringes is constant but this is no longer the case for larger angles.

35.18. **IDENTIFY:** At large distances from the antennas the equation $d\sin\theta=m\lambda,\ m=0,\pm1,\pm2,\ldots$ gives the angles where maximum intensity is observed and $d\sin\theta=(m+\tfrac{1}{2})\lambda,\ m=0,\pm1,\pm2,\ldots$ gives the angles where minimum intensity is observed.

SET UP: $d=12.0\text{ m}$. $\lambda=\dfrac{c}{f}$.

EXECUTE: **(a)** $\lambda=\dfrac{c}{f}=\dfrac{3.00\times10^{8}\text{ m/s}}{107.9\times10^{6}\text{ Hz}}=2.78\text{ m}$. $\sin\theta=\dfrac{m\lambda}{d}=m\left(\dfrac{2.78\text{ m}}{12.0\text{ m}}\right)=m(0.232)$.

$\theta=\pm13.4°,\ \pm27.6°,\ \pm44.1°,\ \pm68.1°$.

(b) $\sin\theta=(m+\tfrac{1}{2})\dfrac{\lambda}{d}=(m+\tfrac{1}{2})(0.232)$. $\theta=\pm6.66°,\ \pm20.4°,\ \pm35.5°,\ \pm54.3°$.

EVALUATE: The angles for zero intensity are approximately midway between those for maximum intensity.

35.19. **IDENTIFY:** Eq.(35.10): $I=I_0\cos^2(\phi/2)$. Eq.(35.11): $\phi=(2\pi/\lambda)(r_2-r_1)$.

SET UP: ϕ is the phase difference and (r_2-r_1) is the path difference.

EXECUTE: **(a)** $I=I_0(\cos 30.0°)^2=0.750I_0$

(b) $60.0°=(\pi/3)\text{ rad}$. $(r_2-r_1)=(\phi/2\pi)\lambda=[(\pi/3)/2\pi]\lambda=\lambda/6=80\text{ nm}$.

EVALUATE: $\phi=360°/6$ and $(r_2-r_1)=\lambda/6$.

35.20. **IDENTIFY:** $\dfrac{\Delta\phi}{2\pi}=\dfrac{\text{path difference}}{\lambda}$ relates the path difference to the phase difference $\Delta\phi$.

SET UP: The sources and point *P* are shown in Figure 35.20.

EXECUTE: $\Delta\phi=2\pi\left(\dfrac{524\text{ cm}-486\text{ cm}}{2\text{ cm}}\right)=119\text{ radians}$

EVALUATE: The distances from *B* to *P* and *A* to *P* aren't important, only the difference in these distances.

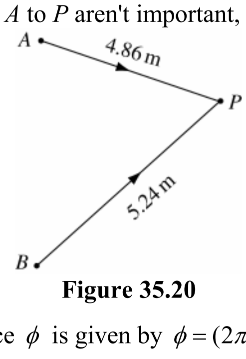

Figure 35.20

35.21. **IDENTIFY** and **SET UP:** The phase difference ϕ is given by $\phi=(2\pi d/\lambda)\sin\theta$ (Eq.35.13.)

EXECUTE: $\phi=[2\pi(0.340\times10^{-3}\text{ m})/(500\times10^{-9}\text{ m})]\sin 23.0°=1670\text{ rad}$

EVALUATE: The *m*th bright fringe occurs when $\phi=2\pi m$, so there are a large number of bright fringes within $23.0°$ from the centerline. Note that Eq.(35.13) gives ϕ in radians.

35.22. **IDENTIFY:** The maximum intensity occurs at all the points of constructive interference. At these points, the path difference between waves from the two transmitters is an integral number of wavelengths.

SET UP: For constructive interference, $\sin\theta = m\lambda/d$.

EXECUTE: **(a)** First find the wavelength of the UHF waves:

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(1575.42 \text{ MHz}) = 0.1904 \text{ m}$$

For maximum intensity $(\pi d \sin\theta)/\lambda = m\pi$, so

$$\sin\theta = m\lambda/d = m[(0.1904 \text{ m})/(5.18 \text{ m})] = 0.03676m$$

The maximum possible m would be for $\theta = 90°$, or $\sin\theta = 1$, so

$$m_{\text{max}} = d/\lambda = (5.18 \text{ m})/(0.1904 \text{ m}) = 27.2$$

which must be ±27 since m is an integer. The total number of maxima is 27 on either side of the central fringe, plus the central fringe, for a total of 27 + 27 + 1 = 55 bright fringes.

(b) Using $\sin\theta = m\lambda/d$, where $m = 0, \pm1, \pm2$, and ±3, we have

$$\sin\theta = m\lambda/d = m[(0.1904 \text{ m})/(5.18 \text{ m})] = 0.03676m$$

$m = 0$: $\sin\theta = 0$, which gives $\theta = 0°$

$m = \pm1$: $\sin\theta = \pm(0.03676)(1)$, which gives $\theta = \pm2.11°$

$m = \pm2$: $\sin\theta = \pm(0.03676)(2)$, which gives $\theta = \pm4.22°$

$m = \pm3$: $\sin\theta = \pm(0.03676)(3)$, which gives $\theta = \pm6.33°$

(c) $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \left(2.00\text{ W/m}^2\right)\cos^2\left[\dfrac{\pi(5.18\text{ m})\sin(4.65°)}{0.1904\text{ m}}\right] = 1.28 \text{ W/m}^2$.

EVALUATE: Notice that $\sin\theta$ increases in integer steps, but θ only increases in integer steps for small θ.

35.23. **(a) IDENTIFY** and **SET UP:** The minima are located at angles θ given by $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$. The first minimum corresponds to $m = 0$. Solve for θ. Then the distance on the screen is $y = R\tan\theta$.

EXECUTE: $\sin\theta = \dfrac{\lambda}{2d} = \dfrac{660\times10^{-9}\text{ m}}{2(0.260\times10^{-3}\text{ m})} = 1.27\times10^{-3}$ and $\theta = 1.27\times10^{-3}$ rad

$y = (0.700\text{ m})\tan(1.27\times10^{-3}\text{ rad}) = 0.889\text{ mm}.$

(b) IDENTIFY and **SET UP:** Eq.(35.15) given the intensity I as a function of the position y on the screen: $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$. Set $I = I_0/2$ and solve for y.

EXECUTE: $I = \frac{1}{2}I_0$ says $\cos^2\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{2}$

$\cos\left(\dfrac{\pi dy}{\lambda R}\right) = \dfrac{1}{\sqrt{2}}$ so $\dfrac{\pi dy}{\lambda R} = \dfrac{\pi}{4}$ rad

$y = \dfrac{\lambda R}{4d} = \dfrac{(660\times10^{-9}\text{ m})(0.700\text{ m})}{4(0.260\times10^{-3}\text{ m})} = 0.444\text{ mm}$

EVALUATE: $I = I_0/2$ at a point on the screen midway between where $I = I_0$ and $I = 0$.

35.24. **IDENTIFY:** Eq. (35.14): $I = I_0\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right)$.

SET UP: The intensity goes to zero when the cosine's argument becomes an odd integer multiple of $\dfrac{\pi}{2}$

EXECUTE: $\dfrac{\pi d}{\lambda}\sin\theta = (m + 1/2)\pi$ gives $d\sin\theta = \lambda(m + 1/2)$, which is Eq. (35.5).

EVALUATE: Section 35.3 shows that the maximum-intensity directions from Eq.(35.14) agree with Eq.(35.4).

35.25. **IDENTIFY:** The intensity decreases as we move away from the central maximum.

SET UP: The intensity is given by $I = I_0\cos^2\left(\dfrac{\pi dy}{\lambda R}\right)$.

EXECUTE: First find the wavelength: $\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(12.5 \text{ MHz}) = 24.00 \text{ m}$

At the farthest the receiver can be placed, $I = I_0/4$, which gives

$$\frac{I_0}{4} = I_0\cos^2\left(\frac{\pi dy}{\lambda R}\right) \Rightarrow \cos^2\left(\frac{\pi dy}{\lambda R}\right) = \frac{1}{4} \Rightarrow \cos\left(\frac{\pi dy}{\lambda R}\right) = \pm\frac{1}{2}$$

The solutions are $\pi dy/\lambda R = \pi/3$ and $2\pi/3$. Using $\pi/3$, we get

$$y = \lambda R/3d = (24.00 \text{ m})(500 \text{ m})/[3(56.0 \text{ m})] = 71.4 \text{ m}$$

It must remain within 71.4 m of point C.

EVALUATE: Using $\pi dy/\lambda R = 2\pi/3$ gives $y = 142.8$ m. But to reach this point, the receiver would have to go beyond 71.4 m from C, where the signal would be too weak, so this second point is not possible.

35.26. IDENTIFY: The phase difference ϕ and the path difference $r_1 - r_2$ are related by $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2)$. The intensity is given by $I = I_0 \cos^2\left(\dfrac{\phi}{2}\right)$.

SET UP: $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8 \text{ m/s}}{1.20\times10^8 \text{ Hz}} = 2.50 \text{ m}$. When the receiver measures zero intensity I_0, $\phi = 0$.

EXECUTE: **(a)** $\phi = \dfrac{2\pi}{\lambda}(r_1 - r_2) = \dfrac{2\pi}{2.50 \text{ m}}(1.8 \text{ m}) = 4.52 \text{ rad}.$

(b) $I = I_0\cos^2\left(\dfrac{\phi}{2}\right) = I_0\cos^2\left(\dfrac{4.52 \text{ rad}}{2}\right) = 0.404 I_0.$

EVALUATE: $(r_1 - r_2)$ is greater than $\lambda/2$, so one minimum has been passed as the receiver is moved.

35.27. IDENTIFY: Consider interference between rays reflected at the upper and lower surfaces of the film. Consider phase difference due to the path difference of $2t$ and any phase differences due to phase changes upon reflection.

SET UP: Consider Figure 35.27.

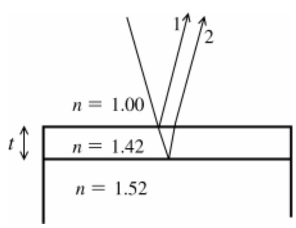

Figure 35.27

Both rays (1) and (2) undergo a $180°$ phase change on reflection, so these is no net phase difference introduced and the condition for destructive interference is $2t = \left(m + \dfrac{1}{2}\right)\lambda.$

EXECUTE: $t = \dfrac{\left(m + \frac{1}{2}\right)\lambda}{2}$; thinnest film says $m = 0$ so $t = \dfrac{\lambda}{4}$

$\lambda = \dfrac{\lambda_0}{1.42}$ and $t = \dfrac{\lambda_0}{4(1.42)} = \dfrac{650\times10^{-9} \text{ m}}{4(1.42)} = 1.14\times10^{-7} \text{ m} = 114 \text{ nm}$

EVALUATE: We compared the path difference to the wavelength in the film, since that is where the path difference occurs.

35.28. IDENTIFY: Require destructive interference for light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air ($n = 1.00$) reflects from the film ($n = 2.62$) and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{505 \text{ nm}}{2.62}$.

EXECUTE: **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{505 \text{ nm}}{2[2.62]}\right) = (96.4 \text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2$, 3 and 4: 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 35.27, where the minimum thickness for destructive interference is $t = \lambda/4n$.

35.29. IDENTIFY: The fringes are produced by interference between light reflected from the top and bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) is therefore $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.29. At a distance *x* from the point of contact of the two plates, the thickness of the air wedge is *t*.

EXECUTE: $\tan\theta = \frac{t}{x}$ so $t = x\tan\theta$. $t_m = (m + \frac{1}{2})\frac{\lambda}{2}$. $x_m = (m + \frac{1}{2})\frac{\lambda}{2\tan\theta}$ and $x_{m+1} = (m + \frac{3}{2})\frac{\lambda}{2\tan\theta}$. The distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \frac{\lambda}{2\tan\theta}$. 15.0 fringes/cm $= \frac{1.00}{\Delta x}$ and $\Delta x = \frac{1.00}{15.0\text{ fringes/cm}} = 0.0667\text{ cm}$. $\tan\theta = \frac{\lambda}{2\Delta x} = \frac{546\times 10^{-9}\text{ m}}{2(0.0667\times 10^{-2}\text{ m})} = 4.09\times 10^{-4}$. The angle of the wedge is $4.09\times 10^{-4}\text{ rad} = 0.0234°$.

EVALUATE: The fringes are equally spaced; Δx is independent of *m*.

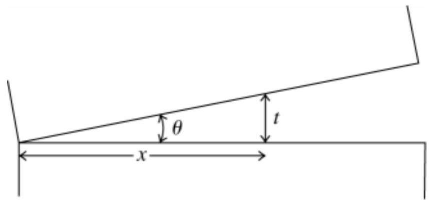

Figure 35.29

35.30. **IDENTIFY:** The fringes are produced by interference between light reflected from the top and from the bottom surfaces of the air wedge. The refractive index of glass is greater than that of air, so the waves reflected from the top surface of the air wedge have no reflection phase shift and the waves reflected from the bottom surface of the air wedge do have a half-cycle reflection phase shift. The condition for constructive interference (bright fringes) therefore is $2t = (m + \frac{1}{2})\lambda$.

SET UP: The geometry of the air wedge is sketched in Figure 35.30.

EXECUTE: $\tan\theta = \frac{0.0800\text{ mm}}{90.0\text{ mm}} = 8.89\times 10^{-4}$. $\tan\theta = \frac{t}{x}$ so $t = (8.89\times 10^{-4})x$. $t_m = (m + \frac{1}{2})\frac{\lambda}{2}$.

$x_m = (m + \frac{1}{2})\frac{\lambda}{2(8.89\times 10^{-4})}$ and $x_{m+1} = (m + \frac{3}{2})\frac{\lambda}{2(8.89\times 10^{-4})}$. The distance along the plate between adjacent fringes is $\Delta x = x_{m+1} - x_m = \frac{\lambda}{2(8.89\times 10^{-4})} = \frac{656\times 10^{-9}\text{ m}}{2(8.89\times 10^{-4})} = 3.69\times 10^{-4}\text{ m} = 0.369\text{ mm}$. The number of fringes per cm is $\frac{1.00}{\Delta x} = \frac{1.00}{0.0369\text{ cm}} = 27.1\text{ fringes/cm}$.

EVALUATE: As $t \to 0$ the interference is destructive and there is a dark fringe at the line of contact between the two plates.

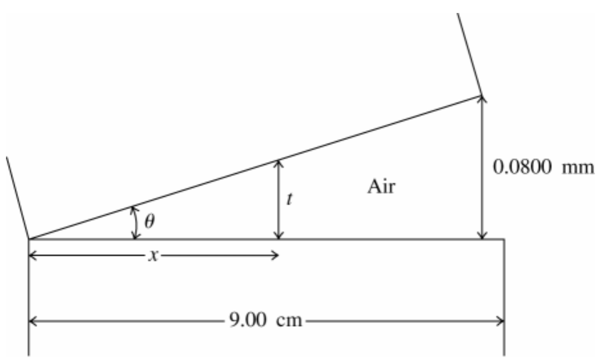

Figure 35.30

35.31. **IDENTIFY:** The light reflected from the top of the TiO_2 film interferes with the light reflected from the top of the glass surface. These waves are out of phase due to the path difference in the film and the phase differences caused by reflection.

SET UP: There is a π phase change at the TiO_2 surface but none at the glass surface, so for destructive interference the path difference must be $m\lambda$ in the film.

EXECUTE: **(a)** Calling *T* the thickness of the film gives $2T = m\lambda_0/n$, which yields $T = m\lambda_0/(2n)$. Substituting the numbers gives

$$T = m\,(520.0\text{ nm})/[2(2.62)] = 99.237m$$

T must be greater than 1036 nm, so $m = 11$, which gives $T = 1091.6$ nm, since we want to know the minimum thickness to add.

$$\Delta T = 1091.6 \text{ nm} - 1036 \text{ nm} = 55.6 \text{ nm}$$

(b) (i) Path difference = $2T = 2(1092 \text{ nm}) = 2184 \text{ nm} = 2180$ nm.
(ii) The wavelength in the film is $\lambda = \lambda_0/n = (520.0 \text{ nm})/2.62 = 198.5$ nm.

$$\text{Path difference} = (2180 \text{ nm})/[(198.5 \text{ nm})/\text{wavelength}] = 11.0 \text{ wavelengths}$$

EVALUATE: Because the path difference in the film is 11.0 wavelengths, the light reflected off the top of the film will be 180° out of phase with the light that traveled through the film and was reflected off the glass due to the phase change at reflection off the top of the film.

35.32. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections. For destructive interference the total phase difference is an integer number of half cycles.
SET UP: The reflection at the top surface of the film produces a half-cycle phase shift. There is no phase shift at the reflection at the bottom surface.
EXECUTE: **(a)** Since there is a half-cycle phase shift at just one of the interfaces, the minimum thickness for constructive interference is $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{550 \text{ nm}}{4(1.85)} = 74.3$ nm.

(b) The next smallest thickness for constructive interference is with another half wavelength thickness added:
$t = \dfrac{3\lambda}{4} = \dfrac{3\lambda_0}{4n} = \dfrac{3(550 \text{ nm})}{4(1.85)} = 223$ nm.

EVALUATE: Note that we must compare the path difference to the wavelength in the film.

35.33. **IDENTIFY:** Consider the interference between rays reflected from the two surfaces of the soap film. Strongly reflected means constructive interference. Consider phase difference due to the path difference of $2t$ and any phase difference due to phase changes upon reflection.
(a) SET UP: Consider Figure 35.33.

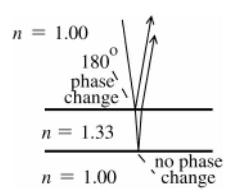

Figure 35.33

There is a 180° phase change when the light is reflected from the outside surface of the bubble and no phase change when the light is reflected from the inside surface.

EXECUTE: The reflections produce a net 180° phase difference and for there to be constructive interference the path difference $2t$ must correspond to a half-integer number of wavelengths to compensate for the $\lambda/2$ shift due to the reflections. Hence the condition for constructive interference is $2t = \left(m + \dfrac{1}{2}\right)(\lambda_0/n),\ m = 0,1,2,\ldots$ Here λ_0 is the wavelength in air and (λ_0/n) is the wavelength in the bubble, where the path difference occurs.

$$\lambda_0 = \frac{2tn}{m+\frac{1}{2}} = \frac{2(290 \text{ nm})(1.33)}{m+\frac{1}{2}} = \frac{771.4 \text{ nm}}{m+\frac{1}{2}}$$

for $m = 0$, $\lambda = 1543$ nm; for $m = 1$, $\lambda = 514$ nm; for $m = 2$, $\lambda = 308$ nm;… Only 514 nm is in the visible region; the color for this wavelength is green.

(b) $\lambda_0 = \dfrac{2tn}{m+\frac{1}{2}} = \dfrac{2(340 \text{ nm})(1.33)}{m+\frac{1}{2}} = \dfrac{904.4 \text{ nm}}{m+\frac{1}{2}}$

for $m = 0$, $\lambda = 1809$ nm; for $m = 1$, $\lambda = 603$ nm; for $m = 2$, $\lambda = 362$ nm;… Only 603 nm is in the visible region; the color for this wavelength is orange.

EVALUATE: The dominant color of the reflected light depends on the thickness of the film. If the bubble has varying thickness at different points, these points will appear to be different colors when the light reflected from the bubble is viewed.

35.34. **IDENTIFY:** The number of waves along the path is the path length divided by the wavelength. The path difference and the reflections determine the phase difference.

SET UP: The path length is $2t = 17.52 \times 10^{-6}$ m. The wavelength in the film is $\lambda = \dfrac{\lambda_0}{n}$.

EXECUTE: **(a)** $\lambda = \dfrac{648 \text{ nm}}{1.35} = 480 \text{ nm}$. The number of waves is $\dfrac{2t}{\lambda} = \dfrac{17.52\times10^{-6} \text{ m}}{480\times10^{-9} \text{ m}} = 36.5$.

(b) The path difference introduces a $\lambda/2$, or $180°$, phase difference. The ray reflected at the top surface of the film undergoes a $180°$ phase shift upon reflection. The reflection at the lower surface introduces no phase shift. Both rays undergo a $180°$ phase shift, one due to reflection and one due to reflection. The two effects cancel and the two rays are in phase as they leave the film.

EVALUATE: Note that we must use the wavelength in the film to determine the number of waves in the film.

35.35. **IDENTIFY:** Require destructive interference between light reflected from the two points on the disc.

SET UP: Both reflections occur for waves in the plastic substrate reflecting from the reflective coating, so they both have the same phase shift upon reflection and the condition for destructive interference (cancellation) is $2t = (m + \frac{1}{2})\lambda$, where t is the depth of the pit. $\lambda = \dfrac{\lambda_0}{n}$. The minimum pit depth is for $m = 0$.

EXECUTE: $2t = \dfrac{\lambda}{2}$. $t = \dfrac{\lambda}{4} = \dfrac{\lambda_0}{4n} = \dfrac{790 \text{ nm}}{4(1.8)} = 110 \text{ nm} = 0.11\ \mu\text{m}$.

EVALUATE: The path difference occurs in the plastic substrate and we must compare the wavelength in the substrate to the path difference.

35.36. **IDENTIFY:** Consider light reflected at the front and rear surfaces of the film.

SET UP: At the front surface of the film, light in air ($n = 1.00$) reflects from the film ($n = 2.62$) and there is a $180°$ phase shift due to the reflection. At the back surface of the film, light in the film ($n = 2.62$) reflects from glass ($n = 1.62$) and there is no phase shift due to reflection. Therefore, there is a net $180°$ phase difference produced by the reflections. The path difference for these two rays is $2t$, where t is the thickness of the film. The wavelength in the film is $\lambda = \dfrac{505 \text{ nm}}{2.62}$.

EXECUTE: **(a)** Since the reflection produces a net $180°$ phase difference, destructive interference of the reflected light occurs when $2t = m\lambda$. $t = m\left(\dfrac{505 \text{ nm}}{2[2.62]}\right) = (96.4 \text{ nm})m$. The minimum thickness is 96.4 nm.

(b) The next three thicknesses are for $m = 2$, 3 and 4: 192 nm, 289 nm and 386 nm.

EVALUATE: The minimum thickness is for $t = \lambda/2n$. Compare this to Problem 34.27, where the minimum thickness for destructive interference is $t = \lambda/4n$.

35.37. **IDENTIFY** and **SET UP:** Apply Eq.(35.19) and calculate y for $m = 1800$.

EXECUTE: Eq.(35.19): $y = m(\lambda/2) = 1800(633\times10^{-9} \text{ m})/2 = 5.70\times10^{-4} \text{ m} = 0.570 \text{ mm}$

EVALUATE: A small displacement of the mirror corresponds to many wavelengths and a large number of fringes cross the line.

35.38. **IDENTIFY:** Apply Eq.(35.19).

SET UP: $m = 818$. Since the fringes move in opposite directions, the two people move the mirror in opposite directions.

EXECUTE: **(a)** For Jan, the total shift was $y_1 = \dfrac{m\lambda_1}{2} = \dfrac{818(6.06\times10^{-7} \text{ m})}{2} = 2.48\times10^{-4}$m. For Linda, the total shift was $y_2 = \dfrac{m\lambda_2}{2} = \dfrac{818(5.02\times10^{-7} \text{ m})}{2} = 2.05\times10^{-4}$ m.

(b) The net displacement of the mirror is the difference of the above values:

$$\Delta y = y_1 - y_2 = 0.248 \text{ mm} - 0.205 \text{ mm} = 0.043 \text{ mm}.$$

EVALUATE: The person using the larger wavelength moves the mirror the greater distance.

35.39. **IDENTIFY:** Consider the interference between light reflected from the top and bottom surfaces of the air film between the lens and the glass plate.

SET UP: For maximum intensity, with a net half-cycle phase shift due to reflections, $2t = \left(m + \dfrac{1}{2}\right)\lambda$. $t = R - \sqrt{R^2 - r^2}$.

EXECUTE: $\dfrac{(2m+1)\lambda}{4} = R - \sqrt{R^2 - r^2} \Rightarrow \sqrt{R^2 - r^2} = R - \dfrac{(2m+1)\lambda}{4}$

$$\Rightarrow R^2 - r^2 = R^2 + \left[\frac{(2m+1)\lambda}{4}\right]^2 - \frac{(2m+1)\lambda R}{2} \Rightarrow r = \sqrt{\frac{(2m+1)\lambda R}{2} - \left[\frac{(2m+1)\lambda}{4}\right]^2}$$

$$\Rightarrow r \approx \sqrt{\frac{(2m+1)\lambda R}{2}}, \text{ for } R \gg \lambda.$$

The second bright ring is when $m = 1$:

$$r \approx \sqrt{\frac{(2(1)+1)\,(5.80\times10^{-7}\,\text{m})\,(0.952\ \text{m})}{2}} = 9.10\times10^{-4}\,\text{m} = 0.910\ \text{mm}.$$

So the diameter of the second bright ring is 1.82 mm.

EVALUATE: The diameter of the m^{th} ring is proportional to $\sqrt{2m+1}$, so the rings get closer together as m increases. This agrees with Figure 35.17b in the textbook.

35.40. **IDENTIFY:** As found in Problem 35.39, the radius of the mth bright ring is $r \approx \sqrt{\dfrac{(2m+1)\lambda R}{2}}$, for $R \gg \lambda$.

SET UP: Introducing a liquid between the lens and the plate just changes the wavelength from λ to $\dfrac{\lambda}{n}$, where n is the refractive index of the liquid.

EXECUTE: $r(n) \approx \sqrt{\dfrac{(2m+1)\lambda R}{2n}} = \dfrac{r}{\sqrt{n}} = \dfrac{0.850\ \text{mm}}{\sqrt{1.33}} = 0.737\ \text{mm}.$

EVALUATE: The refractive index of the water is less than that of the glass plate, so the phase changes on reflection are the same as when air is in the space.

35.41. **IDENTIFY:** The liquid alters the wavelength of the light and that affects the locations of the interference minima.

SET UP: The interference minima are located by $d\sin\theta = (m+\tfrac{1}{2})\lambda$. For a liquid with refractive index n, $\lambda_{\text{liq}} = \dfrac{\lambda_{\text{air}}}{n}$.

EXECUTE: $\dfrac{\sin\theta}{\lambda} = \dfrac{(m+\frac{1}{2})}{d} =$ constant , so $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{liq}}}$. $\dfrac{\sin\theta_{\text{air}}}{\lambda_{\text{air}}} = \dfrac{\sin\theta_{\text{liq}}}{\lambda_{\text{air}}/n}$ and $n = \dfrac{\sin\theta_{\text{air}}}{\sin\theta_{\text{liq}}} = \dfrac{\sin 35.20^\circ}{\sin 19.46^\circ} = 1.730$.

EVALUATE: In the liquid the wavelength is shorter and $\sin\theta = (m+\tfrac{1}{2})\dfrac{\lambda}{d}$ gives a smaller θ than in air, for the same m.

35.42. **IDENTIFY:** As the brass is heated, thermal expansion will cause the two slits to move farther apart.

SET UP: For destructive interference, $d\sin\theta = \lambda/2$. The change in separation due to thermal expansion is $dw = \alpha w_0\,dT$, where w is the distance between the slits.

EXECUTE: The first dark fringe is at $d\sin\theta = \lambda/2 \Rightarrow \sin\theta = \lambda/2d$.

Call $d \equiv w$ for these calculations to avoid confusion with the differential. $\sin\theta = \lambda/2w$

Taking differentials gives $d(\sin\theta) = d(\lambda/2w)$ and $\cos\theta\,d\theta = -\lambda/2\,dw/w^2$.

For thermal expansion, $dw = \alpha w_0\,dT$, which gives $\cos\theta\,d\theta = -\dfrac{\lambda}{2}\dfrac{\alpha w_0 dT}{w_0^2} = -\dfrac{\lambda\alpha dT}{2w_0}$. Solving for $d\theta$ gives

$d\theta = -\dfrac{\lambda\alpha dT}{2w_0\cos\theta_0}$. Get λ: $w_0\sin\theta_0 = \lambda/2 \rightarrow \lambda = 2w_0\sin\theta_0$. Substituting this quantity into the equation for $d\theta$ gives

$$d\theta = -\frac{2w_0\sin\theta_0\alpha dT}{2w_0\cos\theta_0} = -\tan\theta_0\,\alpha dT.$$

$$d\theta = -\tan(32.5^\circ)(2.0\times10^{-5}\ \text{K}^{-1})(115\ \text{K}) = -0.001465\ \text{rad} = -0.084^\circ$$

The minus sign tells us that the dark fringes move closer together.

EVALUATE: We can also see that the dark fringes move closer together because $\sin\theta$ is proportional to $1/d$, so as d increases due to expansion, θ decreases.

35.43. **IDENTIFY:** *Both* frequencies will interfere constructively when the path difference from both of them is an integral number of wavelengths.

SET UP: Constructive interference occurs when $\sin\theta = m\lambda/d$.

EXECUTE: First find the two wavelengths.

$$\lambda_1 = v/f_1 = (344\ \text{m/s})/(900\ \text{Hz}) = 0.3822\ \text{m}$$

$$\lambda_2 = v/f_2 = (344\ \text{m/s})/(1200\ \text{Hz}) = 0.2867\ \text{m}$$

To interfere constructively at the same angle, the angles must be the same, and hence the sines of the angles must be equal. Each sine is of the form $\sin\theta = m\lambda/d$, so we can equate the sines to get

$$m_1\lambda_1/d = m_2\lambda 2/d$$

$$m_1(0.3822\ \text{m}) = m_2(0.2867\ \text{m})$$

$$m_2 = 4/3\ m_1$$

Since both m_1 and m_2 must be integers, the allowed pairs of values of m_1 and m_2 are

$$m_1 = m_2 = 0$$
$$m_1 = 3,\ m_2 = 4$$
$$m_1 = 6,\ m_2 = 8$$
$$m_1 = 9,\ m_2 = 12$$
etc.

For $m_1 = m_2 = 0$, we have $\theta = 0$.
For $m_1 = 3$, $m_2 = 4$, we have $\sin\theta_1 = (3)(0.3822\text{ m})/(2.50\text{ m})$, giving $\theta_1 = 27.3°$
For $m_1 = 6$, $m_2 = 8$, we have $\sin\theta_1 = (6)(0.3822\text{ m})/(2.50\text{ m})$, giving $\theta_1 = 66.5°$
For $m_1 = 9$, $m_2 = 12$, we have $\sin\theta_1 = (9)(0.3822\text{ m})/(2.50\text{ m}) = 1.38 > 1$, so no angle is possible.
EVALUATE: At certain other angles, one frequency will interfere constructively, but the other will not.

35.44. **IDENTIFY:** For destructive interference, $d = r_2 - r_1 = \left(m + \frac{1}{2}\right)\lambda$.

SET UP: $r_2 - r_1 = \sqrt{(200\text{ m})^2 + x^2} - x$

EXECUTE: $(200\text{ m})^2 + x^2 = x^2 + \left[\left(m + \frac{1}{2}\right)\lambda\right]^2 + 2x\left(m + \frac{1}{2}\right)\lambda$.

$x = \dfrac{20{,}000\text{ m}^2}{\left(m + \frac{1}{2}\right)\lambda} - \dfrac{1}{2}\left(m + \dfrac{1}{2}\right)\lambda$. The wavelength is calculated by $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^8\text{ m/s}}{5.80\times10^6\text{ Hz}} = 51.7\text{ m}$.

$m = 0: x = 761\text{ m}$; $m = 1: x = 219\text{ m}$; $m = 2: x = 90.1\text{ m}$; $m = 3$; $x = 20.0\text{ m}$.

EVALUATE: For $m = 3$, $d = 3.5\lambda = 181\text{ m}$. The maximum possible path difference is the separation of 200 m between the sources.

35.45. **IDENTIFY:** The two scratches are parallel slits, so the light that passes through them produces an interference pattern. However the light is traveling through a medium (plastic) that is different from air.
SET UP: The central bright fringe is bordered by a dark fringe on each side of it. At these dark fringes, $d\sin\theta = \frac{1}{2}\lambda/n$, where n is the refractive index of the plastic.
EXECUTE: First use geometry to find the angles at which the two dark fringes occur. At the first dark fringe $\tan\theta = [(5.82\text{ mm})/2]/(3250\text{ mm})$, giving $\theta = \pm 0.0513°$
For destructive interference, we have $d\sin\theta = \frac{1}{2}\lambda/n$ and

$$n = \lambda/(2d\sin\theta) = (632.8\text{ nm})/[2(0.000225\text{ m})(\sin 0.0513°)] = 1.57$$

EVALUATE: The wavelength of the light in the plastic is reduced compared to what it would be in air.

35.46. **IDENTIFY:** Interference occurs due to the path difference of light in the thin film.
SET UP: Originally the path difference was an odd number of half-wavelengths for cancellation to occur. If the path difference decreases by ½ wavelength, it will be a multiple of the wavelength, so constructive interference will occur.
EXECUTE: Calling ΔT the thickness that must be removed, we have

$$\text{path difference} = 2\Delta T = \tfrac{1}{2}\lambda/n \text{ and } \quad \Delta T = \lambda/4n = (525\text{ nm})/[4(1.40)] = 93.75\text{ nm},$$

At 4.20 nm/yr, we have $(4.20\text{ nm/yr})t = 93.75$ nm and $t = 22.3$ yr.
EVALUATE: If you were giving a warranty on this film, you certainly could not give it a "lifetime guarantee"!

35.47. **IDENTIFY** and **SET UP:** If the total phase difference is an integer number of cycles the interference is constructive and if it is a half-integer number of cycles it is destructive.
EXECUTE: **(a)** If the two sources are out of phase by one half-cycle, we must add an extra half a wavelength to the path difference equations Eq.(35.1) and Eq.(35.2). This exactly changes one for the other, for $m \to m + \frac{1}{2}$ and $m + \frac{1}{2} \to m$, since m in any integer.

(b) If one source leads the other by a phase angle ϕ, the fraction of a cycle difference is $\dfrac{\phi}{2\pi}$. Thus the path length difference for the two sources must be adjusted for both destructive and constructive interference, by this amount. So for constructive inference: $r_1 - r_2 = (m + \phi/2\pi)\lambda$, and for destructive interference, $r_1 - r_2 = (m + 1/2 + \phi/2\pi)\lambda$, where in each case $m = 0, \pm1, \pm2, \ldots$

EVALUATE: If $\phi = 0$ these results reduce to Eqs.(35.1) and (35.2).

35.48. **IDENTIFY:** Follow the steps specified in the problem.
SET UP: Use $\cos(\omega t + \phi/2) = \cos(\omega t)\cos(\phi/2) - \sin(\omega t)\sin(\phi/2)$. Then

$2\cos(\phi/2)\cos(\omega t + \phi/2) = 2\cos(\omega t)\cos^2(\phi/2) - 2\sin(\omega t)\sin(\phi/2)\cos(\phi/2)$. Then use $\cos^2(\phi/2) = \dfrac{1 + \cos(\phi)}{2}$ and

$2\sin(\phi/2)\cos(\phi/2) = \sin\phi$. This gives $\cos(\omega t) + (\cos(\omega t)\cos(\phi) - \sin(\omega t)\sin(\phi)) = \cos(\omega t) + \cos(\omega t + \phi)$, using again the trig identity for the cosine of the sum of two angles.

EXECUTE: **(a)** The electric field is the sum of the two fields and can be written as $E_P(t) = E_2(t) + E_1(t) = E\cos(\omega t) + E\cos(\omega t + \phi)$. $E_P(t) = 2E\cos(\phi/2)\cos(\omega t + \phi/2)$.

(b) $E_p(t) = A\cos(\omega t + \phi/2)$, so comparing with part (a), we see that the amplitude of the wave (which is always positive) must be $A = 2E\,|\cos(\phi/2)|$.

(c) To have an interference maximum, $\frac{\phi}{2} = 2\pi m$. So, for example, using $m = 1$, the relative phases are

E_2: 0; E_1: $\phi = 4\pi$; E_p: $\frac{\phi}{2} = 2\pi$, and all waves are in phase.

(d) To have an interference minimum, $\frac{\phi}{2} = \pi\left(m + \frac{1}{2}\right)$. So, for example using $m = 0$, relative phases are

E_2: 0; E_1: $\phi = \pi$; E_p: $\phi/2 = \pi/2$, and the resulting wave is out of phase by a quarter of a cycle from both of the original waves.

(e) The instantaneous magnitude of the Poynting vector is

$$|\vec{\boldsymbol{S}}| = \varepsilon_0 c E_p^2(t) = \varepsilon_0 c(4E^2\cos^2(\phi/2)\cos^2(\omega t + \phi/2)).$$

For a time average, $\cos^2(\omega t + \phi/2) = \frac{1}{2}$, so $|S_{\text{av}}| = 2\varepsilon_0 c E^2\cos^2(\phi/2)$.

EVALUATE: The result of part (e) shows that the intensity at a point depends on the phase difference ϕ at that point for the waves from each source.

35.49. **IDENTIFY:** Follow the steps specified in the problem.

SET UP: The definition of hyperbola is the locus of points such that the difference between P to S_2 and P to S_1 is a constant.

EXECUTE: **(a)** $\Delta r = m\lambda$. $r_1 = \sqrt{x^2 + (y-d)^2}$ and $r_2 = \sqrt{x^2 + (y+d)^2}$.

$\Delta r = \sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = m\lambda$.

(b) For a given m and λ, Δr is a constant and we get a hyperbola. Or, in the case of all m for a given λ, a family of hyperbolas.

(c) $\sqrt{x^2 + (y+d)^2} - \sqrt{x^2 + (y-d)^2} = (m + \frac{1}{2})\lambda.$

EVALUATE: The hyperbolas approach straight lines at large distances from the source.

35.50. **IDENTIFY:** Follow the derivation of Eq.(35.7), but with different amplitudes for the two waves.

SET UP: $\cos(\pi - \phi) = -\cos\phi$

EXECUTE: **(a)** $E_p^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(\pi - \phi) = E^2 + 4E^2 + 4E^2\cos\phi = 5E^2 + 4E^2\cos\phi$

$I = \frac{1}{2}\varepsilon_0 c E_p{}^2 = \varepsilon_0 c\left[\left(\frac{5}{2}E^2\right) + \left(\frac{4}{2}E^2\right)\cos\phi\right]$. $\phi = 0 \Rightarrow I_0 = \frac{9}{2}\varepsilon_0 c E^2$. Therefore, $I = I_0\left[\frac{5}{9} + \frac{4}{9}\cos\phi\right]$.

(b) The graph is shown in Figure 35.50. $I_{\min} = \frac{1}{9}I_0$ which occurs when $\phi = n\pi (n \text{ odd})$.

EVALUATE: The maxima and minima occur at the same points on the screen as when the two sources have the same amplitude, but when the amplitudes are different the intensity is no longer zero at the minima.

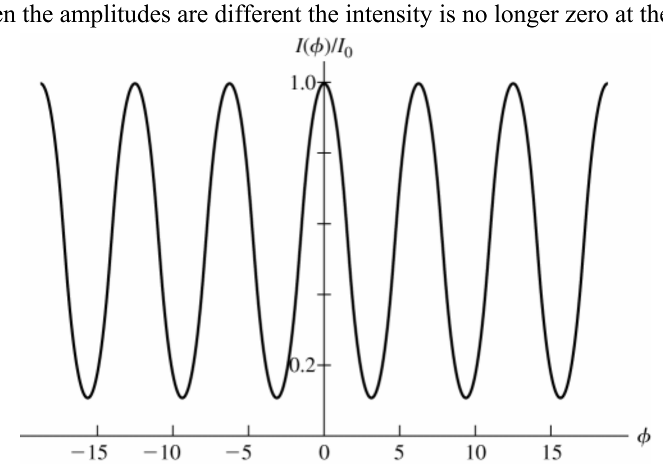

Figure 35.50

35.51. **IDENTIFY** and **SET UP:** Consider interference between rays reflected from the upper and lower surfaces of the film to relate the thickness of the film to the wavelengths for which there is destructive interference. The thermal expansion of the film changes the thickness of the film when the temperature changes.

EXECUTE: For this film on this glass, there is a net $\lambda/2$ phase change due to reflection and the condition for destructive interference is $2t = m(\lambda/n)$, where $n = 1.750$.

Smallest nonzero thickness is given by $t = \lambda/2n$.

At $20.0°\text{C}$, $t_0 = (582.4\text{ nm})/[(2)(1.750)] = 166.4\text{ nm}$.

At $170°\text{C}$, $t_0 = (588.5\text{ nm})/[(2)(1.750)] = 168.1\text{ nm}$.

$t = t_0(1+\alpha\Delta T)$ so

$\alpha = (t - t_0)/(t_0\Delta T) = (1.7\text{ nm})/[(166.4\text{ nm})(150°\text{C})] = 6.8\times10^{-5}(\text{C°})^{-1}$

EVALUATE: When the film is heated its thickness increases, and it takes a larger wavelength in the film to equal $2t$. The value we calculated for α is the same order of magnitude as those given in Table 17.1.

35.52. **IDENTIFY** and **SET UP:** At the $m = 3$ bright fringe for the red light there must be destructive interference at this same θ for the other wavelength.

EXECUTE: For constructive interference: $d\sin\theta = m\lambda_1 \Rightarrow d\sin\theta = 3(700\text{ nm}) = 2100\text{ nm}$. For destructive interference: $d\sin\theta = \left(m+\frac{1}{2}\right)\lambda_2 \Rightarrow \lambda_2 = \frac{d\sin\theta}{m+\frac{1}{2}} = \frac{2100\text{ nm}}{m+\frac{1}{2}}$. So the possible wavelengths are

$\lambda_2 = 600\text{ nm}$, for $m = 3$, and $\lambda_2 = 467\text{ nm}$, for $m = 4$.

EVALUATE: Both d and θ drop out of the calculation since their combination is just the path difference, which is the same for both types of light.

35.53. **IDENTIFY:** Apply $I = I_0\cos\left(\frac{\pi d}{\lambda}\sin\theta\right)$.

SET UP: $I = I_0/2$ when $\frac{\pi d}{\lambda}\sin\theta$ is $\frac{\pi}{4}$ rad , $\frac{3\pi}{4}$ rad ,….

EXECUTE: First we need to find the angles at which the intensity drops by one-half from the value of the m^{th} bright fringe. $I = I_0\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right) = \frac{I_0}{2} \Rightarrow \frac{\pi d}{\lambda}\sin\theta \approx \frac{\pi d\theta_m}{\lambda} = (m+1/2)\frac{\pi}{2}$.

$m = 0 : \theta = \theta_m^- = \frac{\lambda}{4d};\ m = 1 : \theta = \theta_m^+ = \frac{3\lambda}{4d} \Rightarrow \Delta\theta_m = \frac{\lambda}{2d}$.

EVALUATE: There is no dependence on the m-value of the fringe, so all fringes at small angles have the same half-width.

34.54. **IDENTIFY:** Consider the phase difference produced by the path difference and by the reflections.

SET UP: There is just one half-cycle phase change upon reflection, so for constructive interference $2t = (m_1+\frac{1}{2})\lambda_1 = (m_2+\frac{1}{2})\lambda_2$, where these wavelengths are in the glass. The two different wavelengths differ by just one m-value, $m_2 = m_1 - 1$.

EXECUTE: $\left(m_1+\frac{1}{2}\right)\lambda_1 = \left(m_1-\frac{1}{2}\right)\lambda_2 \Rightarrow m_1(\lambda_2-\lambda_1) = \frac{\lambda_1+\lambda_2}{2} \Rightarrow m_1 = \frac{\lambda_1+\lambda_2}{2(\lambda_2-\lambda_1)}$.

$m_1 = \frac{477.0\text{ nm}+540.6\text{ nm}}{2(540.6\text{ nm}-477.0\text{ nm})} = 8.\ \ 2t = \left(8+\frac{1}{2}\right)\frac{\lambda_{01}}{n} \Rightarrow t = \frac{17(477.0\text{ nm})}{4(1.52)} = 1334\text{ nm}.$

EVALUATE: Now that we have t we can calculate all the other wavelengths for which there is constructive interference.

35.55. **IDENTIFY:** Consider the phase difference due to the path difference and due to the reflection of one ray from the glass surface.

(a) SET UP: Consider Figure 35.55

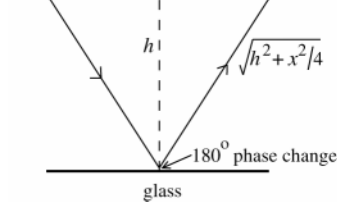

Figure 35.55

path difference $= 2\sqrt{h^2+x^2/4} - x = \sqrt{4h^2+x^2} - x$

Since there is a $180°$ phase change for the reflected ray, the condition for constructive interference is path difference $= \left(m+\frac{1}{2}\right)\lambda$ and the condition for destructive interference is path difference $= m\lambda$.

(b) EXECUTE: Constructive interference: $\left(m+\frac{1}{2}\right)\lambda = \sqrt{4h^2+x^2}-x$ and $\lambda = \dfrac{\sqrt{4h^2+x^2}-x}{m+\frac{1}{2}}$. Longest λ is for $m=0$ and then $\lambda = 2\left(\sqrt{4h^2+x^2}-x\right) = 2\left(\sqrt{4(0.24\text{ m})^2+(0.14\text{ m})^2}-0.14\text{ m}\right) = 0.72\text{ m}$

EVALUATE: For $\lambda = 0.72$ m the path difference is $\lambda/2$.

35.56. **IDENTIFY:** Require constructive interference for the reflection from the top and bottom surfaces of each cytoplasm layer and each guanine layer.

SET UP: At the water (or cytoplasm) to guanine interface, there is a half-cycle phase shift for the reflected light, but there is not one at the guanine to cytoplasm interface. Therefore there will always be one half-cycle phase difference between two neighboring reflected beams, just due to the reflections.

EXECUTE: For the guanine layers:

$$2t_{\text{g}} = (m+\frac{1}{2})\frac{\lambda}{n_{\text{g}}} \Rightarrow \lambda = \frac{2t_{\text{g}}n_{\text{g}}}{(m+\frac{1}{2})} = \frac{2(74\text{ nm})\,(1.80)}{(m+\frac{1}{2})} = \frac{266\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm } (m=0).$$

For the cytoplasm layers:

$$2t_{\text{c}} = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_{\text{c}}} \Rightarrow \lambda = \frac{2t_{\text{c}}n_{\text{c}}}{(m+\frac{1}{2})} = \frac{2(100\text{ nm})\,(1.333)}{(m+\frac{1}{2})} = \frac{267\text{ nm}}{(m+\frac{1}{2})} \Rightarrow \lambda = 533\text{ nm } (m=0).$$

(b) By having many layers the reflection is strengthened, because at each interface some more of the transmitted light gets reflected back, increasing the total percentage reflected.

(c) At different angles, the path length in the layers changes (always to a larger value than the normal incidence case). If the path length changes, then so do the wavelengths that will interfere constructively upon reflection.

EVALUATE: The thickness of the guanine and cytoplasm layers are inversely proportional to their refractive indices $\left(\frac{100}{74} = \frac{1.80}{1.333}\right)$, so both kinds of layers produce constructive interference for the same wavelength in air.

35.57. **IDENTIFY:** The slits will produce an interference pattern, but in the liquid, the wavelength of the light will be less than it was in air.

SET UP: The first bright fringe occurs when $d \sin\theta = \lambda/n$.

EXECUTE: In air: $d\sin 18.0° = \lambda$. In the liquid: $d\sin 12.6° = \lambda/n$. Dividing the equations gives

$$n = (\sin 18.0°)/(\sin 12.6°) = 1.42$$

EVALUATE: It was not necessary to know the spacing of the slits, since it was the same in both air and the liquid.

35.58. **IDENTIFY:** Consider light reflected at the top and bottom surfaces of the film. Wavelengths that are predominant in the transmitted light are those for which there is destructive interference in the reflected light.

SET UP: For the waves reflected at the top surface of the oil film there is a half-cycle reflection phase shift. For the waves reflected at the bottom surface of the oil film there is no reflection phase shift. The condition for constructive interference is $2t = (m+\frac{1}{2})\lambda$. The condition for destructive interference is $2t = m\lambda$. The range of visible wavelengths is approximately 400 nm to 700 nm. In the oil film, $\lambda = \frac{\lambda_0}{n}$.

EXECUTE: **(a)** $2t = (m+\frac{1}{2})\lambda = (m+\frac{1}{2})\frac{\lambda_0}{n}$. $\lambda_0 = \dfrac{2tn}{m+\frac{1}{2}} = \dfrac{2(380\text{ nm})(1.45)}{m+\frac{1}{2}} = \dfrac{1102\text{ nm}}{m+\frac{1}{2}}$.

$m=0$: $\lambda_0 = 2200$ nm. $m=1$: $\lambda_0 = 735$ nm. $m=2$: $\lambda_0 = 441$ nm. $m=3$: $\lambda_0 = 315$ nm. The visible wavelength for which there is constructive interference in the reflected light is 441 nm.

(b) $2t = m\lambda = m\frac{\lambda_0}{n}$. $\lambda_0 = \dfrac{2tn}{m} = \dfrac{1102\text{ nm}}{m}$. $m=1$: $\lambda_0 = 1102$ nm. $m=2$: $\lambda_0 = 551$ nm. $m=3$: $\lambda_0 = 367$ nm.

The visible wavelength for which there is destructive interference in the reflected light is 551 nm. This is the visible wavelength predominant in the transmitted light.

EVALUATE: At a particular wavelength the sum of the intensities of the reflected and transmitted light equals the intensity of the incident light.

35.59. **(a) IDENTIFY:** The wavelength in the glass is decreased by a factor of $1/n$, so for light through the upper slit a shorter path is needed to produce the same phase at the screen. Therefore, the interference pattern is shifted downward on the screen.

(b) SET UP: Consider the total phase difference produced by the path length difference and also by the different wavelength in the glass.

EXECUTE: At a point on the screen located by the angle θ the difference in path length is $d\sin\theta$. This introduces a phase difference of $\phi = \left(\dfrac{2\pi}{\lambda_0}\right)(d\sin\theta)$, where λ_0 is the wavelength of the light in air or vacuum.

In the thickness L of glass the number of wavelengths is $\dfrac{L}{\lambda} = \dfrac{nL}{\lambda_0}$. A corresponding length L of the path of the ray through the lower slit, in air, contains L/λ_0 wavelengths. The phase difference this introduces is $\phi = 2\pi\left(\dfrac{nL}{\lambda_0} - \dfrac{L}{\lambda_0}\right)$ and $\phi = 2\pi(n-1)(L/\lambda_0)$. The total phase difference is the sum of these two,

$\left(\dfrac{2\pi}{\lambda_0}\right)(d\sin\theta) + 2\pi(n-1)(L/\lambda_0) = (2\pi/\lambda_0)(d\sin\theta + L(n-1))$. Eq.(35.10) then gives

$$I = I_0\cos^2\left[\left(\frac{\pi}{\lambda_0}\right)(d\sin\theta + L(n-1))\right].$$

(c) Maxima means $\cos\phi/2 = \pm 1$ and $\phi/2 = m\pi,\ m = 0,\ \pm1,\ \pm2,\ldots$ $(\pi/\lambda_0)(d\sin\theta + L(n-1)) = m\pi$

$$d\sin\theta + L(n-1) = m\lambda_0$$

$$\sin\theta = \frac{m\lambda_0 - L(n-1)}{d}$$

EVALUATE: When $L \to 0$ or $n \to 1$ the effect of the plate goes away and the maxima are located by Eq.(35.4).

35.60. **IDENTIFY:** Dark fringes occur because the path difference is one-half of a wavelength.

SET UP: At the first dark fringe, $d\sin\theta = \lambda/2$. The intensity at any angle θ is given by $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$.

(a) At the first dark fringe, we have

$$d\sin\theta = \lambda/2$$

$$d/\lambda = 2/(2\sin 15.0°) = 1.93$$

(b) $I = I_0\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \dfrac{I_0}{10} \Rightarrow \cos\left(\dfrac{\pi d\sin\theta}{\lambda}\right) = \dfrac{1}{\sqrt{10}}$

$$\frac{\pi d\sin\theta}{\lambda} = \arccos\left(\frac{1}{\sqrt{10}}\right) = 71.57° = 1.249\text{ rad}$$

Using the result from part (a), that $d/\lambda = 1.93$, we have $\pi(1.93)\sin\theta = 1.249$. $\sin\theta = 0.2060$ and

$$\theta = \pm 11.9°$$

EVALUATE: Since the first dark fringes occur at ±15.0°, it is reasonable that at ≈12° the intensity is reduced to only 1/10 of its maximum central value.

35.61. **IDENTIFY:** There are two effects to be considered: first, the expansion of the rod, and second, the change in the rod's refractive index.

SET UP: $\lambda = \dfrac{\lambda_0}{n}$ and $\Delta n = n_0(2.50\times10^{-5}\ (\text{C°})^{-1})\Delta T$. $\Delta L = L_0(5.00\times10^{-6}\ (\text{C°})^{-1})\Delta T$.

EXECUTE: The extra length of rod replaces a little of the air so that the change in the number of wavelengths due to this is given by: $\Delta N_1 = \dfrac{2n_{\text{glass}}\Delta L}{\lambda_0} - \dfrac{2n_{\text{air}}\Delta L}{\lambda_0} = \dfrac{2(n_{\text{glass}}-1)L_0\alpha\Delta T}{\lambda_0}$ and

$$\Delta N_1 = \frac{2(1.48-1)(0.030\text{ m})(5.00\times10^{-6}/\text{C°})(5.00\text{ C°})}{5.89\times10^{-7}\text{ m}} = 1.22.$$

The change in the number of wavelengths due to the change in refractive index of the rod is:

$$\Delta N_2 = \frac{2\Delta n_{\text{glass}}L_0}{\lambda_0} = \frac{2(2.50\times10^{-5}/\text{C°})(5.00\text{ C°/min})(1.00\text{ min})(0.0300\text{ m})}{5.89\times10^{-7}\text{m}} = 12.73.$$

So, the total change in the number of wavelengths as the rod expands is $\Delta N = 12.73 + 1.22 = 14.0$ fringes/minute.

EVALUATE: Both effects increase the number of wavelengths along the length of the rod. Both ΔL and Δn_{glass} are very small and the two effects can be considered separately.

35.62. **IDENTIFY:** Apply Snell's law to the refraction at the two surfaces of the prism. S_1 and S_2 serve as coherent sources so the fringe spacing is $\Delta y = \dfrac{R\lambda}{d}$, where d is the distance between S_1 and S_2.

SET UP: For small angles, $\sin\theta \approx \theta$, with θ expressed in radians.

EXECUTE: **(a)** Since we can approximate the angles of incidence on the prism as being small, Snell's Law tells us that an incident angle of θ on the flat side of the prism enters the prism at an angle of θ/n, where n is the index of refraction of the prism. Similarly on leaving the prism, the in-going angle is $\theta/n - A$ from the normal, and the outgoing angle, relative to the prism, is $n(\lambda/n - A)$. So the beam leaving the prism is at an angle of $\theta' = n(\theta/n - A) + A$ from the optical axis. So $\theta - \theta' = (n-1)A$. At the plane of the source S_0, we can calculate the height of one image above the source: $\dfrac{d}{2} = \tan(\theta - \theta')a \approx (\theta - \theta')a = (n-1)Aa \Rightarrow d = 2aA(n-1)$.

(b) To find the spacing of fringes on a screen, we use

$$\Delta y = \frac{R\lambda}{d} = \frac{R\lambda}{2aA(n-1)} = \frac{(2.00\text{ m} + 0.200\text{ m})\,(5.00\times10^{-7}\text{ m})}{2(0.200\text{ m})\,(3.50\times10^{-3}\text{ rad})\,(1.50-1.00)} = 1.57\times10^{-3}\text{ m}.$$

EVALUATE: The fringe spacing is proportional to the wavelength of the light. The biprism serves as an alternative to two closely spaced narrow slits.