

ELECTROMAGNETIC WAVES

- 32.1. IDENTIFY:** Since the speed is constant, distance $x = ct$.
SET UP: The speed of light is $c = 3.00 \times 10^8$ m/s. $1 \text{ yr} = 3.156 \times 10^7$ s.
EXECUTE: (a) $t = \frac{x}{c} = \frac{3.84 \times 10^8 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \text{ s}$
 (b) $x = ct = (3.00 \times 10^8 \text{ m/s})(8.61 \text{ yr})(3.156 \times 10^7 \text{ s/yr}) = 8.15 \times 10^{16} \text{ m} = 8.15 \times 10^{13} \text{ km}$
EVALUATE: The speed of light is very great. The distance between stars is very large compared to terrestrial distances.
- 32.2. IDENTIFY:** Since the speed is constant the difference in distance is $c\Delta t$.
SET UP: The speed of electromagnetic waves in air is $c = 3.00 \times 10^8$ m/s.
EXECUTE: A total time difference of $0.60 \mu\text{s}$ corresponds to a difference in distance of
 $c\Delta t = (3.00 \times 10^8 \text{ m/s})(0.60 \times 10^{-6} \text{ s}) = 180 \text{ m}$.
EVALUATE: The time delay doesn't depend on the distance from the transmitter to the receiver, it just depends on the difference in the length of the two paths.
- 32.3. IDENTIFY:** Apply $c = f\lambda$.
SET UP: $c = 3.00 \times 10^8$ m/s
EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5000 \text{ m}} = 6.0 \times 10^4 \text{ Hz}$.
 (b) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.0 \text{ m}} = 6.0 \times 10^7 \text{ Hz}$.
 (c) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.0 \times 10^{-6} \text{ m}} = 6.0 \times 10^{13} \text{ Hz}$.
 (d) $f = \frac{c}{\lambda} = \frac{3.0 \times 10^8 \text{ m/s}}{5.0 \times 10^{-9} \text{ m}} = 6.0 \times 10^{16} \text{ Hz}$.
EVALUATE: f increases when λ decreases.
- 32.4. IDENTIFY:** $c = f\lambda$ and $k = \frac{2\pi}{\lambda}$.
SET UP: $c = 3.00 \times 10^8$ m/s.
EXECUTE: (a) $f = \frac{c}{\lambda}$. UVA: $7.50 \times 10^{14} \text{ Hz}$ to $9.38 \times 10^{14} \text{ Hz}$. UVB: $9.38 \times 10^{14} \text{ Hz}$ to $1.07 \times 10^{15} \text{ Hz}$.
 (b) $k = \frac{2\pi}{\lambda}$. UVA: $1.57 \times 10^7 \text{ rad/m}$ to $1.96 \times 10^7 \text{ rad/m}$. UVB: $1.96 \times 10^7 \text{ rad/m}$ to $2.24 \times 10^7 \text{ rad/m}$.
EVALUATE: Larger λ corresponds to smaller f and k .
- 32.5. IDENTIFY:** $c = f\lambda$. $E_{\text{max}} = cB_{\text{max}}$. $k = 2\pi/\lambda$. $\omega = 2\pi f$.
SET UP: Since the wave is traveling in empty space, its wave speed is $c = 3.00 \times 10^8$ m/s.
EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{432 \times 10^{-9} \text{ m}} = 6.94 \times 10^{14} \text{ Hz}$
 (b) $E_{\text{max}} = cB_{\text{max}} = (3.00 \times 10^8 \text{ m/s})(1.25 \times 10^{-6} \text{ T}) = 375 \text{ V/m}$

$$(c) k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{432 \times 10^{-9} \text{ m}} = 1.45 \times 10^7 \text{ rad/m} . \quad \omega = (2\pi \text{ rad})(6.94 \times 10^{14} \text{ Hz}) = 4.36 \times 10^{15} \text{ rad/s} .$$

$$E = E_{\max} \cos(kx - \omega t) = (375 \text{ V/m}) \cos([1.45 \times 10^7 \text{ rad/m}]x - [4.36 \times 10^{15} \text{ rad/s}]t)$$

$$B = B_{\max} \cos(kx - \omega t) = (1.25 \times 10^{-6} \text{ T}) \cos([1.45 \times 10^7 \text{ rad/m}]x - [4.36 \times 10^{15} \text{ rad/s}]t)$$

EVALUATE: The $\cos(kx - \omega t)$ factor is common to both the electric and magnetic field expressions, since these two fields are in phase.

32.6. IDENTIFY: $c = f\lambda$. $E_{\max} = cB_{\max}$. Apply Eqs.(32.17) and (32.19).

SET UP: The speed of the wave is $c = 3.00 \times 10^8 \text{ m/s}$.

$$\text{EXECUTE: (a) } f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{435 \times 10^{-9} \text{ m}} = 6.90 \times 10^{14} \text{ Hz}$$

$$(b) B_{\max} = \frac{E_{\max}}{c} = \frac{2.70 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 9.00 \times 10^{-12} \text{ T}$$

$$(c) k = \frac{2\pi}{\lambda} = 1.44 \times 10^7 \text{ rad/m} . \quad \omega = 2\pi f = 4.34 \times 10^{15} \text{ rad/s} . \text{ If } \vec{E}(z, t) = \hat{i}E_{\max} \cos(kz + \omega t) , \text{ then}$$

$$\vec{B}(z, t) = -\hat{j}B_{\max} \cos(kz + \omega t) , \text{ so that } \vec{E} \times \vec{B} \text{ will be in the } -\hat{k} \text{ direction.}$$

$$\vec{E}(z, t) = \hat{i}(2.70 \times 10^{-3} \text{ V/m}) \cos([1.44 \times 10^7 \text{ rad/s}]z + [4.34 \times 10^{15} \text{ rad/s}]t) \text{ and}$$

$$\vec{B}(z, t) = -\hat{j}(9.00 \times 10^{-12} \text{ T}) \cos([1.44 \times 10^7 \text{ rad/s}]z + [4.34 \times 10^{15} \text{ rad/s}]t) .$$

EVALUATE: The directions of \vec{E} and \vec{B} and of the propagation of the wave are all mutually perpendicular. The argument of the cosine is $kz + \omega t$ since the wave is traveling in the $-z$ -direction. Waves for visible light have very high frequencies.

32.7. IDENTIFY and SET UP: The equations are of the form of Eqs.(32.17), with x replaced by z . \vec{B} is along the y -axis; deduce the direction of \vec{E} .

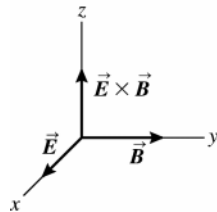
$$\text{EXECUTE: } \omega = 2\pi f = 2\pi(6.10 \times 10^{14} \text{ Hz}) = 3.83 \times 10^{15} \text{ rad/s}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{\omega}{c} = \frac{3.83 \times 10^{15} \text{ rad/s}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \times 10^7 \text{ rad/m}$$

$$B_{\max} = 5.80 \times 10^{-4} \text{ T}$$

$$E_{\max} = cB_{\max} = (3.00 \times 10^8 \text{ m/s})(5.80 \times 10^{-4} \text{ T}) = 1.74 \times 10^5 \text{ V/m}$$

\vec{B} is along the y -axis. $\vec{E} \times \vec{B}$ is in the direction of propagation (the $+z$ -direction). From this we can deduce the direction of \vec{E} , as shown in Figure 32.7.



\vec{E} is along the x -axis.

Figure 32.7

$$\vec{E} = E_{\max} \hat{i} \cos(kz - \omega t) = (1.74 \times 10^5 \text{ V/m}) \hat{i} \cos[(1.28 \times 10^7 \text{ rad/m}]z - (3.83 \times 10^{15} \text{ rad/s}]t]$$

$$\vec{B} = B_{\max} \hat{j} \cos(kz - \omega t) = (5.80 \times 10^{-4} \text{ T}) \hat{j} \cos[(1.28 \times 10^7 \text{ rad/m}]z - (3.83 \times 10^{15} \text{ rad/s}]t]$$

EVALUATE: \vec{E} and \vec{B} are perpendicular and oscillate in phase.

32.8. IDENTIFY: For an electromagnetic wave propagating in the negative x direction, $E = E_{\max} \cos(kx + \omega t)$. $\omega = 2\pi f$

$$\text{and } k = \frac{2\pi}{\lambda} . \quad T = \frac{1}{f} . \quad E_{\max} = cB_{\max} .$$

SET UP: The wave specified in the problem has a different phase, so $E = -E_{\max} \sin(kx + \omega t)$. $E_{\max} = 375 \text{ V/m}$,

$$k = 1.99 \times 10^7 \text{ rad/m} \text{ and } \omega = 5.97 \times 10^{15} \text{ rad/s} .$$

$$\text{EXECUTE: (a) } B_{\max} = \frac{E_{\max}}{c} = 1.25 \mu\text{T} .$$

(b) $f = \frac{\omega}{2\pi} = 9.50 \times 10^{14}$ Hz. $\lambda = \frac{2\pi}{k} = 3.16 \times 10^{-7}$ m = 316 nm. $T = \frac{1}{f} = 1.05 \times 10^{-15}$ s. This wavelength is too short to be visible.

(c) $c = f\lambda = (9.50 \times 10^{14} \text{ Hz})(3.16 \times 10^{-7} \text{ m}) = 3.00 \times 10^8$ m/s. This is what the wave speed should be for an electromagnetic wave propagating in vacuum.

EVALUATE: $c = f\lambda = \left(\frac{\omega}{2\pi}\right)\left(\frac{2\pi}{k}\right) = \frac{\omega}{k}$ is an alternative expression for the wave speed.

32.9. IDENTIFY and SET UP: Compare the $\vec{E}(y, t)$ given in the problem to the general form given by Eq.(32.17). Use the direction of propagation and of \vec{E} to find the direction of \vec{B} .

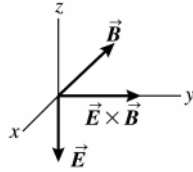
(a) EXECUTE: The equation for the electric field contains the factor $\sin(ky - \omega t)$ so the wave is traveling in the $+y$ -direction. The equation for $\vec{E}(y, t)$ is in terms of $\sin(ky - \omega t)$ rather than $\cos(ky - \omega t)$; the wave is shifted in phase by 90° relative to one with a $\cos(ky - \omega t)$ factor.

(b) $\vec{E}(y, t) = -(3.10 \times 10^5 \text{ V/m})\hat{k} \sin[ky - (2.65 \times 10^{12} \text{ rad/s})t]$

Comparing to Eq.(32.17) gives $\omega = 2.65 \times 10^{12}$ rad/s

$$\omega = 2\pi f = \frac{2\pi c}{\lambda} \text{ so } \lambda = \frac{2\pi c}{\omega} = \frac{2\pi(2.998 \times 10^8 \text{ m/s})}{(2.65 \times 10^{12} \text{ rad/s})} = 7.11 \times 10^{-4} \text{ m}$$

(c)



$\vec{E} \times \vec{B}$ must be in the $+y$ -direction (the direction in which the wave is traveling).
When \vec{E} is in the $-z$ -direction then \vec{B} must be in the $-x$ -direction, as shown in Figure 32.9.

Figure 32.9

$$k = \frac{2\pi}{\lambda} = \frac{\omega}{c} = \frac{2.65 \times 10^{12} \text{ rad/s}}{2.998 \times 10^8 \text{ m/s}} = 8.84 \times 10^3 \text{ rad/m}$$

$$E_{\text{max}} = 3.10 \times 10^5 \text{ V/m}$$

$$\text{Then } B_{\text{max}} = \frac{E_{\text{max}}}{c} = \frac{3.10 \times 10^5 \text{ V/m}}{2.998 \times 10^8 \text{ m/s}} = 1.03 \times 10^{-3} \text{ T}$$

Using Eq.(32.17) and the fact that \vec{B} is in the $-\hat{i}$ direction when \vec{E} is in the $-\hat{k}$ direction,

$$\vec{B} = -(1.03 \times 10^{-3} \text{ T})\hat{i} \sin[(8.84 \times 10^3 \text{ rad/m})y - (2.65 \times 10^{12} \text{ rad/s})t]$$

EVALUATE: \vec{E} and \vec{B} are perpendicular and oscillate in phase.

32.10. IDENTIFY: Apply Eqs.(32.17) and (32.19). $f = c/\lambda$ and $k = 2\pi/\lambda$.

SET UP: The wave in this problem has a different phase, so $B_y(z, t) = B_{\text{max}} \sin(kx + \omega t)$.

EXECUTE: (a) The phase of the wave is given by $kx + \omega t$, so the wave is traveling in the $-x$ direction.

$$(b) k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}, \quad f = \frac{kc}{2\pi} = \frac{(1.38 \times 10^4 \text{ rad/m})(3.0 \times 10^8 \text{ m/s})}{2\pi} = 6.59 \times 10^{11} \text{ Hz}$$

(c) Since the magnetic field is in the $+y$ -direction, and the wave is propagating in the $-x$ -direction, then the electric field is in the $+z$ -direction so that $\vec{E} \times \vec{B}$ will be in the $-x$ -direction.

$$\vec{E}(x, t) = +cB(x, t)\hat{k} = cB_{\text{max}} \sin(kx + \omega t)\hat{k}$$

$$\vec{E}(x, t) = (c(3.25 \times 10^{-9} \text{ T}))\sin((1.38 \times 10^4 \text{ rad/m})x + (4.14 \times 10^{12} \text{ rad/s})t)\hat{k}$$

$$\vec{E}(x, t) = +(2.48 \text{ V/m})\sin((1.38 \times 10^4 \text{ rad/m})x + (4.14 \times 10^{12} \text{ rad/s})t)\hat{k}$$

EVALUATE: \vec{E} and \vec{B} have the same phase and are in perpendicular directions.

32.11. IDENTIFY and SET UP: $c = f\lambda$ allows calculation of λ . $k = 2\pi/\lambda$ and $\omega = 2\pi f$. Eq.(32.18) relates the electric and magnetic field amplitudes.

$$\text{EXECUTE: (a) } c = f\lambda \text{ so } \lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{830 \times 10^3 \text{ Hz}} = 361 \text{ m}$$

$$(b) k = \frac{2\pi}{\lambda} = \frac{2\pi \text{ rad}}{361 \text{ m}} = 0.0174 \text{ rad/m}$$

$$(c) \omega = 2\pi f = (2\pi)(830 \times 10^3 \text{ Hz}) = 5.22 \times 10^6 \text{ rad/s}$$

$$(d) \text{Eq.(32.18): } E_{\max} = cB_{\max} = (2.998 \times 10^8 \text{ m/s})(4.82 \times 10^{-11} \text{ T}) = 0.0144 \text{ V/m}$$

EVALUATE: This wave has a very long wavelength; its frequency is in the AM radio broadcast band. The electric and magnetic fields in the wave are very weak.

32.12. IDENTIFY: $E_{\max} = cB_{\max}$.

SET UP: The magnetic field of the earth is about 10^{-4} T.

$$\text{EXECUTE: } B = \frac{E}{c} = \frac{3.85 \times 10^{-3} \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.28 \times 10^{-11} \text{ T.}$$

EVALUATE: The field is much smaller than the earth's field.

32.13. IDENTIFY and SET UP: $v = f\lambda$ relates frequency and wavelength to the speed of the wave. Use Eq.(32.22) to calculate n and K .

$$\text{EXECUTE: (a) } \lambda = \frac{v}{f} = \frac{2.17 \times 10^8 \text{ m/s}}{5.70 \times 10^{14} \text{ Hz}} = 3.81 \times 10^{-7} \text{ m}$$

$$(b) \lambda = \frac{c}{f} = \frac{2.998 \times 10^8 \text{ m/s}}{5.70 \times 10^{14} \text{ Hz}} = 5.26 \times 10^{-7} \text{ m}$$

$$(c) n = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{2.17 \times 10^8 \text{ m/s}} = 1.38$$

$$(d) n = \sqrt{KK_m} \approx \sqrt{K} \text{ so } K = n^2 = (1.38)^2 = 1.90$$

EVALUATE: In the material $v < c$ and f is the same, so λ is less in the material than in air. $v < c$ always, so n is always greater than unity.

32.14. IDENTIFY: Apply Eq.(32.21). $E_{\max} = cB_{\max}$. $v = f\lambda$. Apply Eq.(32.29) with $\mu = K_m\mu_0$ in place of μ_0 .

SET UP: $K = 3.64$. $K_m = 5.18$

$$\text{EXECUTE: (a) } v = \frac{c}{\sqrt{KK_m}} = \frac{(3.00 \times 10^8 \text{ m/s})}{\sqrt{(3.64)(5.18)}} = 6.91 \times 10^7 \text{ m/s.}$$

$$(b) \lambda = \frac{v}{f} = \frac{6.91 \times 10^7 \text{ m/s}}{65.0 \text{ Hz}} = 1.06 \times 10^6 \text{ m.}$$

$$(c) B_{\max} = \frac{E_{\max}}{v} = \frac{7.20 \times 10^{-3} \text{ V/m}}{6.91 \times 10^7 \text{ m/s}} = 1.04 \times 10^{-10} \text{ T.}$$

$$(d) I = \frac{E_{\max} B_{\max}}{2K_m\mu_0} = \frac{(7.20 \times 10^{-3} \text{ V/m})(1.04 \times 10^{-10} \text{ T})}{2(5.18)\mu_0} = 5.75 \times 10^{-8} \text{ W/m}^2.$$

EVALUATE: The wave travels slower in this material than in air.

32.15. IDENTIFY: $I = P/A$. $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$.

SET UP: The surface area of a sphere of radius r is $A = 4\pi r^2$. $\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$.

$$\text{EXECUTE: (a) } I = \frac{P}{A} = \frac{(0.05)(75 \text{ W})}{4\pi(3.0 \times 10^{-2} \text{ m})^2} = 330 \text{ W/m}^2.$$

$$(b) E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(330 \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 500 \text{ V/m. } B_{\max} = \frac{E_{\max}}{c} = 1.7 \times 10^{-6} \text{ T} = 1.7 \mu\text{T}.$$

EVALUATE: At the surface of the bulb the power radiated by the filament is spread over the surface of the bulb. Our calculation approximates the filament as a point source that radiates uniformly in all directions.

32.16. IDENTIFY and SET UP: The direction of propagation is given by $\vec{E} \times \vec{B}$.

$$\text{EXECUTE: (a) } \hat{S} = \hat{i} \times (-\hat{j}) = -\hat{k}.$$

$$(b) \hat{S} = \hat{j} \times \hat{i} = -\hat{k}.$$

$$(c) \hat{S} = (-\hat{k}) \times (-\hat{i}) = \hat{j}.$$

$$(d) \hat{S} = \hat{i} \times (-\hat{k}) = \hat{j}.$$

EVALUATE: In each case the directions of \vec{E} , \vec{B} and the direction of propagation are all mutually perpendicular.

- 32.17. IDENTIFY:** $E_{\max} = cB_{\max}$. $\vec{E} \times \vec{B}$ is in the direction of propagation.
SET UP: $c = 3.00 \times 10^8$ m/s. $E_{\max} = 4.00$ V/m.
EXECUTE: $B_{\max} = E_{\max} / c = 1.33 \times 10^{-8}$ T. For \vec{E} in the $+x$ -direction, $\vec{E} \times \vec{B}$ is in the $+z$ -direction when \vec{B} is in the $+y$ -direction.
EVALUATE: \vec{E} , \vec{B} and the direction of propagation are all mutually perpendicular.
- 32.18. IDENTIFY:** The intensity of the electromagnetic wave is given by Eq.(32.29): $I = \frac{1}{2} \epsilon_0 c E_{\max}^2 = \epsilon_0 c E_{\text{rms}}^2$. The total energy passing through a window of area A during a time t is IAt .
SET UP: $\epsilon_0 = 8.85 \times 10^{-12}$ F/m
EXECUTE: Energy $= \epsilon_0 c E_{\text{rms}}^2 At = (8.85 \times 10^{-12} \text{ F/m})(3.00 \times 10^8 \text{ m/s})(0.0200 \text{ V/m})^2 (0.500 \text{ m}^2)(30.0 \text{ s}) = 15.9 \text{ mJ}$
EVALUATE: The intensity is proportional to the square of the electric field amplitude.
- 32.19. IDENTIFY and SET UP:** Use Eq.(32.29) to calculate I , Eq.(32.18) to calculate B_{\max} , and use $I = P_{\text{av}} / 4\pi r^2$ to calculate P_{av} .
(a) EXECUTE: $I = \frac{1}{2} \epsilon_0 E_{\max}^2$; $E_{\max} = 0.090$ V/m, so $I = 1.1 \times 10^{-5}$ W/m²
(b) $E_{\max} = cB_{\max}$ so $B_{\max} = E_{\max} / c = 3.0 \times 10^{-10}$ T
(c) $P_{\text{av}} = I(4\pi r^2) = (1.075 \times 10^{-5} \text{ W/m}^2)(4\pi)(2.5 \times 10^3 \text{ m})^2 = 840$ W
(d) EVALUATE: The calculation in part (c) assumes that the transmitter emits uniformly in all directions.
- 32.20. IDENTIFY and SET UP:** $I = P_{\text{av}} / A$ and $I = \epsilon_0 c E_{\text{rms}}^2$.
EXECUTE: **(a)** The average power from the beam is $P_{\text{av}} = IA = (0.800 \text{ W/m}^2)(3.0 \times 10^{-4} \text{ m}^2) = 2.4 \times 10^{-4}$ W.
(b) $E_{\text{rms}} = \sqrt{\frac{I}{\epsilon_0 c}} = \sqrt{\frac{0.800 \text{ W/m}^2}{(8.85 \times 10^{-12} \text{ F/m})(3.00 \times 10^8 \text{ m/s})}} = 17.4$ V/m
EVALUATE: The laser emits radiation only in the direction of the beam.
- 32.21. IDENTIFY:** $I = P_{\text{av}} / A$
SET UP: At a distance r from the star, the radiation from the star is spread over a spherical surface of area $A = 4\pi r^2$.
EXECUTE: $P_{\text{av}} = I(4\pi r^2) = (5.0 \times 10^3 \text{ W/m}^2)(4\pi)(2.0 \times 10^{10} \text{ m})^2 = 2.5 \times 10^{25}$ J
EVALUATE: The intensity decreases with distance from the star as $1/r^2$.
- 32.22. IDENTIFY and SET UP:** $c = f\lambda$, $E_{\max} = cB_{\max}$ and $I = E_{\max} B_{\max} / 2\mu_0$
EXECUTE: **(a)** $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{0.354 \text{ m}} = 8.47 \times 10^8$ Hz.
(b) $B_{\max} = \frac{E_{\max}}{c} = \frac{0.0540 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 1.80 \times 10^{-10}$ T.
(c) $I = S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0} = \frac{(0.0540 \text{ V/m})(1.80 \times 10^{-10} \text{ T})}{2\mu_0} = 3.87 \times 10^{-6}$ W/m².
EVALUATE: Alternatively, $I = \frac{1}{2} \epsilon_0 c E_{\max}^2$.
- 32.23. IDENTIFY:** $P_{\text{av}} = IA$ and $I = \frac{1}{2} \epsilon_0 c E_{\max}^2$
SET UP: The surface area of a sphere is $A = 4\pi r^2$.
EXECUTE: $P_{\text{av}} = S_{\text{av}} A = \left(\frac{E_{\max}^2}{2c\mu_0} \right) (4\pi r^2)$. $E_{\max} = \sqrt{\frac{P_{\text{av}} c \mu_0}{2\pi r^2}} = \sqrt{\frac{(60.0 \text{ W})(3.00 \times 10^8 \text{ m/s}) \mu_0}{2\pi (5.00 \text{ m})^2}} = 12.0$ V/m.
 $B_{\max} = \frac{E_{\max}}{c} = \frac{12.0 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.00 \times 10^{-8}$ T.
EVALUATE: E_{\max} and B_{\max} are both inversely proportional to the distance from the source.
- 32.24. IDENTIFY:** The Poynting vector is $\vec{S} = \vec{E} \times \vec{B}$.
SET UP: The electric field is in the $+y$ -direction, and the magnetic field is in the $+z$ -direction.
 $\cos^2 \phi = \frac{1}{2}(1 + \cos 2\phi)$
EXECUTE: **(a)** $\hat{S} = \hat{E} \times \hat{B} = (-\hat{j}) \times \hat{k} = -\hat{i}$. The Poynting vector is in the $-x$ -direction, which is the direction of propagation of the wave.

(b) $S(x, t) = \frac{E(x, t)B(x, t)}{\mu_0} = \frac{E_{\max} B_{\max}}{\mu_0} \cos^2(kx + \omega t) = \frac{E_{\max} B_{\max}}{2\mu_0} (1 + \cos(2(\omega t + kx)))$. But over one period, the cosine function averages to zero, so we have $S_{\text{av}} = \frac{E_{\max} B_{\max}}{2\mu_0}$. This is Eq.(32.29).

EVALUATE: We can also show that these two results also apply to the wave represented by Eq.(32.17).

32.25. IDENTIFY: Use the radiation pressure to find the intensity, and then $P_{\text{av}} = I(4\pi r^2)$.

SET UP: For a perfectly absorbing surface, $p_{\text{rad}} = \frac{I}{c}$

EXECUTE: $p_{\text{rad}} = I/c$ so $I = cp_{\text{rad}} = 2.70 \times 10^3 \text{ W/m}^2$. Then

$$P_{\text{av}} = I(4\pi r^2) = (2.70 \times 10^3 \text{ W/m}^2)(4\pi)(5.0 \text{ m})^2 = 8.5 \times 10^5 \text{ W}.$$

EVALUATE: Even though the source is very intense the radiation pressure 5.0 m from the surface is very small.

32.26. IDENTIFY: The intensity and the energy density of an electromagnetic wave depends on the amplitudes of the electric and magnetic fields.

SET UP: Intensity is $I = P_{\text{av}}/A$, and the average power is $P_{\text{av}} = 2I/c$, where $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. The energy density is $u = \epsilon_0 E^2$.

EXECUTE: (a) $I = P_{\text{av}}/A = \frac{316,000 \text{ W}}{2\pi(5000 \text{ m})^2} = 0.00201 \text{ W/m}^2$. $P_{\text{av}} = 2I/c = \frac{2(0.00201 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} = 1.34 \times 10^{-11} \text{ Pa}$

(b) $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$ gives

$$E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.00201 \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 1.23 \text{ N/C}$$

$$B_{\max} = E_{\max}/c = (1.23 \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = 4.10 \times 10^{-9} \text{ T}$$

(c) $u = \epsilon_0 E^2$, so $u_{\text{av}} = \epsilon_0 (E_{\text{av}})^2$ and $E_{\text{av}} = \frac{E_{\max}}{\sqrt{2}}$, so

$$u_{\text{av}} = \frac{\epsilon_0 E_{\max}^2}{2} = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(1.23 \text{ N/C})^2}{2} = 6.69 \times 10^{-12} \text{ J/m}^3$$

(d) As was shown in Section 32.4, the energy density is the same for the electric and magnetic fields, so each one has 50% of the energy density.

EVALUATE: Compared to most laboratory fields, the electric and magnetic fields in ordinary radiowaves are extremely weak and carry very little energy.

32.27. IDENTIFY and SET UP: Use Eqs.(32.30) and (32.31).

EXECUTE: (a) By Eq.(32.30) the average momentum density is $\frac{dp}{dV} = \frac{S_{\text{av}}}{c^2} = \frac{I}{c^2}$

$$\frac{dp}{dV} = \frac{0.78 \times 10^3 \text{ W/m}^2}{(2.998 \times 10^8 \text{ m/s})^2} = 8.7 \times 10^{-15} \text{ kg/m}^2 \cdot \text{s}$$

(b) By Eq.(32.31) the average momentum flow rate per unit area is $\frac{S_{\text{av}}}{c} = \frac{I}{c} = \frac{0.78 \times 10^3 \text{ W/m}^2}{2.998 \times 10^8 \text{ m/s}} = 2.6 \times 10^{-6} \text{ Pa}$

EVALUATE: The radiation pressure that the sunlight would exert on an absorbing or reflecting surface is very small.

32.28. IDENTIFY: Apply Eqs.(32.32) and (32.33). The average momentum density is given by Eq.(32.30), with S replaced by $S_{\text{av}} = I$.

SET UP: $1 \text{ atm} = 1.013 \times 10^5 \text{ Pa}$

EXECUTE: (a) Absorbed light: $p_{\text{rad}} = \frac{I}{c} = \frac{2500 \text{ W/m}^2}{3.0 \times 10^8 \text{ m/s}} = 8.33 \times 10^{-6} \text{ Pa}$. Then

$$p_{\text{rad}} = \frac{8.33 \times 10^{-6} \text{ Pa}}{1.013 \times 10^5 \text{ Pa/atm}} = 8.23 \times 10^{-11} \text{ atm}.$$

(b) Reflecting light: $p_{\text{rad}} = \frac{2I}{c} = \frac{2(2500 \text{ W/m}^2)}{3.0 \times 10^8 \text{ m/s}} = 1.67 \times 10^{-5} \text{ Pa}$. Then

$$p_{\text{rad}} = \frac{1.67 \times 10^{-5} \text{ Pa}}{1.013 \times 10^5 \text{ Pa/atm}} = 1.65 \times 10^{-10} \text{ atm}.$$

(c) The momentum density is $\frac{dp}{dV} = \frac{S_{\text{av}}}{c^2} = \frac{2500 \text{ W/m}^2}{(3.0 \times 10^8 \text{ m/s})^2} = 2.78 \times 10^{-14} \text{ kg/m}^2 \cdot \text{s}$.

EVALUATE: The factor of 2 in p_{rad} for the reflecting surface arises because the momentum vector totally reverses direction upon reflection. Thus the *change* in momentum is twice the original momentum.

32.29. IDENTIFY: Apply Eq.(32.4) and (32.9).

SET UP: Eq.(32.26) is $S = \epsilon_0 c E^2$.

EXECUTE: $S = \frac{\epsilon_0}{\sqrt{\epsilon_0 \mu_0}} E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E^2 = \sqrt{\frac{\epsilon_0}{\mu_0}} E c \frac{E}{c} = c \sqrt{\frac{\epsilon_0}{\mu_0}} E B = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \sqrt{\frac{\epsilon_0}{\mu_0}} E B = \frac{E B}{\mu_0} = \frac{E^2}{\mu_0 c} = \epsilon_0 c E^2$

EVALUATE: We can also write $S = \epsilon_0 c (cB)^2 = \epsilon_0 c^3 B^2$. S can be written solely in terms of E or solely in terms of B .

32.30. IDENTIFY: The electric field at the nodes is zero, so there is no force on a point charge placed at a node.

SET UP: The location of the nodes is given by Eq.(32.36), where x is the distance from one of the planes.
 $\lambda = c/f$.

EXECUTE: $\Delta x_{\text{nodes}} = \frac{\lambda}{2} = \frac{c}{2f} = \frac{3.00 \times 10^8 \text{ m/s}}{2(7.50 \times 10^8 \text{ Hz})} = 0.200 \text{ m} = 20.0 \text{ cm}$. There must be nodes at the planes, which

are 80.0 cm apart, and there are two nodes between the planes, each 20.0 cm from a plane. It is at 20 cm, 40 cm, and 60 cm from one plane that a point charge will remain at rest, since the electric fields there are zero.

EVALUATE: The magnetic field amplitude at these points isn't zero, but the magnetic field doesn't exert a force on a stationary charge.

32.31. IDENTIFY and SET UP: Apply Eqs.(32.36) and (32.37).

EXECUTE: (a) By Eq.(32.37) we see that the nodal planes of the \vec{B} field are a distance $\lambda/2$ apart, so $\lambda/2 = 3.55 \text{ mm}$ and $\lambda = 7.10 \text{ mm}$.

(b) By Eq.(32.36) we see that the nodal planes of the \vec{E} field are also a distance $\lambda/2 = 3.55 \text{ mm}$ apart.

(c) $v = f\lambda = (2.20 \times 10^{10} \text{ Hz})(7.10 \times 10^{-3} \text{ m}) = 1.56 \times 10^8 \text{ m/s}$.

EVALUATE: The spacing between the nodes of \vec{E} is the same as the spacing between the nodes of \vec{B} . Note that $v < c$, as it must.

32.32. IDENTIFY: The nodal planes of \vec{E} and \vec{B} are located by Eqs.(32.26) and (32.27).

SET UP: $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{75.0 \times 10^6 \text{ Hz}} = 4.00 \text{ m}$

EXECUTE: (a) $\Delta x = \frac{\lambda}{2} = 2.00 \text{ m}$.

(b) The distance between the electric and magnetic nodal planes is one-quarter of a wavelength, so is

$$\frac{\lambda}{4} = \frac{\Delta x}{2} = \frac{2.00 \text{ m}}{2} = 1.00 \text{ m}.$$

EVALUATE: The nodal planes of \vec{B} are separated by a distance $\lambda/2$ and are midway between the nodal planes of \vec{E} .

32.33. (a) IDENTIFY and SET UP: The distance between adjacent nodal planes of \vec{B} is $\lambda/2$. There is an antinodal plane of \vec{B} midway between any two adjacent nodal planes, so the distance between a nodal plane and an adjacent antinodal plane is $\lambda/4$. Use $v = f\lambda$ to calculate λ .

EXECUTE: $\lambda = \frac{v}{f} = \frac{2.10 \times 10^8 \text{ m/s}}{1.20 \times 10^{10} \text{ Hz}} = 0.0175 \text{ m}$

$$\frac{\lambda}{4} = \frac{0.0175 \text{ m}}{4} = 4.38 \times 10^{-3} \text{ m} = 4.38 \text{ mm}$$

(b) IDENTIFY and SET UP: The nodal planes of \vec{E} are at $x = 0, \lambda/2, \lambda, 3\lambda/2, \dots$, so the antinodal planes of \vec{E} are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$. The nodal planes of \vec{B} are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$, so the antinodal planes of \vec{B} are at $\lambda/2, \lambda, 3\lambda/2, \dots$.

EXECUTE: The distance between adjacent antinodal planes of \vec{E} and antinodal planes of \vec{B} is therefore $\lambda/4 = 4.38 \text{ mm}$.

(c) From Eqs.(32.36) and (32.37) the distance between adjacent nodal planes of \vec{E} and \vec{B} is $\lambda/4 = 4.38 \text{ mm}$.

EVALUATE: The nodes of \vec{E} coincide with the antinodes of \vec{B} , and conversely. The nodes of \vec{B} and the nodes of \vec{E} are equally spaced.

32.34. IDENTIFY: Evaluate the derivatives of the expressions for $E_y(x, t)$ and $B_z(x, t)$ that are given in Eqs.(32.34) and (32.35).

SET UP: $\frac{\partial}{\partial x} \sin kx = k \cos kx$, $\frac{\partial}{\partial t} \sin \omega t = \omega \cos \omega t$. $\frac{\partial}{\partial x} \cos kx = -k \sin kx$, $\frac{\partial}{\partial t} \cos \omega t = -\omega \sin \omega t$.

EXECUTE: (a) $\frac{\partial^2 E_y(x, t)}{\partial x^2} = \frac{\partial^2}{\partial x^2} (-2E_{\max} \sin kx \sin \omega t) = \frac{\partial}{\partial x} (-2kE_{\max} \cos kx \sin \omega t)$ and

$$\frac{\partial^2 E_y(x, t)}{\partial x^2} = 2k^2 E_{\max} \sin kx \sin \omega t = \frac{\omega^2}{c^2} 2E_{\max} \sin kx \sin \omega t = \epsilon_0 \mu_0 \frac{\partial^2 E_y(x, t)}{\partial t^2}.$$

Similarly: $\frac{\partial^2 B_z(x, t)}{\partial x^2} = \frac{\partial^2}{\partial x^2} (-2B_{\max} \cos kx \cos \omega t) = \frac{\partial}{\partial x} (+2kB_{\max} \sin kx \cos \omega t)$ and

$$\frac{\partial^2 B_z(x, t)}{\partial x^2} = 2k^2 B_{\max} \cos kx \cos \omega t = \frac{\omega^2}{c^2} 2B_{\max} \cos kx \cos \omega t = \epsilon_0 \mu_0 \frac{\partial^2 B_z(x, t)}{\partial t^2}.$$

(b) $\frac{\partial E_y(x, t)}{\partial x} = \frac{\partial}{\partial x} (-2E_{\max} \sin kx \sin \omega t) = -2kE_{\max} \cos kx \sin \omega t$.

$$\frac{\partial E_y(x, t)}{\partial x} = -\frac{\omega}{c} 2E_{\max} \cos kx \sin \omega t = -\omega 2 \frac{E_{\max}}{c} \cos kx \sin \omega t = -\omega 2B_{\max} \cos kx \sin \omega t.$$

$$\frac{\partial E_y(x, t)}{\partial x} = +\frac{\partial}{\partial t} (2B_{\max} \cos kx \cos \omega t) = -\frac{\partial B_z(x, t)}{\partial t}.$$

Similarly: $-\frac{\partial B_z(x, t)}{\partial x} = \frac{\partial}{\partial x} (+2B_{\max} \cos kx \cos \omega t) = -2kB_{\max} \sin kx \cos \omega t$.

$$-\frac{\partial B_z(x, t)}{\partial x} = -\frac{\omega}{c} 2B_{\max} \sin kx \cos \omega t = -\frac{\omega}{c^2} 2cB_{\max} \sin kx \cos \omega t.$$

$$-\frac{\partial B_z(x, t)}{\partial x} = -\epsilon_0 \mu_0 \omega 2E_{\max} \sin kx \cos \omega t = \epsilon_0 \mu_0 \frac{\partial}{\partial t} (-2E_{\max} \sin kx \sin \omega t) = \epsilon_0 \mu_0 \frac{\partial E_y(x, t)}{\partial t}.$$

EVALUATE: The standing waves are linear superpositions of two traveling waves of the same k and ω .

32.35. IDENTIFY: The nodal and antinodal planes are each spaced one-half wavelength apart.

SET UP: $2\frac{1}{2}$ wavelengths fit in the oven, so $(2\frac{1}{2})\lambda = L$, and the frequency of these waves obeys the equation $f\lambda = c$.

EXECUTE: (a) Since $(2\frac{1}{2})\lambda = L$, we have $L = (5/2)(12.2 \text{ cm}) = 30.5 \text{ cm}$.

(b) Solving for the frequency gives $f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(0.122 \text{ m}) = 2.46 \times 10^9 \text{ Hz}$.

(c) $L = 35.5 \text{ cm}$ in this case. $(2\frac{1}{2})\lambda = L$, so $\lambda = 2L/5 = 2(35.5 \text{ cm})/5 = 14.2 \text{ cm}$.

$$f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(0.142 \text{ m}) = 2.11 \times 10^9 \text{ Hz}$$

EVALUATE: Since microwaves have a reasonably large wavelength, microwave ovens can have a convenient size for household kitchens. Ovens using radiowaves would need to be far too large, while ovens using visible light would have to be microscopic.

32.36. IDENTIFY: Evaluate the partial derivatives of the expressions for $E_y(x, t)$ and $B_z(x, t)$.

SET UP: $\frac{\partial}{\partial x} \sin(kx - \omega t) = k \cos(kx - \omega t)$, $\frac{\partial}{\partial t} \sin(kx - \omega t) = -\omega \cos(kx - \omega t)$. $\frac{\partial}{\partial x} \cos(kx - \omega t) = -k \sin(kx - \omega t)$,

$$\frac{\partial}{\partial t} \cos(kx - \omega t) = \omega \sin(kx - \omega t)$$

EXECUTE: Assume $\vec{E} = E_{\max} \hat{j} \sin(kx - \omega t)$ and $\vec{B} = B_{\max} \hat{k} \sin(kx - \omega t + \phi)$, with $-\pi < \phi < \pi$. Eq. (32.12) is

$$\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}. \text{ This gives } kE_{\max} \cos(kx - \omega t) = +\omega B_{\max} \cos(kx - \omega t + \phi), \text{ so } \phi = 0, \text{ and } kE_{\max} = \omega B_{\max}, \text{ so}$$

$$E_{\max} = \frac{\omega}{k} B_{\max} = \frac{2\pi f}{2\pi/\omega} B_{\max} = f\omega B_{\max} = cB_{\max}. \text{ Similarly for Eq.(32.14), } -\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t} \text{ gives}$$

$$-kB_{\max} \cos(kx - \omega t + \phi) = -\epsilon_0 \mu_0 \omega E_{\max} \cos(kx - \omega t), \text{ so } \phi = 0 \text{ and } kB_{\max} = \epsilon_0 \mu_0 \omega E_{\max}, \text{ so}$$

$$B_{\max} = \frac{\epsilon_0 \mu_0 \omega}{k} E_{\max} = \frac{2\pi f}{c^2 2\pi/\omega} E_{\max} = \frac{f\omega}{c^2} E_{\max} = \frac{1}{c} E_{\max}.$$

EVALUATE: The \vec{E} and \vec{B} fields must oscillate in phase.

32.37. IDENTIFY and SET UP: Take partial derivatives of Eqs.(32.12) and (32.14), as specified in the problem.

EXECUTE: Eq.(32.12): $\frac{\partial E_y}{\partial x} = -\frac{\partial B_z}{\partial t}$

Taking $\frac{\partial}{\partial t}$ of both sides of this equation gives $\frac{\partial^2 E_y}{\partial x \partial t} = -\frac{\partial^2 B_z}{\partial t^2}$. Eq.(32.14) says $-\frac{\partial B_z}{\partial x} = \epsilon_0 \mu_0 \frac{\partial E_y}{\partial t}$. Taking $\frac{\partial}{\partial x}$ of both sides of this equation gives $-\frac{\partial^2 B_z}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 E_y}{\partial t \partial x}$, so $\frac{\partial^2 E_y}{\partial t \partial x} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 B_z}{\partial x^2}$. But $\frac{\partial^2 E_y}{\partial x \partial t} = \frac{\partial^2 E_y}{\partial t \partial x}$ (The order in which the partial derivatives are taken doesn't change the result.) So $-\frac{\partial^2 B_z}{\partial t^2} = -\frac{1}{\epsilon_0 \mu_0} \frac{\partial^2 B_z}{\partial x^2}$ and $\frac{\partial^2 B_z}{\partial x^2} = \epsilon_0 \mu_0 \frac{\partial^2 B_z}{\partial t^2}$, as was to be shown.

EVALUATE: Both fields, electric and magnetic, satisfy the wave equation, Eq.(32.10). We have also shown that both fields propagate with the same speed $v = 1/\sqrt{\epsilon_0 \mu_0}$.

32.38. IDENTIFY: The average energy density in the electric field is $u_{E,av} = \frac{1}{2} \epsilon_0 (E^2)_{av}$ and the average energy density in the magnetic field is $u_{B,av} = \frac{1}{2\mu_0} (B^2)_{av}$.

SET UP: $(\cos^2(kx - \omega t))_{av} = \frac{1}{2}$.

EXECUTE: $E_y(x, t) = E_{max} \cos(kx - \omega t)$. $u_E = \frac{1}{2} \epsilon_0 E^2 = \frac{1}{2} \epsilon_0 E_{max}^2 \cos^2(kx - \omega t)$ and $u_{E,av} = \frac{1}{4} \epsilon_0 E_{max}^2$.

$B_z(x, t) = B_{max} \cos(kx - \omega t)$, so $u_B = \frac{1}{2\mu_0} B_z^2 = \frac{1}{2\mu_0} B_{max}^2 \cos^2(kx - \omega t)$ and $u_{B,av} = \frac{1}{4\mu_0} B_{max}^2$. $E_{max} = cB_{max}$, so

$u_{E,av} = \frac{1}{4} \epsilon_0 c^2 B_{max}^2$. $c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$, so $u_{E,av} = \frac{1}{2\mu_0} B_{max}^2$, which equals $u_{B,av}$.

EVALUATE: Our result allows us to write $u_{av} = 2u_{E,av} = \frac{1}{2} \epsilon_0 E_{max}^2$ and $u_{av} = 2u_{B,av} = \frac{1}{2\mu_0} B_{max}^2$.

32.39. IDENTIFY: The intensity of an electromagnetic wave depends on the amplitude of the electric and magnetic fields. Such a wave exerts a force because it carries energy.

SET UP: The intensity of the wave is $I = P_{av}/A = \frac{1}{2} \epsilon_0 c E_{max}^2$, and the force is $F = P_{av} A$ where $P_{av} = I/c$.

EXECUTE: (a) $I = P_{av}/A = (25,000 \text{ W})/[4\pi(575 \text{ m})^2] = 0.00602 \text{ W/m}^2$

(b) $I = \frac{1}{2} \epsilon_0 c E_{max}^2$, so $E_{max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(0.00602 \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 2.13 \text{ N/C}$.

$B_{max} = E_{max}/c = (2.13 \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = 7.10 \times 10^{-9} \text{ T}$

(c) $F = P_{av} A = (I/c)A = (0.00602 \text{ W/m}^2)(0.150 \text{ m})(0.400 \text{ m})/(3.00 \times 10^8 \text{ m/s}) = 1.20 \times 10^{-12} \text{ N}$

EVALUATE: The fields are very weak compared to ordinary laboratory fields, and the force is hardly worth worrying about!

32.40. IDENTIFY: $c = f\lambda$. $E_{max} = cB_{max}$. $I = \frac{1}{2} \epsilon_0 c E_{max}^2$. For a totally absorbing surface the radiation pressure is $\frac{I}{c}$.

SET UP: The wave speed in air is $c = 3.00 \times 10^8 \text{ m/s}$.

EXECUTE: (a) $f = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m/s}}{3.84 \times 10^{-2} \text{ m}} = 7.81 \times 10^9 \text{ Hz}$

(b) $B_{max} = \frac{E_{max}}{c} = \frac{1.35 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 4.50 \times 10^{-9} \text{ T}$

(c) $I = \frac{1}{2} \epsilon_0 c E_{max}^2 = \frac{1}{2} (8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})(1.35 \text{ V/m})^2 = 2.42 \times 10^{-3} \text{ W/m}^2$

(d) $F = (\text{pressure})A = \frac{IA}{c} = \frac{(2.42 \times 10^{-3} \text{ W/m}^2)(0.240 \text{ m}^2)}{3.00 \times 10^8 \text{ m/s}} = 1.94 \times 10^{-12} \text{ N}$

EVALUATE: The intensity depends only on the amplitudes of the electric and magnetic fields and is independent of the wavelength of the light.

32.41. (a) IDENTIFY and SET UP: Calculate I and then use Eq.(32.29) to calculate E_{\max} and Eq.(32.18) to calculate B_{\max} .

EXECUTE: The intensity is power per unit area: $I = \frac{P}{A} = \frac{3.20 \times 10^{-3} \text{ W}}{\pi(1.25 \times 10^{-3} \text{ m})^2} = 652 \text{ W/m}^2$.

$$I = \frac{E_{\max}^2}{2\mu_0 c}, \text{ so } E_{\max} = \sqrt{2\mu_0 c I}$$

$$E_{\max} = \sqrt{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(2.998 \times 10^8 \text{ m/s})(652 \text{ W/m}^2)} = 701 \text{ V/m}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{701 \text{ V/m}}{2.998 \times 10^8 \text{ m/s}} = 2.34 \times 10^{-6} \text{ T}$$

EVALUATE: The magnetic field amplitude is quite small.

(b) IDENTIFY and SET UP: Eqs.(24.11) and (30.10) give the energy density in terms of the electric and magnetic field values at any time. For sinusoidal fields average over E^2 and B^2 to get the average energy densities.

EXECUTE: The energy density in the electric field is $u_E = \frac{1}{2}\epsilon_0 E^2$. $E = E_{\max} \cos(kx - \omega t)$ and the average value of $\cos^2(kx - \omega t)$ is $\frac{1}{2}$. The average energy density in the electric field then is

$u_{E,av} = \frac{1}{4}\epsilon_0 E_{\max}^2 = \frac{1}{4}(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(701 \text{ V/m})^2 = 1.09 \times 10^{-6} \text{ J/m}^3$. The energy density in the magnetic field

is $u_B = \frac{B^2}{2\mu_0}$. The average value is $u_{B,av} = \frac{B_{\max}^2}{4\mu_0} = \frac{(2.34 \times 10^{-6} \text{ T})^2}{4(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})} = 1.09 \times 10^{-6} \text{ J/m}^3$.

EVALUATE: Our result agrees with the statement in Section 32.4 that the average energy density for the electric field is the same as the average energy density for the magnetic field.

(c) IDENTIFY and SET UP: The total energy in this length of beam is the total energy density

$u_{av} = u_{E,av} + u_{B,av} = 2.18 \times 10^{-6} \text{ J/m}^3$ times the volume of this part of the beam.

EXECUTE: $U = u_{av} LA = (2.18 \times 10^{-6} \text{ J/m}^3)(1.00 \text{ m})\pi(1.25 \times 10^{-3} \text{ m})^2 = 1.07 \times 10^{-11} \text{ J}$.

EVALUATE: This quantity can also be calculated as the power output times the time it takes the light to travel $L = 1.00 \text{ m}$: $U = P\left(\frac{L}{c}\right) = (3.20 \times 10^{-3} \text{ W})\left(\frac{1.00 \text{ m}}{2.998 \times 10^8 \text{ m/s}}\right) = 1.07 \times 10^{-11} \text{ J}$, which checks.

32.42. IDENTIFY: Use the gaussian surface specified in the hint.

SET UP: The wave is in free space, so in Gauss's law for the electric field, $Q_{\text{encl}} = 0$ and $\oint \vec{E} \cdot d\vec{A} = 0$. Gauss's law for the magnetic field says $\oint \vec{B} \cdot d\vec{A} = 0$

EXECUTE: Use a gaussian surface such that the front surface is ahead of the wave front (no electric or magnetic fields) and the back face is behind the wave front, as shown in Figure 32.42. $\oint \vec{E} \cdot d\vec{A} = E_x A = \frac{Q_{\text{encl}}}{\epsilon_0} = 0$, so $E_x = 0$.

$\oint \vec{B} \cdot d\vec{A} = B_x A = 0$ and $B_x = 0$.

EVALUATE: The wave must be transverse, since there are no components of the electric or magnetic field in the direction of propagation.

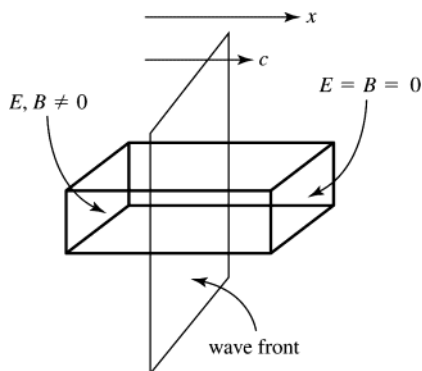


Figure 32.42

32.43. IDENTIFY: $I = P_{\text{av}} / A$. For an absorbing surface, the radiation pressure is $p_{\text{rad}} = \frac{I}{c}$

SET UP: Assume the electromagnetic waves are formed at the center of the sun, so at a distance r from the center of the sun $I = P_{\text{av}} / (4\pi r^2)$.

EXECUTE: (a) At the sun's surface: $I = \frac{P_{\text{av}}}{4\pi R^2} = \frac{3.9 \times 10^{26} \text{ W}}{4\pi(6.96 \times 10^8 \text{ m})^2} = 6.4 \times 10^7 \text{ W/m}^2$ and

$$p_{\text{rad}} = \frac{I}{c} = \frac{6.4 \times 10^7 \text{ W/m}^2}{3.00 \times 10^8 \text{ m/s}} = 0.21 \text{ Pa}.$$

(b) Halfway out from the sun's center, the intensity is 4 times more intense, and so is the radiation pressure: $I = 2.6 \times 10^8 \text{ W/m}^2$ and $p_{\text{rad}} = 0.85 \text{ Pa}$. At the top of the earth's atmosphere, the measured sunlight intensity is $1400 \text{ W/m}^2 = 5 \times 10^{-6} \text{ Pa}$, which is about 100,000 times less than the values above.

EVALUATE: (b) The gas pressure at the sun's surface is 50,000 times greater than the radiation pressure, and halfway out of the sun the gas pressure is believed to be about 6×10^{13} times greater than the radiation pressure. Therefore it is reasonable to ignore radiation pressure when modeling the sun's interior structure.

32.44. IDENTIFY: $I = \frac{P}{A}$. $I = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$.

SET UP: $3.00 \times 10^8 \text{ m/s}$

EXECUTE: $I = \frac{P}{A} = \frac{2.80 \times 10^3 \text{ W}}{36.0 \text{ m}^2} = 77.8 \text{ W/m}^2$.

$$E_{\text{max}} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(77.8 \text{ W/m}^2)}{(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 242 \text{ N/C}.$$

EVALUATE: This value of E_{max} is similar to the electric field amplitude in ordinary light sources.

32.45. IDENTIFY: The same intensity light falls on both reflectors, but the force on the reflecting surface will be twice as great as the force on the absorbing surface. Therefore there will be a net torque about the rotation axis.

SET UP: For a totally absorbing surface, $F = P_{\text{av}} A = (I/c)A$, while for a totally reflecting surface the force will be twice as great. The intensity of the wave is $I = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$. Once we have the torque, we can use the rotational form of Newton's second law, $\tau_{\text{net}} = I\alpha$, to find the angular acceleration.

EXECUTE: The force on the absorbing reflector is $F_{\text{Abs}} = p_{\text{av}} A = (I/c)A = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2 \frac{A}{c} = \frac{1}{2} \epsilon_0 A E_{\text{max}}^2$

For a totally reflecting surface, the force will be twice as great, which is $\epsilon_0 c E_{\text{max}}^2$. The net torque is therefore

$$\tau_{\text{net}} = F_{\text{Refl}}(L/2) - F_{\text{Abs}}(L/2) = \epsilon_0 A E_{\text{max}}^2 L/4$$

Newton's 2nd law for rotation gives $\tau_{\text{net}} = I\alpha$. $\epsilon_0 A E_{\text{max}}^2 L/4 = 2m(L/2)^2 \alpha$

$$\text{Solving for } \alpha \text{ gives } \alpha = \epsilon_0 A E_{\text{max}}^2 / (2mL) = \frac{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(0.0150 \text{ m})^2 (1.25 \text{ N/C})^2}{(2)(0.00400 \text{ kg})(1.00 \text{ m})} = 3.89 \times 10^{-13} \text{ rad/s}^2$$

EVALUATE: This is an extremely small angular acceleration. To achieve a larger value, we would have to greatly increase the intensity of the light wave or decrease the mass of the reflectors.

32.46. IDENTIFY: For light of intensity I_{abs} incident on a totally absorbing surface, the radiation pressure is

$$p_{\text{rad,abs}} = \frac{I_{\text{abs}}}{c}. \text{ For light of intensity } I_{\text{refl}} \text{ incident on a totally reflecting surface, } p_{\text{rad,refl}} = \frac{2I_{\text{refl}}}{c}.$$

SET UP: The total radiation pressure is $p_{\text{rad}} = p_{\text{rad,abs}} + p_{\text{rad,refl}}$. $I_{\text{abs}} = wI$ and $I_{\text{refl}} = (1-w)I$

EXECUTE: (a) $p_{\text{rad}} = p_{\text{rad,abs}} + p_{\text{rad,refl}} = \frac{I_{\text{abs}}}{c} + \frac{2I_{\text{refl}}}{c} = \frac{wI}{c} + \frac{2(1-w)I}{c} = \frac{(2-w)I}{c}$.

(b) (i) For totally absorbing $w = 1$ so $p_{\text{rad}} = \frac{I}{c}$. (ii) For totally reflecting $w = 0$ so $p_{\text{rad}} = \frac{2I}{c}$. These are just equations

32.32 and 32.33.

(c) For $w = 0.9$ and $I = 1.40 \times 10^2 \text{ W/m}^2$, $p_{\text{rad}} = \frac{(2-0.9)(1.40 \times 10^2 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} = 5.13 \times 10^{-6} \text{ Pa}$. For $w = 0.1$ and

$$I = 1.40 \times 10^3 \text{ W/m}^2, p_{\text{rad}} = \frac{(2-0.1)(1.40 \times 10^2 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} = 8.87 \times 10^{-6} \text{ Pa}.$$

EVALUATE: The radiation pressure is greater when a larger fraction is reflected.

32.47. IDENTIFY and SET UP: In the wire the electric field is related to the current density by Eq.(25.7). Use Ampere's law to calculate \vec{B} . The Poynting vector is given by Eq.(32.28) and the equation that follows it relates the energy flow through a surface to \vec{S} .

EXECUTE: (a) The direction of \vec{E} is parallel to the axis of the cylinder, in the direction of the current. From Eq.(25.7), $E = \rho J = \rho I / \pi a^2$. (E is uniform across the cross section of the conductor.)

(b) A cross-sectional view of the conductor is given in Figure 32.47a; take the current to be coming out of the page.

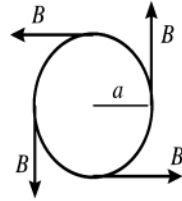


Figure 32.47a

Apply Ampere's law to a circle of radius a .

$$\oint \vec{B} \cdot d\vec{l} = B(2\pi a)$$

$$I_{\text{encl}} = I$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{encl}} \text{ gives } B(2\pi a) = \mu_0 I \text{ and } B = \frac{\mu_0 I}{2\pi a}$$

The direction of \vec{B} is counterclockwise around the circle.

(c) The directions of \vec{E} and \vec{B} are shown in Figure 32.47b.

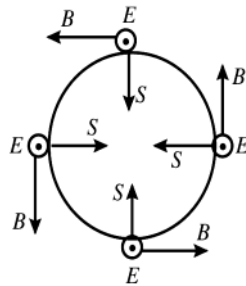


Figure 32.47b

The direction of $\vec{S} = \frac{1}{\mu_0} \vec{E} \times \vec{B}$

is radially inward.

$$S = \frac{1}{\mu_0} EB = \frac{1}{\mu_0} \left(\frac{\rho I}{\pi a^2} \right) \left(\frac{\mu_0 I}{2\pi a} \right)$$

$$S = \frac{\rho I^2}{2\pi^2 a^3}$$

(d) **EVALUATE:** Since S is constant over the surface of the conductor, the rate of energy flow P is given by S times the surface of a length l of the conductor: $P = SA = S(2\pi al) = \frac{\rho I^2}{2\pi^2 a^3} (2\pi al) = \frac{\rho I^2 l}{\pi a^2}$. But $R = \frac{\rho l}{\pi a^2}$, so the result from the Poynting vector is $P = RI^2$. This agrees with $P_R = I^2 R$, the rate at which electrical energy is being dissipated by the resistance of the wire. Since \vec{S} is radially inward at the surface of the wire and has magnitude equal to the rate at which electrical energy is being dissipated in the wire, this energy can be thought of as entering through the cylindrical sides of the conductor.

32.48. IDENTIFY: The intensity of the wave, not the electric field strength, obeys an inverse-square distance law.

SET UP: The intensity is inversely proportional to the distance from the source, and it depends on the amplitude of the electric field by $I = S_{\text{av}} = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$.

EXECUTE: Since $I = \frac{1}{2} \epsilon_0 c E_{\text{max}}^2$, $E_{\text{max}} \propto \sqrt{I}$. A point at 20.0 cm (0.200 m) from the source is 50 times closer to the source than a point that is 10.0 m from it. Since $I \propto 1/r^2$ and $(0.200 \text{ m}) / (10.0 \text{ m}) = 1/50$, we have $I_{0.20} = 50^2 I_{10}$. Since $E_{\text{max}} \propto \sqrt{I}$, we have $E_{0.20} = 50 E_{10} = (50)(1.50 \text{ N/C}) = 75.0 \text{ N/C}$.

EVALUATE: While the intensity increases by a factor of $50^2 = 2500$, the amplitude of the wave only increases by a factor of 50. Recall that the intensity of *any* wave is proportional to the *square* of its amplitude.

32.49. IDENTIFY and SET UP: The magnitude of the induced emf is given by Faraday's law: $|\mathcal{E}| = \left| \frac{d\Phi_B}{dt} \right|$. To calculate

$d\Phi_B / dt$ we need dB / dt at the antenna. Use the total power output to calculate I and then combine Eq.(32.29) and (32.18) to calculate B_{max} . The time dependence of B is given by Eq.(32.17).

EXECUTE: $\Phi_B = B\pi R^2$, where $R = 0.0900 \text{ m}$ is the radius of the loop. (This assumes that the magnetic field is uniform across the loop, an excellent approximation.) $|\mathcal{E}| = \pi R^2 \left| \frac{dB}{dt} \right|$

$$B = B_{\text{max}} \cos(kx - \omega t) \text{ so } \left| \frac{dB}{dt} \right| = B_{\text{max}} \omega \sin(kx - \omega t)$$

The maximum value of $\left| \frac{dB}{dt} \right|$ is $B_{\text{max}} \omega$, so $|\mathcal{E}|_{\text{max}} = \pi R^2 B_{\text{max}} \omega$.

$$R = 0.0900 \text{ m}, \omega = 2\pi f = 2\pi(95.0 \times 10^6 \text{ Hz}) = 5.97 \times 10^8 \text{ rad/s}$$

Calculate the intensity I at this distance from the source, and from that the magnetic field amplitude B_{\max} :

$$I = \frac{P}{4\pi r^2} = \frac{55.0 \times 10^3 \text{ W}}{4\pi(2.50 \times 10^3 \text{ m})^2} = 7.00 \times 10^{-4} \text{ W/m}^2. \quad I = \frac{E_{\max}^2}{2\mu_0 c} = \frac{(cB_{\max})^2}{2\mu_0 c} = \frac{c}{2\mu_0} B_{\max}^2$$

$$\text{Thus } B_{\max} = \sqrt{\frac{2\mu_0 I}{c}} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(7.00 \times 10^{-4} \text{ W/m}^2)}{2.998 \times 10^8 \text{ m/s}}} = 2.42 \times 10^{-9} \text{ T. Then}$$

$$|\mathcal{E}|_{\max} = \pi R^2 B_{\max} \omega = \pi(0.0900 \text{ m})^2 (2.42 \times 10^{-9} \text{ T})(5.97 \times 10^8 \text{ rad/s}) = 0.0368 \text{ V.}$$

EVALUATE: An induced emf of this magnitude is easily detected.

32.50. IDENTIFY: The nodal planes are one-half wavelength apart.

SET UP: The nodal planes of B are at $x = \lambda/4, 3\lambda/4, 5\lambda/4, \dots$, which are $\lambda/2$ apart.

EXECUTE: (a) The wavelength is $\lambda = c/f = (3.000 \times 10^8 \text{ m/s})/(110.0 \times 10^6 \text{ Hz}) = 2.727 \text{ m}$. So the nodal planes are at $(2.727 \text{ m})/2 = 1.364 \text{ m}$ apart.

(b) For the nodal planes of E , we have $\lambda_n = 2L/n$, so $L = n\lambda/2 = (8)(2.727 \text{ m})/2 = 10.91 \text{ m}$

EVALUATE: Because radiowaves have long wavelengths, the distances involved are easily measurable using ordinary metersticks.

32.51. IDENTIFY and SET UP: Find the force on you due to the momentum carried off by the light. Express this force in terms of the radiated power of the flashlight. Use this force to calculate your acceleration and use a constant acceleration equation to find the time.

(a) **EXECUTE:** $p_{\text{rad}} = I/c$ and $F = p_{\text{rad}} A$ gives $F = IA/c = P_{\text{av}}/c$

$$a_x = F/m = P_{\text{av}}/(mc) = (200 \text{ W})/[(150 \text{ kg})(3.00 \times 10^8 \text{ m/s})] = 4.44 \times 10^{-9} \text{ m/s}^2$$

$$\text{Then } x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2 \text{ gives } t = \sqrt{2(x - x_0)/a_x} = \sqrt{2(16.0 \text{ m})/(4.44 \times 10^{-9} \text{ m/s}^2)} = 8.49 \times 10^4 \text{ s} = 23.6 \text{ h}$$

EVALUATE: The radiation force is very small. In the calculation we have ignored any other forces on you.

(b) You could throw the flashlight in the direction away from the ship. By conservation of linear momentum you would move toward the ship with the same magnitude of momentum as you gave the flashlight.

32.52. IDENTIFY: $P_{\text{av}} = IA$ and $I = \frac{1}{2}\epsilon_0 c E_{\max}^2$. $E_{\max} = cB_{\max}$

SET UP: The power carried by the current i is $P = Vi$.

$$\text{EXECUTE: } I = \frac{P_{\text{av}}}{A} = \frac{1}{2}\epsilon_0 c E_{\max}^2 \text{ and } E_{\max} = \sqrt{\frac{2P_{\text{av}}}{A\epsilon_0 c}} = \sqrt{\frac{2Vi}{A\epsilon_0 c}} = \sqrt{\frac{2(5.00 \times 10^5 \text{ V})(1000 \text{ A})}{(100 \text{ m}^2)\epsilon_0(3.00 \times 10^8 \text{ m/s})}} = 6.14 \times 10^4 \text{ V/m.}$$

$$B_{\max} = \frac{E_{\max}}{c} = \frac{6.14 \times 10^4 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 2.05 \times 10^{-4} \text{ T.}$$

EVALUATE: $I = Vi/A = \frac{(5.00 \times 10^5 \text{ V})(1000 \text{ A})}{100 \text{ m}^2} = 5.00 \times 10^6 \text{ W/m}^2$. This is a very intense beam spread over a

large area.

32.53. IDENTIFY: The orbiting satellite obeys Newton's second law of motion. The intensity of the electromagnetic waves it transmits obeys the inverse-square distance law, and the intensity of the waves depends on the amplitude of the electric and magnetic fields.

SET UP: Newton's second law applied to the satellite gives $mv^2/R = GmM/r^2$, where M is the mass of the Earth and m is the mass of the satellite. The intensity I of the wave is $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\max}^2$, and by definition, $I = P_{\text{av}}/A$.

EXECUTE: (a) The period of the orbit is 12 hr. Applying Newton's 2nd law to the satellite gives $mv^2/R = GmM/r^2$,

which gives $\frac{m(2\pi r/T)^2}{r} = \frac{GmM}{r^2}$. Solving for r , we get

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{1/3} = \left[\frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.97 \times 10^{24} \text{ kg})(12 \times 3600 \text{ s})^2}{4\pi^2}\right]^{1/3} = 2.66 \times 10^7 \text{ m}$$

The height above the surface is $h = 2.66 \times 10^7 \text{ m} - 6.38 \times 10^6 \text{ m} = 2.02 \times 10^7 \text{ m}$. The satellite only radiates its energy to the lower hemisphere, so the area is 1/2 that of a sphere. Thus, from the definition of intensity, the intensity at the ground is

$$I = P_{\text{av}}/A = P_{\text{av}}/(2\pi h^2) = (25.0 \text{ W})/[2\pi(2.02 \times 10^7 \text{ m})^2] = 9.75 \times 10^{-15} \text{ W/m}^2$$

(b) $I = S_{\text{av}} = \frac{1}{2}\epsilon_0 c E_{\max}^2$, so $E_{\max} = \sqrt{\frac{2I}{\epsilon_0 c}} = \sqrt{\frac{2(9.75 \times 10^{-15} \text{ W/m}^2)}{(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(3.00 \times 10^8 \text{ m/s})}} = 2.71 \times 10^{-6} \text{ N/C}$

$$B_{\max} = E_{\max}/c = (2.71 \times 10^{-6} \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = 9.03 \times 10^{-15} \text{ T}$$

$$t = d/c = (2.02 \times 10^7 \text{ m})/(3.00 \times 10^8 \text{ m/s}) = 0.0673 \text{ s}$$

$$(c) P_{\text{av}} = I/c = (9.75 \times 10^{-15} \text{ W/m}^2)/(3.00 \times 10^8 \text{ m/s}) = 3.25 \times 10^{-23} \text{ Pa}$$

$$(d) \lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(1575.42 \times 10^6 \text{ Hz}) = 0.190 \text{ m}$$

EVALUATE: The fields and pressures due to these waves are very small compared to typical laboratory quantities.

32.54. IDENTIFY: For a totally reflective surface the radiation pressure is $\frac{2I}{c}$. Find the force due to this pressure and

express the force in terms of the power output P of the sun. The gravitational force of the sun is $F_g = G \frac{mM_{\text{sun}}}{r^2}$.

SET UP: The mass of the sun is $M_{\text{sun}} = 1.99 \times 10^{30} \text{ kg}$. $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

EXECUTE: (a) The sail should be reflective, to produce the maximum radiation pressure.

(b) $F_{\text{rad}} = \left(\frac{2I}{c}\right)A$, where A is the area of the sail. $I = \frac{P}{4\pi r^2}$, where r is the distance of the sail from the sun.

$$F_{\text{rad}} = \left(\frac{2A}{c}\right)\left(\frac{P}{4\pi r^2}\right) = \frac{PA}{2\pi r^2 c}. F_{\text{rad}} = F_g \text{ so } \frac{PA}{2\pi r^2 c} = G \frac{mM_{\text{sun}}}{r^2}.$$

$$A = \frac{2\pi c G m M_{\text{sun}}}{P} = \frac{2\pi(3.00 \times 10^8 \text{ m/s})(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(10,000 \text{ kg})(1.99 \times 10^{30} \text{ kg})}{3.9 \times 10^{26} \text{ W}}.$$

$$A = 6.42 \times 10^6 \text{ m}^2 = 6.42 \text{ km}^2.$$

(c) Both the gravitational force and the radiation pressure are inversely proportional to the square of the distance from the sun, so this distance divides out when we set $F_{\text{rad}} = F_g$.

EVALUATE: A very large sail is needed, just to overcome the gravitational pull of the sun.

32.55. IDENTIFY and SET UP: The gravitational force is given by Eq.(12.2). Express the mass of the particle in terms of its density and volume. The radiation pressure is given by Eq.(32.32); relate the power output L of the sun to the intensity at a distance r . The radiation force is the pressure times the cross sectional area of the particle.

EXECUTE: (a) The gravitational force is $F_g = G \frac{mM}{r^2}$. The mass of the dust particle is $m = \rho V = \rho \frac{4}{3} \pi R^3$. Thus

$$F_g = \frac{4\rho G \pi M R^3}{3r^2}.$$

(b) For a totally absorbing surface $p_{\text{rad}} = \frac{I}{c}$. If L is the power output of the sun, the intensity of the solar radiation

a distance r from the sun is $I = \frac{L}{4\pi r^2}$. Thus $p_{\text{rad}} = \frac{L}{4\pi c r^2}$. The force F_{rad} that corresponds to p_{rad} is in the

direction of propagation of the radiation, so $F_{\text{rad}} = p_{\text{rad}} A_{\perp}$, where $A_{\perp} = \pi R^2$ is the component of area of the particle

perpendicular to the radiation direction. Thus $F_{\text{rad}} = \left(\frac{L}{4\pi c r^2}\right)(\pi R^2) = \frac{LR^2}{4cr^2}$.

(c) $F_g = F_{\text{rad}}$

$$\frac{4\rho G \pi M R^3}{3r^2} = \frac{LR^2}{4cr^2}$$

$$\left(\frac{4\rho G \pi M}{3}\right)R = \frac{L}{4c} \text{ and } R = \frac{3L}{16c\rho G \pi M}$$

$$R = \frac{3(3.9 \times 10^{26} \text{ W})}{16(2.998 \times 10^8 \text{ m/s})(3000 \text{ kg/m}^3)(6.673 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)\pi(1.99 \times 10^{30} \text{ kg})}$$

$$R = 1.9 \times 10^{-7} \text{ m} = 0.19 \text{ } \mu\text{m}.$$

EVALUATE: The gravitational force and the radiation force both have a r^{-2} dependence on the distance from the sun, so this distance divides out in the calculation of R .

(d) $\frac{F_{\text{rad}}}{F_g} = \left(\frac{LR^2}{4cr^2}\right)\left(\frac{3r^2}{4\rho G \pi M R^3}\right) = \frac{3L}{16c\rho G \pi M R}$. F_{rad} is proportional to R^2 and F_g is proportional to R^3 , so this

ratio is proportional to $1/R$. If $R < 0.20 \text{ } \mu\text{m}$ then $F_{\text{rad}} > F_g$ and the radiation force will drive the particles out of the solar system.

32.56. IDENTIFY: The electron has acceleration $a = \frac{v^2}{R}$.

SET UP: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$. An electron has $|q| = e = 1.60 \times 10^{-19} \text{ C}$.

EXECUTE: For the electron in the classical hydrogen atom, its acceleration is

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(13.6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{(9.11 \times 10^{-31} \text{ kg})(5.29 \times 10^{-11} \text{ m})} = 9.03 \times 10^{22} \text{ m/s}^2. \text{ Then using the formula for the rate of energy}$$

emission given in Problem 32.57:

$$\frac{dE}{dt} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = \frac{(1.60 \times 10^{-19} \text{ C})^2 (9.03 \times 10^{22} \text{ m/s}^2)^2}{6\pi\epsilon_0 (3.00 \times 10^8 \text{ m/s})^3} = 4.64 \times 10^{-8} \text{ J/s} = 2.89 \times 10^{11} \text{ eV/s}. \text{ This large value of } \frac{dE}{dt}$$

would mean that the electron would almost immediately lose all its energy!

EVALUATE: The classical physics result in Problem 32.57 must not apply to electrons in atoms.

32.57. IDENTIFY: The orbiting particle has acceleration $a = \frac{v^2}{R}$.

SET UP: $K = \frac{1}{2}mv^2$. An electron has mass $m_e = 9.11 \times 10^{-31} \text{ kg}$ and a proton has mass $m_p = 1.67 \times 10^{-27} \text{ kg}$.

EXECUTE: (a)
$$\left[\frac{q^2 a^2}{6\pi\epsilon_0 c^3} \right] = \frac{C^2 (\text{m/s}^2)^2}{(C^2/\text{N} \cdot \text{m}^2) (\text{m/s})^3} = \frac{\text{N} \cdot \text{m}}{\text{s}} = \frac{\text{J}}{\text{s}} = \text{W} = \left[\frac{dE}{dt} \right].$$

(b) For a proton moving in a circle, the acceleration is

$$a = \frac{v^2}{R} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mR} = \frac{2(6.00 \times 10^6 \text{ eV})(1.6 \times 10^{-19} \text{ J/eV})}{(1.67 \times 10^{-27} \text{ kg})(0.75 \text{ m})} = 1.53 \times 10^{15} \text{ m/s}^2. \text{ The rate at which it emits energy because of}$$

its acceleration is
$$\frac{dE}{dt} = \frac{q^2 a^2}{6\pi\epsilon_0 c^3} = \frac{(1.6 \times 10^{-19} \text{ C})^2 (1.53 \times 10^{15} \text{ m/s}^2)^2}{6\pi\epsilon_0 (3.0 \times 10^8 \text{ m/s})^3} = 1.33 \times 10^{-23} \text{ J/s} = 8.32 \times 10^{-5} \text{ eV/s}.$$

Therefore, the fraction of its energy that it radiates every second is
$$\frac{(dE/dt)(1 \text{ s})}{E} = \frac{8.32 \times 10^{-5} \text{ eV}}{6.00 \times 10^6 \text{ eV}} = 1.39 \times 10^{-11}.$$

(c) Carry out the same calculations as in part (b), but now for an electron at the same speed and radius. That means the electron's acceleration is the same as the proton, and thus so is the rate at which it emits energy, since they also have the same charge. However, the electron's initial energy differs from the proton's by the ratio of their masses:

$$E_e = E_p \frac{m_e}{m_p} = (6.00 \times 10^6 \text{ eV}) \frac{(9.11 \times 10^{-31} \text{ kg})}{(1.67 \times 10^{-27} \text{ kg})} = 3273 \text{ eV}. \text{ Therefore, the fraction of its energy that it radiates every}$$

second is
$$\frac{(dE/dt)(1 \text{ s})}{E} = \frac{8.32 \times 10^{-5} \text{ eV}}{3273 \text{ eV}} = 2.54 \times 10^{-8}.$$

EVALUATE: The proton has speed
$$v = \sqrt{\frac{2E}{m_p}} = \sqrt{\frac{2(6.00 \times 10^6 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{1.67 \times 10^{-27} \text{ kg}}} = 3.39 \times 10^7 \text{ m/s}. \text{ The electron}$$

has the same speed and kinetic energy 3.27 keV. The particles in the accelerator radiate at a much smaller rate than the electron in Problem 32.56 does, because in the accelerator the orbit radius is very much larger than in the atom, so the acceleration is much less.

32.58. IDENTIFY and SET UP: Follow the steps specified in the problem.

EXECUTE: (a) $E_y(x, t) = E_{\max} e^{-k_c x} \sin(k_c x - \omega t)$.

$$\frac{\partial E_y}{\partial x} = E_{\max} (-k_c) e^{-k_c x} \sin(k_c x - \omega t) + E_{\max} (+k_c) e^{-k_c x} \cos(k_c x - \omega t)$$

$$\frac{\partial^2 E_y}{\partial x^2} = E_{\max} (+k_c^2) e^{-k_c x} \sin(k_c x - \omega t) + E_{\max} (-k_c^2) e^{-k_c x} \cos(k_c x - \omega t) \\ + E_{\max} (-k_c^2) e^{-k_c x} \cos(k_c x - \omega t) + E_{\max} (-k_c^2) e^{-k_c x} \sin(k_c x - \omega t).$$

$$\frac{\partial^2 E_y}{\partial x^2} = -2E_{\max} k_c^2 e^{-k_c x} \cos(k_c x - \omega t). \quad \frac{\partial E_y}{\partial t} = E_{\max} e^{-k_c x} \omega \cos(k_c x - \omega t).$$

Setting $\frac{\partial^2 E_y}{\partial x^2} = \frac{\mu \partial E_y}{\rho \partial t}$ gives $2E_{\max} k_c^2 e^{-k_c x} \cos(k_c x - \omega t) = E_{\max} e^{-k_c x} \omega \cos(k_c x - \omega t)$. This will only be true if

$$\frac{2k_c^2}{\omega} = \frac{\mu}{\rho}, \text{ or } k_c = \sqrt{\frac{\omega \mu}{2\rho}}.$$

(b) The energy in the wave is dissipated by the $i^2 R$ heating of the conductor.

(c) $E_y = \frac{E_{y0}}{e} \Rightarrow k_c x = 1, x = \frac{1}{k_c} = \sqrt{\frac{2\rho}{\omega \mu}} = \sqrt{\frac{2(1.72 \times 10^{-8} \text{ } \Omega \cdot \text{m})}{2\pi(1.0 \times 10^6 \text{ Hz})\mu_0}} = 6.60 \times 10^{-5} \text{ m}.$

EVALUATE: The lower the frequency of the waves, the greater is the distance they can penetrate into a conductor. A dielectric (insulator) has a much larger resistivity and these waves can penetrate a greater distance in these materials.

